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MODELLING OF SHORT DURATION RAINFALL (SDR) INTENSITY EQUATIONS FOR ERZURUM

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ABSTRACT

The scope of this study is to develop a rainfall intensity-duration-frequency (IDF) equation for some return periods at Erzurum rainfall station. The maximum annual rainfall values for 5, 10, 15, 30 and 60 minutes are statistically analyzed for the period 1956 – 2004 by using some statistical distributions such as the Generalized Extreme Values (GEV), Gumbel, Normal, Two-parameter Lognormal, Three-parameter Lognormal, Gamma, Pearson type III and Log-Pearson type III distributions. χ^2 goodness-of-fit test was used to choose the best statistical distribution among all distributions. IDF equation constants and coefficients of correlation (R) for each emprical functions are calculated using nonlinear estimation method for each return periods (T = 2, 5, 10, 25, 50, 75 and 100 years). The most suitable IDF equation is observed that $i_{max}(t) = A/(t+C)^B$, except for T=100 years, because of the highest coefficients of correlation.

Key Words : Statistical distribution function, Rainfall intensity equations.

ERZURUM İÇİN KISA SÜRELİ YAĞIŞ ŞİDDETİ DENKLEMLERİNİN MODELLENMESİ

ÖZET

Bu çalışmanın amacı, Erzurum yağış istasyonunda bazı dönüş periyodları için bir yağış şiddeti-süre-tekerrür (IDF) denklemi geliştirmektir. 5, 10, 15, 30 ve 60 dakikalar için maksimum yıllık yağış değerleri Genelleştirilmiş Ekstrem Değerler (GEV), Gumbel, Normal, iki parametreli Lognormal, üç parametreli Lognormal, Gamma, Pearson tip III ve Log-Pearson tip III dağılımları gibi bazı istatistiksel dağılımlar kullanılarak 1956-2004 aralığı için istatistiksel olarak analiz edildi. Bütün dağılımlar arasından en iyi istatistiksel dağılımı seçmek için χ^2 uyum testi kullanıldı. Her amprik fonksiyon için IDF denklem sabitleri ve korelasyon katsayıları (R) bütün dönüş periyodları için (T = 2, 5, 10, 25, 50, 75 ve 100 yıl) lineer olmayan tahmin metodu kullanılarak hesaplandı. En yüksek korelasyon katsayısından dolayı, T = 100 yıl hariç, en uygun IDF denkleminin $i_{max}(t) = A/(t+C)^B$ olduğu gözlendi.

Anahtar Kelimeler : İstatistiksel dağılım fonksiyonu, Yağış şiddet denklemleri

1. INTRODUCTION

Hydraulic structures are designed to control stormwater volumes and flows need some quantitative criteria to determine their size. The volume or flow rate which must be stored or conveyed by the system can be related mathematically to the precipitation for stormwater system design. Two important stormwater parameters, intensity and duration, can be statistically related to the frequency of occurence. The resulting graphical representation is the intensity-duration-frequency (IDF) curve.

A suitable rainfall IDF relationship is commonly required for planning and design of water resources projects. There has been considerable attention to identify the IDF relationship. Many procedures and formulas, mainly emprical, have been proposed in the literature (Yarnell, 1935; Chow, 1954; Bell, 1969; Chen, 1983; Aron *et al.*, 1987). On the other hand, a mathematical approach has been proposed by Burlando and Rosso (1996) and Koutsoyiannis *et al*. (1998). Division of the rainfall duration into three groups (short, intermediate, long durations) leads to regionalization of IDF relationships into different geographical areas for several counties (Froehlich, 1995; Hanson, 1995; Ferro, Froechlich, 1997; Garcia-Batual and Schneider, 2001).

Record of observed maximum annual rainfall values for durations of 5, 10, 15, 30, and 60 minutes are obtained from DMI (Turkish State Meteorological Service) for Erzurum rainfall station. This rainfall station is located 39^0 55' north and 41^0 16' east with 1869 meter elevation. It measures daily precipitation, mean temperature, min. and max. temperature, snow density, snow depth, radiation. Maximum annual rainfall values have been fitted to the Log-Pearson type III, two parameter Lognormal, Pearson type III and Gumbel distributions.

2. MATERIALS AND METHODS

2. 1. Statistical Distributions

All the parameters are estimated by the method of moments and maximum likelihood approached by considering the Weibull formula in all calculation as, (Weibull, 1939a, 1939b).

$$F = k/(n+1) \tag{1}$$

All statistical distributions and their functions that were analysed in this study are shown in Table 1.

Table 1. Statistical Distributions and it's Functions.

Statistical Distributions	Functions
Generalized Extreme Value (Gev); (Jenkinson, 1969)	$f(x)=\frac{1}{\alpha}.\left[1-k\left(\frac{x-u}{\alpha}\right)\right]^{1/k-1}.e^{-\left[1-k\left(\frac{x-u}{\alpha}\right)\right]^{1/k}}$
The Extreme Value Type I (Gumbel) (Gumbel, 1958; Chow, 1964; Yevjevich, 1972).	$f(x)=\frac{1}{\alpha}\exp\left[\left(-\frac{x-u}{\alpha}\right)-\exp\left(-\frac{x-u}{\alpha}\right)\right]$
Normal An early application of the normal distribution to hydrologic variables was presented by Hazen (1914), who introduced the normal probability paper for analysis of the hydrologic data.	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{\frac{1}{2\sigma^2}(x-\mu)^2}$
Two-Parameter Lognormal	$f(x)=\frac{1}{x.\sigma_y\sqrt{2\pi}}.\exp\left[-\frac{\left(\text{In}x-\mu_y\right)^2}{2\sigma_y{}^2}\right]$
Three-Parameter Lognormal The three-parameter lognormal distribution is similar to the two-parameter lognormal distribution, except that x is shifted by an amount (m) which represents a lower bound. The normally distributed variable becomes log (x – m) with the pdf (Kite, 1977)	$f(x)=\frac{1}{(x-a).\sigma_y\sqrt{2\pi}}.\exp\left[-\frac{\left(\text{In}(x-a)-\mu_y\right)^2}{2\sigma_y{}^2}\right]$
Two-Parameter Gamma	$f(x)=\frac{1}{\alpha^\beta\Gamma(\beta)}x^{\beta-1}e^{-(x/\alpha)}$
The Pearson type III (Pearson, K., 1930)	$f(x)=\frac{1}{\alpha.\Gamma(\beta)}\left(\frac{x-\gamma}{\alpha}\right)^{\beta-1}e^{-\left(\frac{x-\gamma}{\alpha}\right)}$
Log-Pearson type III	$f(x)=\frac{1}{\alpha.x.\Gamma(\beta)}\left[\frac{\text{In}(x)-\gamma}{\alpha}\right]^{\beta-1}e^{-\left\{\frac{\text{In}(x)-\gamma}{\alpha}\right\}}$

2. 2. Goodness-of-Fit Test

It is neccessary to consider a test to determine if a sample record has a specified theoretical distribution. The test is based on to decide how good a fit is between the frequency of occurence observations in an observed sample and the expected frequencies obtained from the hypothesized distribution. A gooness-of-fit test between observed and expected frequencies is based on the Chi-square quantity, which is defined as,

$$\chi^2 = \sum_{i=1}^{k} (o_i - e_i)^2 / e_i \qquad (2)$$

Where χ^2 is a random variable, whose sampling distribution is approximated very closely by the Chi-square distribution. The symbols o_i and e_i represent the observed and expected frequencies, respectively, for the *i*- th class interval in the histogram.

If the observed frequencies are close to the corresponding expected frequencies, the χ^2 value will be small, indicating a good fit; otherwise it is a poor fit. A good fit leads to the acceptance of H_0 (null hypothesis), whereas a poor fit leads to its rejection. The critical region will, therfore, fall in the right tail of the chi-square distribution. For a level of significance equal to α, the critical value χ^2_α is found from readily available Chi-square tables, and $\chi^2 > \chi^2_\alpha$ constitutes the critical region (Walpole and Myers, 1978).

2. 3. Applications

Erzurum, the regional capital with a population of 400 000 and the leading mountain resort in Eastern Anatolia, is situated in a very high valley with an altitude of some 2000 metres. Erzurum has only 453 mm/year making water availability a problem of major concern. Additionally, large interannual variability in precipitation, makes the problem of climate forecasting for this region an important one.

Flooding in the Eastern Anatolia Region is probably the most severe hydrometeorological hazards because of the snowmelt in spring season in Turkey. As a results of these floods, landslides, roads, bridges, residental and agricultural areas are damaged. For instance, one of the most catastrophic flood events was on the 10 August 2005 causing 3 deaths, houses and office to be demolished or heavily damaged in Erzurum.

Stormwater is water from rain that flows accross the land surface. They frequently affect people but typically go unnoticed unless they fail. The consequences of failure range from nuisance flooding of yards, basements, and roadway travel lanes, through temporary road or bridge closure and minor property damage, to widespread destruction and even loss of life.

The decision criterion described here should not be used unless each of the expected frequencies is at least equal to 5. Herein, α (the significance level) is chosen as 0.05. All statistical distributions for different durations at each station are performed by the goodness-of-fit χ^2 test. The Chi-square values and probability values of the statistic distributions are compared. One can then decided the fitting distribution for durations (5, 10, 15, 30, 60 min.) by the smallest Chi-square distribution value. In Table 2, the Chi-square values and the corresponding probability for 5 minutes duration of statistical distributions are shown for Erzurum rainfall records. The Log-Pearson type III distribution is chosen because its χ^2 value is the least and the probability value (p) is the highest for 5 minutes.

Table 2. Goodness of Fit Tests To All Statistical Distributions For 5 Min. Duration In Erzurum.

ERZURUM (5 min.)		
DISRIBUTION	Chi-square	p
GEV (Max. Likehood)	4.46	0.3469
GEV (Moment)	5.63	0.2282
EV I (Max. Likehood)	3.29	0.6550
EV I (Moment)	9.15	0.1034
NORMAL (Max. Likehood)	17.73	0.0033
LOGNORMAL (2P)	5.63	0.3435
LOGNORMAL (3P) (Max. Likelihood)	5.63	0.2282
LOGNORMAL (3 P) (Moment)	5.63	0.2282
GAMMA (Max. Likehood)	8.76	0.1192
GAMMA (Moment)	9.93	0.0773
PEARSON TYPE III (Max. Likehood)	7.20	0.1259
PEARSON TYPE III (Moment)	4.46	0.3469
LOGPEARSON TYPE III (Moment)	5.63	0.2282

The chosen distributions and parameters for Erzurum are shown in Table 3 (Şenocak, 2004).

Table 3. Best fitted Distributions and Estimated Statistical Parameters in Erzurum.

t (min)	Statistical Distributions	α	β	μ	σ	u
5	Log Pearson type III	-7.61893	4.40505	-	-	1.17367
10	Two-parameters Lognormal	-	-	1.77787	0.589613	-
15	Pearson type III	0.510565	5.08066	-	-	-1.76569
30	EV I	3.72939	7.76197	-	-	-
60	two-parameters Lognormal	-	-	2.38308	0.421342	-

2. 4. IDF Emprical Forms

Eight types of rainfall–duration equations are investigated for evaluting short duration rainfall equation parameters for Erzurum rainfall station, Turkey. The equations are empirical and show that rainfall intensity is a decreasing function of rainfall duration for a given period.

It is found that several commonly used functions in the literature of hydrology can be applied to the rainfall data in this study. In all the following equations A, B and C parameters are to be estimated by nonlinear estimation procedure. The highest correlation coefficient indicates the most appropriate IDF equation for the return period.

Widely used two-parameter IDF function is given as follows,

$$i_{max}(t) = A/(t+B) \qquad (3)$$

Probably one of the oldest functions used for IDF representation is given as (Bernard, 1932)

$$i_{max}(t) = A/t^{B} \qquad (4)$$

Another two parameter function is the Fuller equation as,

$$i_{max} = A - B.In(t) \qquad (5)$$

Equations (3) and (4), can be regarded as particular cases of more general three parameter functions as follows

$$i_{max}(t) = A/(t+C)^{B} \qquad (6)$$

and

$$i_{max}(t) = A/(t^{B}+C) \qquad (7)$$

Equation (6) is proposed in the early 30's by Sherman (1932) and Yarnell (1935). These expressions are particular cases of the general 4-parameter intensity – duration relationship for a specified return period cited recently in Koutsoyiannis et al. (1998).

In equations (6) and (7), the B and C parameters are independent when function (4) is used. Equations (6) and (7) have been widely used in hydrology (Keifer and Chu, 1957; Aron et al., 1987)

Other additional 3-parameter equations which are used in Alicante by Garcia-Bartual and Schneider, 2001 for comparative purposes are,

$$i_{max}(t) = A + B/(t+C) \qquad (8)$$

and

$$i_{max}(t) = A/(B+t.C) \qquad (9)$$

The equation used and recommended in the U.K. Flood Studies Report (Keers and Wescott, 1977) is also used in this study is given as,

$$i_{max}(t) = A/(1+B.t)^{C} \qquad (10)$$

5. RESULTS

The precipitation estimates are obtained for short durations at different return periods for the station in Trabzon city. For example, Table 5 presents the precipitation estimates for short durations at different return periods for the station at Erzurum.

The rainfall intensities in Table 6 are calculated by using the following formula

$$i = dP/dt \qquad (11)$$

The parameters of the IDF equations and coefficient of correlation for different return periods (2, 5, 10, 25, 50, 75 and 100 years) are calculated by using nonlinear estimation method (Table 7).

Table 5. Precipitations for Short Durations in Different Return Periods for Station Erzurum.

Precipitation (mm)					
T \ t	5 min	10 min	15 min	30 min	60 min
T = 2 years	4.4	5.9	7.5	9.1	10.8
T = 5 years	6.8	9.7	11.6	13.4	15.5
T = 10 years	8.1	12.6	14.1	16.2	18.6
T = 25 years	9.5	16.6	17.1	19.7	22.7
T = 50 years	10.3	19.9	19.2	22.3	25.8
T = 75 years	10.7	21.9	20.4	23.8	27.6
T = 100 years	11.0	23.3	21.2	24.9	28.9

Table 6. Rainfall Intensities for Short Durations in Different Return Periods for Station Erzurum.

Rainfall Intensity (mm / h)					
T \ t	5 min	10 min	15 min	30 min	60 min
T = 2 years	52.2	35.5	30.2	18.3	10.8
T = 5 years	81.2	58.3	46.3	26.7	15.5
T = 10 years	97.2	75.6	56.4	32.3	18.6
T = 25 years	113.8	99.7	68.3	39.4	22.7
T = 50 years	123.8	119.2	76.8	44.6	25.8
T = 75 years	128.8	131.2	81.5	47.7	27.6
T = 100 years	132.1	140.0	84.8	49.8	28.9

Table 7. Parameter Estimation With Nonlinear Estimation Method for the Station Erzurum.

T	Function	A	B	C	R
2	i=A / [(t + C) ^ B]	286.05	0.78	3.89	0.998
5	i=A / [(t + C) ^ B]	1207.00	1.04	8.50	0.999
10	i=A / [(t + C) ^ B]	5407.93	1.33	15.39	0.998
25	i=A / [(t + C) ^ B]	127300.58	1.94	31.63	0.989
50	i=A / [(t + C) ^ B]	8081663.25	2.71	53.59	0.975
75	i=A / [(t + C) ^ B]	103428980.84	3.16	67.44	0.963
100	i=A / [(1 + B * t) ^ C]	175.42	0.01	4.10	0.954

For a given return period, the best fit is obtained by the nearest R value to 1.00. For example, the IDF curves of the Erzurum station are drawn (Figure 1).

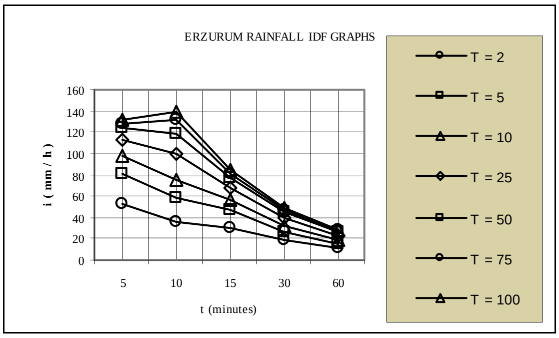

Figure 1. IDF Curves for Erzurum

4. DISCUSSION AND CONCLUSION

IDF equation constants and coefficients of correlation are calculated by using maximum rainfall intensities for each return periods (2, 5, 10, 25, 50, 75 and 100 years).

It is concluded that $i_{max}(t) = A/\left(t+C\right)^{B}$ IDF equation is appropriate, except for T=100 years, because of the highest coefficient of correlation. These IDF equation can be used for hydrological applications to estimate maximum rainfall intensity values for short time durations, as it is the case of planning and design of water resources projects in Erzurum, Turkey.

Besides 3 emprical functions of 2-parameters and 5 emprical functions with 3- parameters were used to represent intensity – duration relationships for Trabzon city. In general, the 3-parameter functions showed acceptable fitting to the rainfall intensity quantiles. χ^2 goodness-of-fit test used to choose the best statistical distribution among all distributions.

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