

OPTIMAL-TUNING OF PID CONTROLLER GAINS USING GENETIC ALGORITHMS

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ABSTRACT

This paper presents a method of optimum parameter tuning of a PID controller to be used in driving an inertial load by a dc motor thorough a gearbox. Specifically, the method uses genetic algorithms to determine the optimum controller parameters by minimizing the sum of the integral of the squared error and the squared controller output deviated from its steady state value. The paper suggests the use of Ziegler-Nichols settings to form the intervals for the controller parameters in which the population to be formed. The results obtained from the genetic algorithms are compared with the ones from Ziegler-Nichols in both figures and tabular form. Comparatively better results are obtained in the genetic algorithm case.

Key Words : PID, Controller tuning, Optimal-tuning, Genetic algorithms, DC motor control

GENETİK ALGORİTMA KULLANILARAK PID KONTROLÇÜ KAZANÇLARININ OPTİMUM AYARLANMASI

ÖZET

Makale, bir dişli kutusuyla tahrik edilen ataletli bir kütlelin kontrolünde kullanılacak PID kazançlarının optimum ayarlanmasına ait bir metodu sunar. Özellikle yöntem, hata kareleriyle kontrolcü çıkışının kararlı haldeki değeri farkının karesi toplamının integralini mizimize ederek optimum kontrolcü parametrelerini tanımlamak için genetik algoritmaları kullanır. Makale, popülasyonun oluşturulacağı kontrolcü parametre aralığını oluşturmak için, Ziegler-Nichols yönteminin önerdiği değerlerin kullanımını önerir. Genetik algoritmalarından elde edilen sonuçlar ile Ziegler-Nichols sonuçları hem tablolar hem de grafikler şeklinde karşılaştırılmıştır. Genetik algoritmalar kullanıldığında daha iyi sonuçlar elde edilmiştir.

Anahtar Kelimeler :

1. INTRODUCTION

At the beginning of the new millennium, PID controllers continue to be the main components of the industrial control applications. New methods were developed through the improvement of the shortcomings of PID controllers in the last century. However, because of their simple and useful nature, they still present powerful solutions to the industrial control processes (Åström et al., 2001).

It is certainly one of the most important topics in the design of PID controllers is the adjustment of controller parameters according to a certain tuning criteria. It is beyond the scope of this paper to cite all the investigations in the literature on the tuning of PID controllers, however some of these methods were included here to give an idea to the reader. For example, the Ziegler-Nichols formulation is a classical tuning method which found a wide range of applications in the controller design process.

However, computing the gains does not always give best results because the tuning criteria presume a one-fourth reduction in the first two-peaks (Goodwin et al., 2001). Hence, the industrial controllers designed with this method should be tuned further before a use (Wu and Huang, 1997). Another approach in the design of controllers is the frequency response techniques for tuning the controller parameters for optimum gain and phase margins (Åström and Hägglund, 1984; Ho et al., 1999; Wang and Shao, 2000). The magnitude optimum multiple integration tuning method (Vrancic et al., 1999; Vrancic et al., 2001) was also used to get a non-oscillatory closed-loop response. In the last decade, artificial intelligence applications were also introduced to the PID controllers. Particularly, the fuzzy (He et al., 1993; Misir et al., 1996; Blanchett et al., 2000), the neuro-fuzzy (Chen and Linkens, 1998), the fuzzy-genetic (Bandyopadhyay et al., 2001), and the neuro-genetic (Lima and Ruano, 2000) approaches have been utilized.

In this study, a simple effective method is developed to optimally tune PID controller gains using both Ziegler-Nichols (ZN) and Genetic Algorithms (GA). Since the determination of the lower and upper bounds for the PID parameter populations is difficult without making a large number of experiments

(Grefenstette, 1986) in GA, ZN rules can easily be used to obtain the vicinity of the proportional, integral and derivative gains. Therefore, the ranges are initially defined from ZN rules. Then, these parameters are tuned optimally with respect to the objective function stated as “sum of the integral of the squared error and the squared controller output deviated from its steady state value” (Wilton, 1999). The main advantage of using genetic algorithms in the tuning of controllers rather than using ZN rules is the adaptability to any constraints desired by a designer. Other advantages include—but not limited to— their ease to be used in linear or nonlinear, and continuous or discrete systems, or any combination of them. According to the results presented in this study, better results were obtained by using GA compared with the method using ZN in terms of both step response and controller output.

2. MATHEMATICAL MODEL

The physical system to be controlled is composed of an inertial load driven by a DC motor through a gear box with a reduction ratio of $N: 1$. The schematic illustration of the system is presented in Figure 1. The system is to integrate a PID controller in a closed loop with a unity feedback.

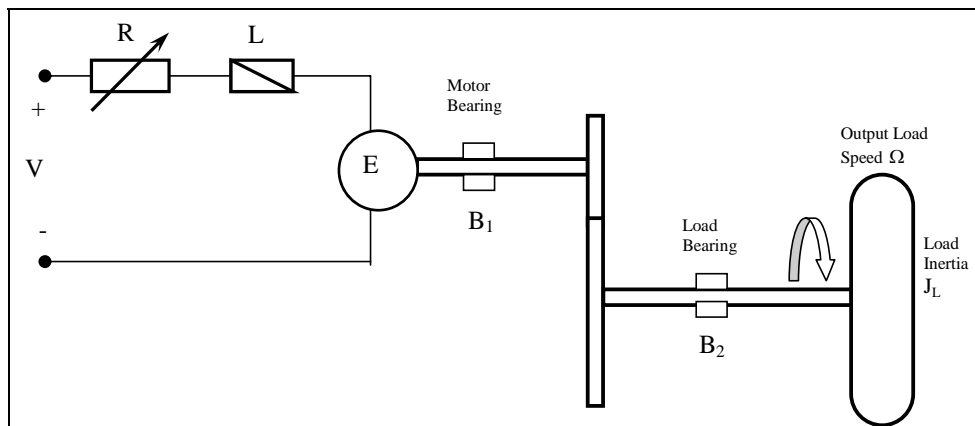


Figure1. A DC motor driven load

The armature equation can be written using Kirhhhoff's voltage law as,

$$v(t) = E + L \frac{di}{dt} + Ri, \quad (1)$$

Where v is the armature voltage, E is the back electromotive force (e.m.f.) generated by the armature winding, L is the inductance of the

armature winding, R is the resistance of the armature, and i is the armature current. The back e.m.f. is directly related to the motor speed, and hence

$$E = K_e \omega \quad (2)$$

Where K_e is the coefficient of e.m.f., and is taken as 0.2 V.s/rad and ω the motor angular speed (rad/sec).

Defining the load angular speed as Ω (rad/s), the speed ratio can be written as

$$N = \frac{\omega}{\Omega} \quad (3)$$

The armature current which is proportional to the motor torque T_M can be written as

$$i = \frac{1}{K_t} T_M \quad (4)$$

Where K_t for the motor under investigation is 0.2 N.m/A. The motor torque T_M can be written from the balance between the power required by the load and that supplied from the motor as

$$T_M = \left(\frac{J_L + J_M N^2 \eta}{N \eta} \right) \frac{d\Omega}{dt} + \left(\frac{B_2 + B_1 N^2 \eta}{N \eta} \right) \Omega \quad (5)$$

Combining the equations (4) and (5), and plugging the current expression and its time derivative into the equation (1) result in a differential equation describing the input-output relation of the above system. Using the parameter values listed below (Fraser and Milne, 1994), one can obtain the differential equation describing the system as

$$\frac{d^2 \Omega}{dt^2} + 5.382 \frac{d\Omega}{dt} + 4.029 \Omega = 0.1059 V \quad (6)$$

and the transfer function as

$$G(s) = \frac{\Omega(s)}{V(s)} = \frac{0.1059}{s^2 + 5.382s + 4.029} \quad (7)$$

Where the parameter values are shown in Table 1.

Table 1. Descriptions and Values for the Parameters Used in the Formulations

Description	Symbols	Units	Numerical Values
Motor armature inertia	J_m	kg.m ²	0.1
Armature resistance	R	ohms	0.5
Armature inductance	L	H	0.1
Motor bearing friction	B_1	N.m.s/rad	0.05
Load bearing friction	B_2	N.m.s/rad	200
Load inertia	J_L	kg.m ²	800
Gear ratio	K_d	-	100/1
Transmission efficiency	η	-	0.90

3. DESIGN OF A PID CONTROLLER

A schematic of a block diagram representing a simple single input single output system with a negative unity feedback is shown in Figure 2.

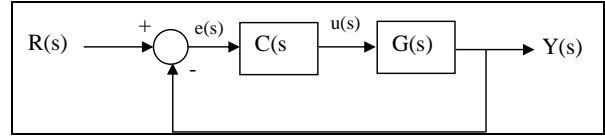


Figure 2. A block diagram representing a SISO system

The PID controller transfer function relating the error $e(s)$ and the controller output $u(s)$ is given as,

$$C(s) = K_P \left(1 + \frac{1}{T_i s} + T_d s \right) \quad (8)$$

Where, T_i and T_d are the reset and the derivative times, respectively. The first term in the Eqn. 8 represents proportionality effect on the error signal, whereas the second and the third represent the integral and the derivative effects. The reader should refer to the references representing the details of the controller parameter selection such as (Fraser and Milne, 1994; Goodwin et al., 2001).

One of the most effective ways for describing the selection of controller gains is to use open loop step response as suggested by Ziegler-Nichols. However, this method intends to have a ratio of 4:1 for the first and second peaks in the closed loop response curve, which results in an oscillatory response (Goodwin et al., 2001). Since the optimum settings for this controller are desired, these parameter settings provide designers an excellent starting point for the parameter tuning.

As a first step, the unit step response of the system (Eqn. 7) is obtained (see Figure 3). The parameters required for this study can easily be found using this response as suggested by Ziegler-Nichols. The dead time and the time constant are respectively found to be $T_1 = 0.1$ s and $T_2 = 1.9$ s, and the controller parameters found from the Ziegler-Nichols rules were presented in Table 2.

Table 2. Controller parameters defined from two methods

	K_P	T_d (sec)	T_i (sec)
Ziegler-Nichols	824.43	0.05	0.2
Genetic Algorithm	397.3	0.13	0.58

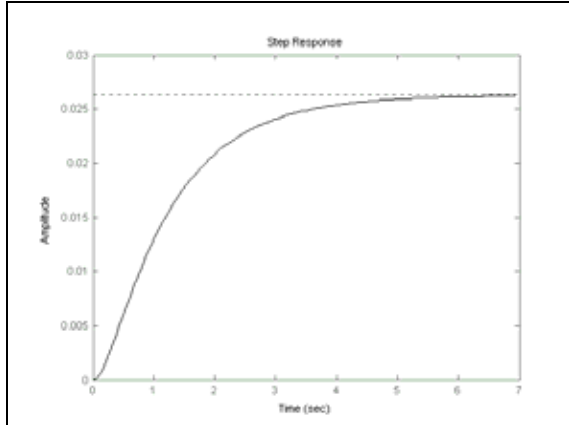


Figure 3. Open loop step response of the system

4. OPTIMUM TUNING

In the design of PID controllers, Ziegler-Nichols settings give an oscillatory response of the system; hence the parameters found by this method cannot be implemented in the designs as computed. However, it can be used with the genetic algorithm to form possible interval for the controller parameter set. Since determining precise intervals for these gains have not been developed yet, it is wise to use these gains as the first step towards tuning the controller. For example, the lower and upper bounds for the design parameters can be formed around the values found from Ziegler-Nichols method, such as one-third for lower bound and three-fold for the upper bound

$$\text{as } K_p \in [\frac{K_p}{3}, 3K_p], \quad T_i \in [\frac{T_i}{3}, 3T_i], \quad \text{and} \\ T_d \in [\frac{T_d}{3}, 3T_d].$$

One of the most commonly used methods in tuning of controller gains is to minimize the integral of the squared error due to a unit step change in the reference input. This tuning, however, may lead to excessive controller output swings that cause the system disturbances. Another preferred method is to minimize the integral of the squared error and the squared controller output deviation from its final value u_∞ , simultaneously.

$$J = \int_0^\infty [e(t)^2 + A_c^2(u(t) - u_\infty)^2] dt \quad (9)$$

Where A_c is a weighting parameter providing an adjustment in between the squared controller input

and output difference, which has a desirable effect on the robustness and overshoot. The input error is relative to reference input, whereas the output error is due to its steady-state output (Wilton, 1999).

The objective function given above was minimized by a genetic algorithm coded in MATLAB® using Goldberg’s (1989) algorithm. A constant population size of 30 with a string length of 30 was used in the GA by using a crossover probability, P_c , of 0.001, and a mutation probability rate, P_m , of 0.002. The optimized parameters were presented in Table 2.

5. RESULTS AND DISCUSSION

In this study, a mathematical model of a dc motor driven inertial load with a gearbox was developed and simulated both to design a PID controller and to tune it. The Ziegler-Nichols rules were used to form the intervals for the design parameters in genetic algorithms to tune the controller by minimizing an objective function described in the previous sections.

The controller gains were computed by using both the Zeigler-Nichols rules and the genetic algorithms. The gains found from both methods were listed in Table 2. Since driving conclusions from the gains was not trivial, step responses were drawn for each case and commonly accepted performance criteria such as overshoot and settling times were computed for the comparisons. Step responses and controller outputs for the two designs were presented in the Figure 4 and 5. It is clearly shown in the figures that the genetic optimum (GA) solutions (present) are less oscillatory than those of Ziegler-Nichols (Z-N) design in both step response and controller output. Although a comparatively smaller rise time (T_r) and settling time (T_s) were obtained from Ziegler-Nichols, genetic algorithm solution had a very small overshoot (OS) with respect to the final value on the step steady-state. These results were presented in Table 3. In conclusion, superior results were obtained in terms of system performance and controller output by using genetic algorithms for tuning PID controllers when these values compared on the tables and figures.

Table 3. Response Characteristics of the System for a Unit Step

	Overshoot (%)	T_r (sec)	T_s (sec)
Ziegler-Nichols	% 58	0.13	1.60
Genetic Algorithm	% 21	0.29	1.68

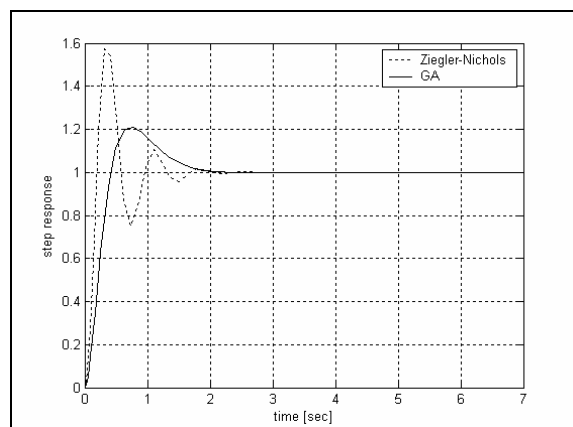


Figure 4. Closed loop unit step response of the system with the controllers

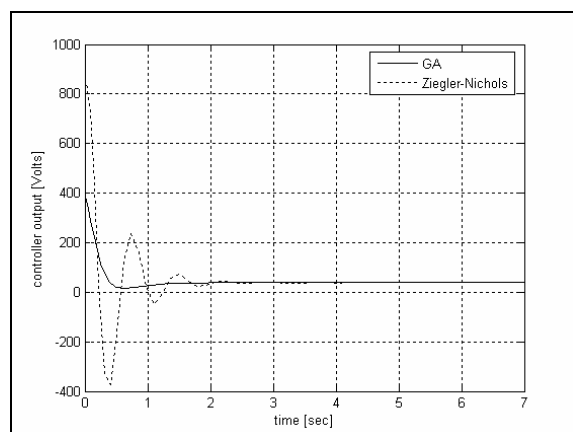


Figure 5. Controller outputs for the system with the controllers

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