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CALCULATION OF DYNAMIC COEFFICIENTS FOR FLUID FILM JOURNAL BEARINGS

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ABSTRACT

An analytical method for calculating the stiffness and damping coefficients for journal bearings is briefly reviewed with author's contributions. These coefficients are required for stability calculations of rotor-bearing systems. The Reynolds equation governing the pressure field within a bearing is obtained by simplifying the Navier-Stokes equations in Cartesian coordinates. It is then solved analytically using the short bearing approximation to obtain pressures in the fluid film. Integration of the pressure obtained by the solution of the Reynolds equation defines the bearing forces and the corresponding dynamic stiffness and damping coefficients of short circular bearings. The results presented in this work can be used in the stability calculations of rotor systems with short circular bearings.

Key Words : Rotor, Bearing, Reynolds equation, Dynamic coefficients, Stability

AKIŞKAN FİLM ŞAFT YATAKLARININ DİNAMİK KATSAYILARININ HESAPLANMASI

ÖZET

Şaft yataklarının rijidlik ve sönüm katsayılarını hesaplamak için analitik bir metot yazarın katkılarıyla birlikte kısaca derleniyor. Bu katsayılar rotor-rulman sistemlerinin kararlılık hesaplamaları için gereklidir. Bir rulman içindeki basıncı veren Reynolds denklemi kartezyen koordinatlarda Navier-Stokes denklemlerinin sadeleştirilmesiyle elde ediliyor. Daha sonra bu denklem akışkan filmdeki basınçları elde etmek için kısa rulman yaklaşımını kullanarak analitik olarak çözülüyor. Reynolds denkleminin çözümünden elde edilen basıncın integrasyonu rulman kuvvetlerini ve bunlara karşılık gelen kısa çembersel rulmanların dinamik rijidlik ve sönüm katsayılarını tanımlar. Bu çalışmada sunulan sonuçlar kısa çembersel rulmanlara sahip rotor sistemlerinin kararlılık hesaplamalarında kullanılabilir.

Anahtar Kelimeler : Rotor, Rulman, Reynolds denklemi, Dinamik katsayılar, Kararlılık

1. INTRODUCTION

The need for higher speed, yet reliable operation of rotating machinery continues to increase. A key factor in achiving this objective continues to be the ability to accurately predict the dynamic response and stability of a rotor-bearing system.

All rotating machinery is supported by one or more bearings which play a vital part of the entire system, since it is the component that permits the relative motion between the stationary and moving parts. There are two general types of bearings which are commonly used in rotor-bearing system applications. These are fluid-film bearings and rolling-element bearings.

Fluid-film bearings have been succesfully used for millennia, and may well be the most used machine element in our civilization. Today large numbers of fluid-film bearings are used in myriad of applications from small electric motors, to automobile and aircraft piston engines, to large steam turbines for electric power generation.

It is well known that the fluid-film bearings play a significant role in the dynamic behavior of the rotor (Goodwin 1989; Rao, 1983; Yücel, 2003). Because the thin film that separates the moving surfaces supports the rotor load, it acts like a spring and provides damping due to squeeze film effect. The stifness and damping properties of the fluid-film significantly alter the critical speeds and out-of-balance responce of a rotor. In addition, rotor instability occurs, which is a self-exited vibration arising out of the bearing fluid-film effects, and this is an important factor to be considered in the rotor design.

Many theoretical studies, numerical calculations and experimental measurements have been carried out to determine the effect of self-exited vibration in rotor-bearing systems due to bearing fluid-film effects. The appended reference list gives some, but far from all of the papers that mark the development of the concept of bearing fluid-film effects.

In this work, a method for calculating the stiffness and damping coefficients of short circular bearings is presented. These stiffness and damping coefficients are necessary for the stability analysis of the rotor-bearing systems. The Reynolds equation governing the pressure field within a bearing is derived from the Navier-Stokes equations in section 2. Analitical solution of the Reynolds equation using the short bearing approximation is given in section 3. Integration of the pressure field gives the bearing forces acting on a rotor. Formulas for the stiffness and damping coefficients are presented in section 4 and the results are summurized in section 5.

2. DERIVATION OF THE REYNOLDS EQUATION

Our interest in this section is the derivation of a governing equation for the pressure field within a bearing. The pressure field will then be integrated to determine the bearing forces acting on a rotor (shaft).

The components used in the theory of hydrodynamic bearings are the bearing journal (shaft), the lubricant and the bearing bushing which is a plain cylindrical sleeve wrapped around the journal. The bearing bushing is rigidly supported. Figure 1 shows the 360 degree plain journal bearing layout. The journal rotates with angular velocity Ω and is statically or dynamically loaded in the radial direction. Figure 2 illustrates the cross-section of a plain journal bearing. The bearing center is at the midpoint of the bearing length L and is denoted by C_b. The point C_b is the origin of the coordinate system X, Y, Z. The bearing journal has radius R and center C_j in the XY-plane. It is assumed for simplicity that the journal remains parallel to the bearing bushing and undergoes only tranlational motion parallel to the XY-plane (in addition to the rotation). In other words, any moments arising from journal tilt are neglected.

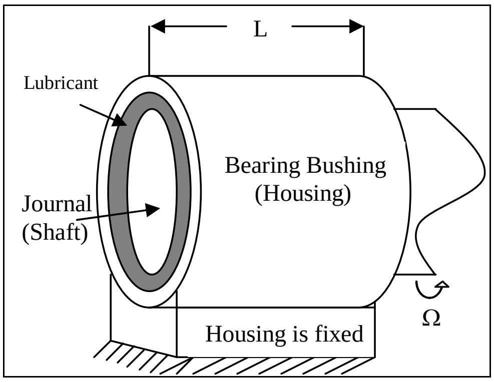

Figure 1. 360 degree plain journal bearing layout

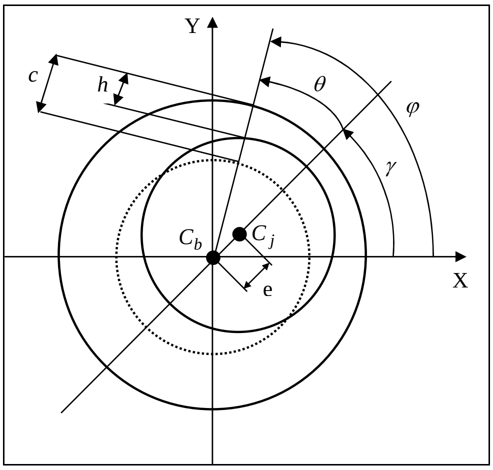

Figure 2. Diagramatic layout of assumed clearance geometry

The journal is considered unloaded when C_j and C_b are coincident. The radial distance between the journal surface and the bearing bushing for a central journal is denoted by c. The position of the journal is defined by the eccentricity e and the angle γ. For an accentric journal the fluid film tickness is given by (Yucel, 2000).

$$h(\varphi) = c - e\cos(\varphi - \gamma), \qquad (1)$$

and for a moving journal with e(t) and $\gamma(t)$

$$h(\varphi, t) = c - e(t)\cos(\varphi - \gamma(t)). \qquad (2)$$

The following simplifying assumptions are made in the derivation of the governing equation for the pressure field within a bearing:

1. The curvature of fluid film is negligible.
2. The lubricant is massless and incompressible.
3. The lubricant is Newtonian and its viscosity is constant in the whole of the fluid-film.
4. The flow is laminar.
5. The pressure of the lubricant is constant in the radial direction.
6. The fluid-film thickness is small compared to the journal radius.

In view of the assumption one, we can consider the fluid filled area of the bearing as the flow region between an infinite rigid plate and an ondulating second surface as shown in Figure 3.

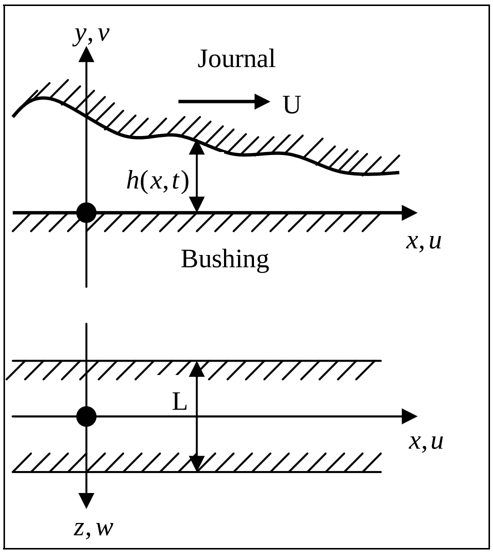

Figure 3. Coordinates of fluid-film

The infinite plate corresponds to the bearing bushing. The ondulating surface corresponds to the journal surface and moves to the right with speed $U = R\Omega$. The distance between the two surfaces is represented by $h(x,t)$ where x measures the distance along the periphery of the bearing bushing.

We use the other assumptions listed above to simplify the study of the fluid flow between the two surfaces. Using Cartesian coordinates and neglecting body forces, the Navier-Stokes equations can be stated as;

$$\begin{aligned}
&\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \\
&-\frac{1}{\rho}\frac{\partial p}{\partial x} + \eta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) \\
&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = \\
&-\frac{1}{\rho}\frac{\partial p}{\partial y} + \eta\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right), \\
&\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = \\
&-\frac{1}{\rho}\frac{\partial p}{\partial z} + \eta\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)
\end{aligned} \qquad (3)$$

Where, ρ and η are the fluid density and kinematic viscosity, respectively. Further, p is the pressure, and u, v, w the fluid velocity components in the x, y, z directions. The flow field model is completed by the incompressible fluid continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \qquad (4)$$

The spatial coordinates are nondimensionalized via

$$\bar{x} = \frac{x}{R} \quad , \quad \bar{y} = \frac{y}{c} \quad , \quad \bar{z} = \frac{z}{R}. \qquad (5)$$

The velocity components are also nondimensionalized via

$$\bar{u} = \frac{u}{R\Omega} \quad , \quad \bar{v} = \left(\frac{R}{c}\right)\frac{v}{R\Omega} \quad , \quad \bar{w} = \frac{w}{R\Omega}. \qquad (6)$$

The nondimensionalization is completed by

$$\bar{p} = \Re e\left(\frac{c}{R}\right)\frac{p}{\rho(R\Omega)^2}, \quad \Re e = \frac{cR\Omega}{\eta}, \quad \bar{t} = \Omega t. \qquad (7)$$

Substituting the nondimensionalized variables into Eqns. (3), we obtain

$$\Re e\left(\frac{c}{R}\right)\left[\frac{\partial\bar{u}}{\partial\bar{t}}+\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}\right)\right]=$$
$$-\frac{\partial\bar{p}}{\partial\bar{x}}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}+\left(\frac{c}{R}\right)^2\left(\frac{\partial^2\bar{u}}{\partial\bar{x}^2}+\frac{\partial^2\bar{u}}{\partial\bar{z}^2}\right)$$
$$\left(\frac{c}{R}\right)^2\left[\begin{array}{l}\dfrac{c^2\Omega}{\eta}\dfrac{\partial\bar{v}}{\partial\bar{t}}+\Re e\dfrac{c}{R}\left(\bar{u}\dfrac{\partial\bar{v}}{\partial\bar{x}}+\bar{v}\dfrac{\partial\bar{v}}{\partial\bar{y}}+\bar{w}\dfrac{\partial\bar{v}}{\partial\bar{z}}\right)\\ -\dfrac{\partial^2\bar{v}}{\partial\bar{y}^2}-\left(\dfrac{c}{R}\right)^2\left(\dfrac{\partial^2\bar{v}}{\partial\bar{x}^2}+\dfrac{\partial^2\bar{v}}{\partial\bar{z}^2}\right)\end{array}\right]. \quad (8)$$
$$=-\frac{\partial\bar{p}}{\partial\bar{y}}$$
$$\Re e\left(\frac{c}{R}\right)\left[\frac{\partial\bar{w}}{\partial\bar{t}}+\left(\bar{u}\frac{\partial\bar{w}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}\right)\right]=$$
$$-\frac{\partial\bar{p}}{\partial\bar{z}}+\frac{\partial^2\bar{w}}{\partial\bar{y}^2}+\left(\frac{c}{R}\right)^2\left(\frac{\partial^2\bar{w}}{\partial\bar{x}^2}+\frac{\partial^2\bar{w}}{\partial\bar{z}^2}\right)$$

Neglecting second order terms in (c/R), the above equations simplify to;

$$\Re e\left(\frac{c}{R}\right)\left[\frac{\partial\bar{u}}{\partial\bar{t}}+\left(\bar{u}\frac{\partial\bar{u}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{u}}{\partial\bar{z}}\right)\right]=$$
$$-\frac{\partial\bar{p}}{\partial\bar{x}}+\frac{\partial^2\bar{u}}{\partial\bar{y}^2}$$
$$0=-\frac{\partial\bar{p}}{\partial\bar{y}} \quad . \qquad (9)$$
$$\Re e\left(\frac{c}{R}\right)\left[\frac{\partial\bar{w}}{\partial\bar{t}}+\left(\bar{u}\frac{\partial\bar{w}}{\partial\bar{x}}+\bar{v}\frac{\partial\bar{w}}{\partial\bar{y}}+\bar{w}\frac{\partial\bar{w}}{\partial\bar{z}}\right)\right]=$$
$$-\frac{\partial\bar{p}}{\partial\bar{z}}+\frac{\partial^2\bar{w}}{\partial\bar{y}^2}$$

Hence, because of the small (c/R) ratio, the pressure gradient across the film is entirely negligible.

In analyzing hydrodynamic bearings, the temporal and acceleration terms on the left of Eqns. (9) are neglected since these terms are arguably small provided that $\Re e(c/R)$ is less than one. Hence, the dimensional governing equations become;

$$\frac{\partial^2 u}{\partial y^2}=\frac{1}{\mu}\frac{\partial p}{\partial x}, \qquad \frac{\partial^2 w}{\partial y^2}=\frac{1}{\mu}\frac{\partial p}{\partial z} \qquad (10)$$

Where, μ is the absolute viscosity. From Figure 3, the boundary conditions can be stated as;

$$u=0 \ , \ w=0 \ \text{ at } \ y=0$$
$$u=U \ , \ w=0 \ \text{ at } \ y=h \quad . \qquad (11)$$

Integrating Eqns. (10) with respect to y and employing the above boundary conditions, we obtain

$$u=\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2-hy\right)+\frac{U}{h}y$$
$$w=\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(y^2-hy\right) \quad . \qquad (12)$$

We note that u is the sum of the flow due to circumferential pressure gradient $\partial p/\partial x$ and the flow due to the no-slip boundary conditions. Note further that the pressure gradients $\partial p/\partial x$ and $\partial p/\partial z$ are independent of y. Substitution from Eqns. (12) into the continuity Eqn. (4) yields.

$$\frac{\partial v}{\partial y}=-\frac{\partial}{\partial x}\left[\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2-hy\right)+\frac{U}{h}y\right]$$
$$-\frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(y^2-hy\right)\right] \quad . \qquad (13)$$

Integration with respect to y from y = 0 to y = h yields.

$$v=\frac{\partial h}{\partial t}=-\int_0^h\frac{\partial}{\partial x}\left[\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2-hy\right)+\frac{U}{h}y\right]dy$$
$$-\int_0^h\frac{\partial}{\partial z}\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(y^2-hy\right)\right]dy \quad . \qquad (14)$$

In the above equation, we consider that $v=\partial h/\partial t$ at the journal surface. Applying Leibniz's rule for the differentiation of integrals,

$$\int_0^{h(x)}\frac{\partial}{\partial x}\left[f(y,x)\right]dy=\frac{\partial}{\partial x}\int_0^{h(x)}f(y,x)dy-f(h,x)\frac{dh}{dx},$$

to the first integral of Eqn. (14), we obtain

$$\frac{\partial h}{\partial t}=-\frac{\partial}{\partial x}\int_0^h\left[\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(y^2-hy\right)+\frac{U}{h}y\right]dy$$
$$-U\frac{\partial h}{\partial x}-\frac{\partial}{\partial z}\int_0^h\left[\frac{1}{2\mu}\frac{\partial p}{\partial z}\left(y^2-hy\right)\right]dy \quad . \qquad (15)$$

Since μ, $\partial p/\partial x$ and $\partial p/\partial z$ are independent of y, the indicated integrations can be completed to yield the following laminar flow Reynolds equation for an isoviscous incompressible fluid:

$$\frac{\partial}{\partial x}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6U\frac{\partial h}{\partial x}+12\frac{\partial h}{\partial t}. \quad (16)$$

With this equation and appropriate boundary and initial conditions the pressure p in the fluid-film is determined.

3. SOLUTION OF THE REYNOLDS EQUATION

The Reynolds equation derived in the previous section is usually solved on the computer using finite difference or finite element methods. However, there are two approximations to Eqn. (16) that have closed-form analytical solutions (Kirk and Gunter, 1976; Szeri, 1980; Szeri, 1987; Hamrock, 1994). These are known as the short bearing and long bearing approximations. For bearings that are short in the axial direction, the pressure change in the circumferential direction is neglected. On the other hand, for a long bearing, the pressure change in the axial direction is neglected. In this work, we only consider the short bearing approximation for the solution of the Reynolds equation given by Eqn. (16). Hence, we remove the first term on the left hand side of Eqn. (16). With $\partial p/\partial x=0$, the Reynolds equation takes the following form:

$$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial p}{\partial z}\right)=6U\frac{\partial h}{\partial x}+12\frac{\partial h}{\partial t}. \quad (17)$$

With $x=R\varphi$ and $h(\varphi,t)$ from Eq. (2), we can write

$$\begin{aligned}\frac{\partial h}{\partial x}&=\frac{1}{R}e\sin(\varphi-\gamma)\\ \frac{\partial h}{\partial t}&=-e\frac{\partial\gamma}{\partial t}\sin(\varphi-\gamma)-\frac{\partial e}{\partial t}\cos(\varphi-\gamma)\end{aligned}. \quad (18)$$

Substituting Eqs. (18) into Eq. (17) and using $\theta=\varphi-\gamma$, we obtain

$$\frac{\partial^2 p}{\partial z^2}=\frac{6\mu}{h^3}\left[e\left(\Omega-2\dot{\gamma}\right)\sin\theta-2\dot{e}\cos\theta\right], \quad (19)$$

where the dot indicates the derivative with respect to time. Here, we note that $\dot{e}$ and $e\dot{\gamma}$ correspond to the radial and tangential velocities of the journal in the bearing, respectively. By integration, one obtains the pressure function

$$p(\theta,z,t)=\frac{3\mu}{h^3}\left[e\left(\Omega-2\dot{\gamma}\right)\sin\theta-2\dot{e}\cos\theta\right]z^2+\tilde{C}z+\tilde{D}, \quad (20)$$

where $\tilde{C}$ and $\tilde{D}$ are the integration constants to be determined using the boundary conditions $p=0$ at $z=\pm L/2$. Making use of these boundary conditions results in

$$p(\theta,z,t)=\frac{3\mu}{h^3}\left(z^2-\frac{L^2}{4}\right)\left[e\left(\Omega-2\dot{\gamma}\right)\sin\theta-2\dot{e}\cos\theta\right]. \quad (21)$$

Integrating the pressure obtained by the solution of Reynolds equation, the fluid-film force is determined. The integration must be done very carefully since a subambient pressure will not be permitted to exist in the fluid film (Kirk and Gunter, 1976). Integrating Eqn. (21) over z gives the fluid film force per unit length in the circumferential direction as

$$q(\theta,t)=\int_{-L/2}^{L/2}p(\theta,z,t)\,dz=\frac{\mu L^3\Omega}{2c^2}q^*(\theta,t), \quad (22)$$

where

$$q^*(\theta,t)=\frac{\varepsilon\left(\frac{2\dot{\gamma}}{\Omega}-1\right)\sin\theta+\frac{2\dot{\varepsilon}}{\Omega}\cos\theta}{\left(1-\varepsilon\cos\theta\right)^3}. \quad (23)$$

In the above equation, ε is the eccentricity ratio ($\varepsilon=e/c$). In the static case, we obtain

$$q^*(\theta)=\frac{-\varepsilon\sin\theta}{\left(1-\varepsilon\cos\theta\right)^3}. \quad (24)$$

Figure 4 shows the behavior of $q^*(\theta)$ for several values of the eccentricity ratio ε.

As can be seen from the figure, in the region $0<\theta<\pi$, $q^*(\theta)$ is negative. Therefore, one assumes in this region that $q^*(\theta)=0$ (Childs, 1993; Kramer, 1993). Possitive pressures occur in the lower half as a result of the eccentricity vector e in the plane $X'Y'$ (see Figure 5). The resultant of forces $dF=qRd\theta$ loads the bearing shell and the journal. The corresponding static equilibrium force on the journal is shown as F_0.

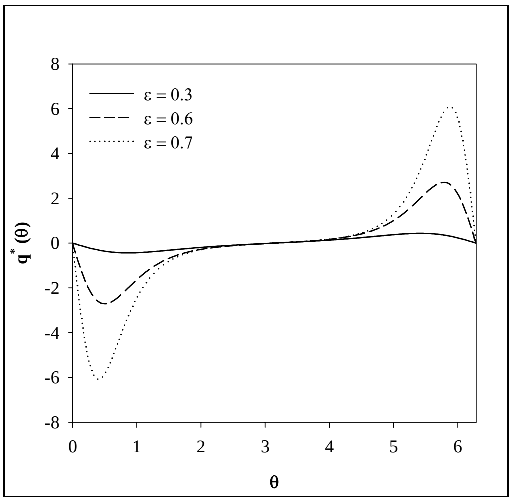

Figure 4. Fluid film force per unit length in the circumferential direction for several values of ε

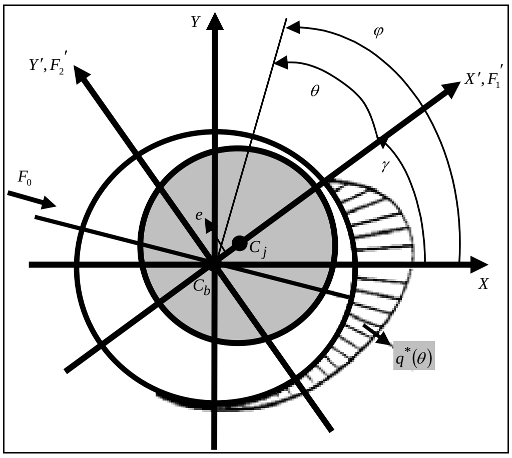

Figure 5. Forces due to static loading for short circular bearing

Now, we consider a horizontally supported shaft with static loadings as shown in Figure 6. If F_1 and F_2 are the force components in the X and Y directions, respectively, then we have

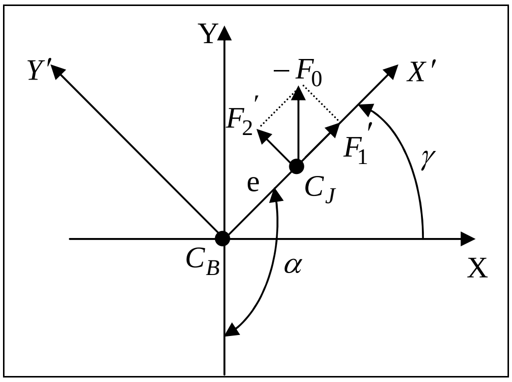

Figure 6. Special case of a static load

$$F_1 = 0 = F_1' \cos\gamma - F_2' \sin\gamma \quad , \qquad F_2 = -F_0 = F_1' \sin\gamma + F_2' \cos\gamma \tag{25}$$

where

$$F_1' = \int_{\pi}^{2\pi} q(\theta)\cos\theta \, R d\theta = F_\mu \frac{2\varepsilon^2}{\left(1-\varepsilon^2\right)^2} \quad . \qquad F_2' = \int_{\pi}^{2\pi} q(\theta)\sin\theta \, R d\theta = -F_\mu \frac{\pi\varepsilon}{2\left(1-\varepsilon^2\right)^{3/2}} \tag{26}$$

In equations (26) , F_μ is given by

$$F_\mu = \frac{\mu L^3 \Omega R}{2c^2} . \tag{27}$$

The angle γ shown in Figure 6 is obtained from the first of equations (25) as

$$\tan\gamma = \frac{F_1'}{F_2'} = -\frac{4\varepsilon}{\pi\sqrt{1-\varepsilon^2}} . \tag{28}$$

The angle α between the vertical direction (load direction) and the line of centers (shown in Figure 6) is related to γ by $\alpha = \gamma + \pi/2$ and consequently is given by

$$\tan\alpha = -\frac{F_2'}{F_1'} = -\frac{\pi\sqrt{1-\varepsilon^2}}{4\varepsilon} . \tag{29}$$

From the second of equations (25), from equations (26) and (28) and using

$$\sin\gamma = \frac{\tan\gamma}{\sqrt{1+\tan^2\gamma}} , \cos\gamma = \frac{1}{\sqrt{1+\tan^2\gamma}} , \tag{30}$$

it follows that the relative static force is given by

$$\frac{F_0}{F_\mu} = S^* = \frac{\varepsilon\sqrt{\pi^2 - \pi^2\varepsilon^2 + 16\varepsilon^2}}{2\left(1-\varepsilon^2\right)^2} . \tag{31}$$

This ratio is refered to as the bearing Summerfeld number.

In the dynamic case, the components of the fluid film force, that is of the journal force in the $X'Y'$-system shown in Figure 5, are

$$F_1' = \int_{\pi}^{2\pi} q(\theta,t)\cos\theta\, Rd\theta = F_\mu\, f_1(\varepsilon,\dot{\varepsilon},\dot{\gamma})$$
$$F_2' = \int_{\pi}^{2\pi} q(\theta,t)\sin\theta\, Rd\theta = F_\mu\, f_2(\varepsilon,\dot{\varepsilon},\dot{\gamma}) \quad (32)$$

where

$$f_1(\varepsilon,\dot{\varepsilon},\dot{\gamma}) = \left(1-\frac{2\dot{\gamma}}{\Omega}\right)\frac{2\varepsilon^2}{\left(1-\varepsilon^2\right)^2} + \frac{\pi\dot{\varepsilon}\left(1+2\varepsilon^2\right)}{\Omega\left(1-\varepsilon^2\right)^{5/2}}$$
$$f_2(\varepsilon,\dot{\varepsilon},\dot{\gamma}) = -\left(1-\frac{2\dot{\gamma}}{\Omega}\right)\frac{\pi\varepsilon}{2\left(1-\varepsilon^2\right)^{3/2}} - \frac{4\varepsilon\dot{\varepsilon}}{\Omega\left(1-\varepsilon^2\right)^2} \quad (33)$$

As it was mentioned earlier the force components relative to the XY-system are

$$F_1 = F_1'\cos\gamma - F_2'\sin\gamma$$
$$F_2 = F_1'\sin\gamma + F_2'\cos\gamma \quad (34)$$

With equations (32) to (34) the explicit relations for the magnitude and directions of the journal force are obtained as functions of positions ε,γ and of velocities $\dot{\varepsilon},\dot{\gamma}$ of the journal center C_j.

4. DETERMINATION OF STIFFNESS AND DAMPING COEFFICIENTS

For rotordynamic analysis we need to calculate the perturbations in the forces (F_1, F_2) which arise due to a small vibration of the journal center about its equilibrium position (X_0, Y_0) under the static load (F_{10}, F_{20}) (see Fig. 7).

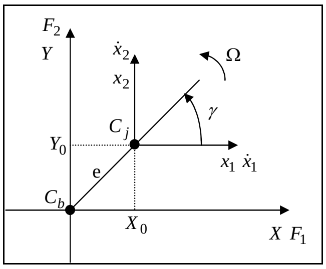

Figure 7. Coordinates for stiffness and damping coefficients for short circular bearings

In general the relation between the force and the motion is non-linear. For small displacements x_1, x_2 and velocities $\dot{x}_1, \dot{x}_2$ of the journal center we linearize the forces F_1 and F_2 as follows

$$F_1 = F_{10} + \left.\frac{\partial F_1}{\partial x_1}\right|_0 x_1 + \left.\frac{\partial F_1}{\partial x_2}\right|_0 x_2 + \left.\frac{\partial F_1}{\partial \dot{x}_1}\right|_0 \dot{x}_1 + \left.\frac{\partial F_1}{\partial \dot{x}_2}\right|_0 \dot{x}_2$$
$$F_2 = F_{20} + \left.\frac{\partial F_2}{\partial x_1}\right|_0 x_1 + \left.\frac{\partial F_2}{\partial x_2}\right|_0 x_2 + \left.\frac{\partial F_2}{\partial \dot{x}_1}\right|_0 \dot{x}_1 + \left.\frac{\partial F_2}{\partial \dot{x}_2}\right|_0 \dot{x}_2 \quad (35)$$

where the partial diffrentials with respect to displacement and velocity are the stiffness and damping coefficients, respectively. These coefficients are to be calculated at the static equilibrium and given by

$$K_{ij} = \left.\frac{\partial F_i}{\partial x_j}\right|_0, \quad C_{ij} = \left.\frac{\partial F_i}{\partial \dot{x}_j}\right|_0, \quad i = 1,2, \quad j = 1,2\,. \quad (36)$$

From equations (34), F_1 and F_2 are functions of ε,γ and $\dot{\varepsilon},\dot{\gamma}$. Thus, the required differentials are

$$\frac{\partial F_i}{\partial x_j} = \frac{\partial F_i}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial x_j} + \frac{\partial F_i}{\partial \gamma}\frac{\partial \gamma}{\partial x_j} + \frac{\partial F_i}{\partial \dot{\varepsilon}}\frac{\partial \dot{\varepsilon}}{\partial x_j} + \frac{\partial F_i}{\partial \dot{\gamma}}\frac{\partial \dot{\gamma}}{\partial x_j}$$
$$\frac{\partial F_i}{\partial \dot{x}_j} = \frac{\partial F_i}{\partial \varepsilon}\frac{\partial \varepsilon}{\partial \dot{x}_j} + \frac{\partial F_i}{\partial \gamma}\frac{\partial \gamma}{\partial \dot{x}_j} + \frac{\partial F_i}{\partial \dot{\varepsilon}}\frac{\partial \dot{\varepsilon}}{\partial \dot{x}_j} + \frac{\partial F_i}{\partial \dot{\gamma}}\frac{\partial \dot{\gamma}}{\partial \dot{x}_j} \quad (37)$$

The displacements dx_1, dx_2 in the XY-system correspond to the displacements $de, d\gamma$ as follows

$$dx_1 = dX_0 = \cos\gamma\, de - e\sin\gamma\, d\gamma$$
$$dx_2 = dY_0 = \sin\gamma\, de + e\cos\gamma\, d\gamma \quad (38)$$

Equations (38) can be solved for $de, d\gamma$ to produce

$$de = \cos\gamma\, dx_1 + \sin\gamma\, dx_2$$
$$e d\gamma = -\sin\gamma\, dx_1 + \cos\gamma\, dx_2 \quad (39)$$

Considering that the derivatives of equations (37) are to be calculated at the static equilibrium, and remembering that $\varepsilon = e/c$, we can write

$$\begin{aligned} \frac{\partial \varepsilon}{\partial x_1} &= \frac{\partial \dot{\varepsilon}}{\partial \dot{x}_1} = \frac{\cos\gamma}{h_0} \quad , \quad \frac{\partial \gamma}{\partial x_1} = \frac{\partial \dot{\gamma}}{\partial \dot{x}_1} = -\frac{\sin\gamma}{e} \\ \frac{\partial \varepsilon}{\partial x_2} &= \frac{\partial \dot{\varepsilon}}{\partial \dot{x}_2} = \frac{\sin\gamma}{h_0} \quad , \quad \frac{\partial \gamma}{\partial x_2} = \frac{\partial \dot{\gamma}}{\partial \dot{x}_2} = \frac{\cos\gamma}{e} \end{aligned} \quad . \tag{40}$$

Here, we note that under the condition of static equilibrium, the first two terms on the right of the second equations of (37) vanish. In addition, the last two terms on the right of the first equations of (37) vanish since the derivatives $\partial\dot{\varepsilon}/\partial x_j$ and $\partial\dot{\gamma}/\partial x_j$ vanish in view of equations (39).

With these relationships, one finally obtains after transformation the following relationships for the stiffness and damping coefficients of the short circular bearing

$$K_{ij} = \frac{F_0}{h_0} K_{ij}^* \quad , \quad C_{ij} = \frac{F_0}{h_0 \Omega} C_{ij}^* , \tag{41}$$

where K_{ij}^* and C_{ij}^* are the dimensionless stiffness and damping coefficients, respectively. They are given by

$$\begin{aligned} K_{11}^* &= \left[2\pi^2 + \left(16-\pi^2\right)\varepsilon^2 \right]\Psi(\varepsilon) \\ K_{12}^* &= \frac{\pi}{4}\frac{\pi^2 - 2\pi^2\varepsilon^2 - \left(16-\pi^2\right)\varepsilon^4}{\varepsilon\left(1-\varepsilon^2\right)^{1/2}}\Psi(\varepsilon) \\ K_{21}^* &= -\frac{\pi}{4}\frac{\pi^2 + \left(32+\pi^2\right)\varepsilon^2 + \left(32-2\pi^2\right)\varepsilon^4}{\varepsilon\left(1-\varepsilon^2\right)^{1/2}} \\ & x\Psi(\varepsilon) \\ K_{22}^* &= \frac{\pi^2 + \left(32+\pi^2\right)\varepsilon^2 + \left(32-2\pi^2\right)\varepsilon^4}{1-\varepsilon^2}\Psi(\varepsilon) \end{aligned} \tag{42}$$

$$\begin{aligned} C_{11}^* &= \frac{\pi\left(1-\varepsilon^2\right)^{1/2}}{2\varepsilon}\left[\pi^2 + \left(2\pi^2-16\right)\varepsilon^2 \right]\Psi(\varepsilon) \\ C_{12}^* &= C_{21}^* = -\left[2\pi^2 + \left(4\pi^2-32\right)\varepsilon^2 \right]\Psi(\varepsilon) \\ C_{22}^* &= \frac{\pi}{2}\frac{\pi^2 + \left(48-2\pi^2\right)\varepsilon^2 + \pi^2\varepsilon^4}{\varepsilon\left(1-\varepsilon^2\right)^{1/2}}\Psi(\varepsilon) \end{aligned} \quad , \tag{43}$$

where

$$\Psi(\varepsilon) = \frac{4}{\left[\pi^2 + \left(16-\pi^2\right)\varepsilon^2\right]^{3/2}} . \tag{44}$$

Note that a MAPLE worksheet has been used to derive the above coefficients. Figure 8 and Figure 9 show the plots of the dimensionless stiffness and damping coefficients versus the eccentricity ratio, respectively.

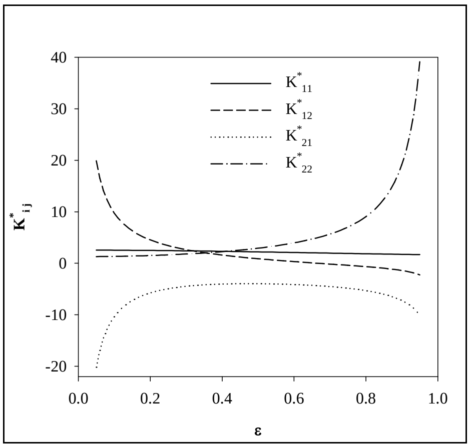

Figure 8. Dimensionless stiffness coefficients for short circular bearing

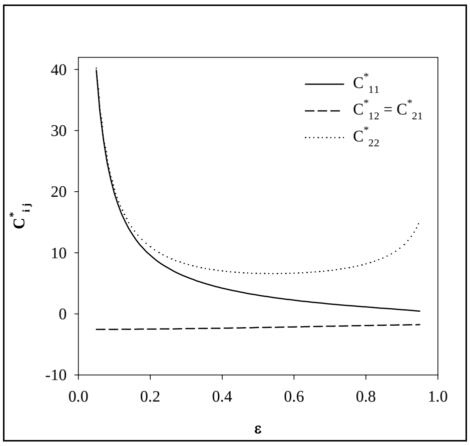

Figure 9. Dimensionless damping coefficients for short circular bearing

These coefficients can now be inserted into the equation of motion of the journal. And then a stability analysis can be performed. This subject will not be considered in this work.

5. CONCLUSIONS

In this work, an analitycal method for computing the stiffness and damping coefficients of short circular bearings has been summurized. The concept of stiffness and damping coefficients for journal bearings has proven very fruitful, and modern rotor dynamic calculations for unbalance response, damped natural frequencies, and stability are based on this concept.

In recent years, extensive experimental works have been carried out to measure the bearing coefficients. Many rusults show good agrement with the theoretical values. That is why many authors are still using these coefficients obtained by the method just described in this work in their stability analysis.

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