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SAFE MOVEMENT OF HAZARDOUS MATERIALS THROUGH HEURISTIC HYBRID APPROACH: TABU SEARCH AND GAME THEORY APLICATION

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ABSTRACT

The safe movement of hazardous materials is receiving increased attention due to growing environmental awareness of the potential health affects of a release causing incident. A novel approach developed in this paper through a game theory interpretation provides a risk-averse solution to the hazardous materials transportation problem. The dispatcher minimizes the expected maximum disutility subject to worst possible set of link failure probabilities, assuming that one link in the network fails. The expected cost at the Nash equilibrium is a useful measure to evaluate the routing strategies for the safe movement of hazardous materials**.**

Key Words : *Transportation of hazardous materials, Routing problem, Game theory.*

YAKLAŞIMSAL HİBRİD METODLA GÜVENLİ TEHLİKELİ MADDE TAŞIMACILIĞI: TABU ARAŞTIRMA VE OYUN TEORİSİ UYGULAMASI

ÖZET

Tehlikeli madde taşımacılığında oluşacak herhangi bir kazanın gerek insan sağlığına gerekse de çevreye olacak olan etkilerine karşı oluşan duyarlılığın artmasıyla, bu çeşit taşımacılığın güvenliğine yönelik kaygılar artmıştır. Bu makalede tehlikeli madde taşımacılığı oyun teorisi çerçevesinde en az riski içerecek çözüm metoduyla modellenmiştir. Taşımacı organizasyon şebekedeki linklerden maksimum olumsuz etkiyi oluşturacak olanın üzerinde bir kazanın olduğu varsayımıyla beklenen maksimum maliyeti minimize etmeye çalışarak Nash dengesi çerçevesinde en güvenli taşıma güzergâhını belirlemektedir.

Anahtar Kelimeler : *Tehlikeli madde taşımacılığı, Araç yönlendirme, Oyun teorisi.*

1. INTRODUCTION

Hazardous materials (hazmats) are part of our every day life. As a consequence of industrial development, huge quantities of hazardous materials are yearly produced and obviously the production of them goes together with their transportation. For most members of industrial societies life without hazmats is inconceivable. Hazardous materials are defined as those which are found to be in a quantity and form that may pose an undesirable risk to public health and safety or property when transported (Hobeika et al., 1986).

In this sense, hazardous materials or dangerous goods include explosives, gases, flammable liquids and solids, oxidising substances, poisonous and infectious substances, radioactive materials, corrosive substances and hazardous wastes (Erkut and Verter, 1998).

In any debate about the transportation of dangerous goods where safety is of primary concern, it is very important that there is a full understanding of the magnitude of the risks involved and the causes and major contributors so that properly informed decisions can be made. A special interest area in hazardous materials transportation is the determination of best routing strategies to reduce the risk of Low-Probability, High-Consequence (LPHC) incidents. This study develops a new approach to the solution of the effective routing strategy for the transportation of this type hazardous materials. The solution approach is based on the combination of the game theory and tabu search. The solution is a mixed strategy Nash Equilibrium to a 2-player, non-cooperative, zero-sum game.

2. STATEMENT OF THE PROBLEM

The risk models for routing of hazardous materials proposed hitherto assume that sufficient historical data exist to determine the frequency and consequences of the release incidents, and that past observations can adequately be used to estimate future expectations. Hence, the quality of the proposed models is heavily depended on the data available. However, due to the rare occurrence of incidents involving hazmat shipments, the quality and accuracy of the data are open to discussion.

2. 1. Vehicle Routing Problem

The best strategy for routing corresponds to the best path for the shipment. In real life, there are problems which require the determination of the best routes from a central depot to several demand points (customers) by a set of vehicle routes, the well known Vehicle Routing Problem (VRP). To illustrate this, consider the following problem. Assume that a petrol company has a big refinery, used as a depot, and a fleet of vehicles located at this depot which are used to carry the petrol to the numerous service stations scattered all over the region (country). The problem here is to find the least expected cost routes for a fleet of vehicles, located at the refinery, in such a way that each service station is visited once and only once by exactly one vehicle, taking into account the fact that the total demand of any vehicle route must not exceed the capacity of the vehicle servicing that particular route. The algorithm that developed targets the solution of this problem.

The vehicle routing problem is an easy problem to describe but hard to solve. A problem of size Y can be solved in *polynomial time* if the number of iterations is a polynomial function of Y. Non-deterministic polynomial time problems (abbreviated as NP-problem) are those whose solutions can be checked in polynomial time (even if it takes exponential time to find the solution). However, NP-hard problems are not solvable in polynomial time. Almost all vehicle routing problems are NP-hard and, hence, unlikely to be solvable in polynomial time (Lenstra et al., 1981).

A problem is easy if it has an algorithm with time complexity $O(Y^{\iota})$ for a constant ι, where Y is the size of the problem. Here, $O(Y^{\iota})$ is a function such that:

$$O(Y^{\iota}) = a_{\iota} Y^{\iota} a_{\iota-1} Y^{\iota-1} + \dots + a_1 Y + a_0 \qquad (1)$$

Where, a_i are constants.

Such complexity is said to be of *polynomial order*, and the algorithm is called a polynomial time algorithm. On the other hand, if any algorithm for the problem requires a complexity not bounded by a polynomial function in Y, it is considered to be difficult or intractable. Typical orders that are not polynomial are $O(Y^{\log Y})$, $O(\iota Y)$, and $(Y!)$. Computational complexity theory has provided strong evidence that the solution for some optimisation problems belonging to the NP-class is likely to require a computation time that grows exponentially with problem size. Thus, the attention paid to the study of exact algorithms for NP-hard problems in general is likely to diminish as the size of the problem size increases.

3. WHY SHOULD WE USE HEURISTIC (APPROXIMATE) ALGORITHMS?

As the VRP is an NP-hard problem, the exact algorithms for the solution of VRP can only solve problems of small size (Laporte,1998). In practice, however, algorithms are required to solve difficult problems of large size and cannot be solved in polynomial time. It may not be possible to solve these problems exactly using even the most sophisticated algorithms. In this sense, the importance of approximate (heuristic) algorithms is recognisable. These algorithms can provide near-optimal solutions for large-sized problems in reasonable amounts of computation time and with reasonable storage space requirements.

Heuristic algorithms are, hence, widely used to solve such problems in a reasonable amount of time. Accordingly, for the solution of real-world vehicle routing problems heuristic algorithms are developed along with some exact algorithms for relatively small problems. The size of the problem which can be solved using exact algorithms is severely limited.

A large number of NP-hard combinatorial problems of practical size can only be solved efficiently by heuristic methods, rather than by exact methods. The solution of these problems can only be obtained if all the possible combinations of the decisions and variables are explored. Heuristics play an effective role in such problems by indicating a way forward to reduce the number of evaluations and to obtain solutions within reasonable time constraints. Almost all large-sized instances of combinatorial optimisation problems, such as travelling salesman problem and other routing problems, many kinds of flow and job shop scheduling problems, can be solved effectively by heuristics.

3. 1. Tabu Search

As a local search metaheuristic, initially introduced by (Osman, 1993), Tabu Search (TS) is a metastrategy iterative procedure that can guide almost any type of local steepest-descent research to explore its solution space more widely by building extended *neighbourhood* with particular emphasis on avoiding being caught in a local optimum. A solution's neighbourhood is the set of solutions that can be reached by a *move*. A transition from a feasible solution to a transformed feasible solution is referred to as a move.

Tabu search is based on the general principle of intelligent problem solving and was partly motivated by the observation that human behaviour appears to operate with a random element that leads to inconsistent behaviour given similar circumstances. The resulting tendency to deviate from a charted course might be regretted as a source of error but can also prove to be a source of gain. Tabu search operates in a similar way with the exception that new courses are not chosen randomly. Instead, tabu search proceeds according to the supposition that there is no point in accepting a new solution unless it is to avoid a path already investigated. This insures that new regions of a problem solution space are investigated with the goal of avoiding local minima and ultimately finding the desired solution. Tabu search along with other metaheuristics, such as Simulated Annealing (Osman, 1993), shares the ability to guide the search for iterative improvements. In this context, tabu search provides a guiding framework for exploring the solution space beyond points where an embedded heuristic would become trapped in a local minimum.

3. 2. Game Theoretic Approach for Vehicle Routing Problem

In this research a new approach is proposed to incorporate the risk averseness in hazardous materials routing. As given by (Bell, 2000), in order to represent the rational behaviour in this situation a two-player, zero-sum, non-cooperative game is visualised between a route planner (dispatcher) who tries to find the best possible tour(s) for each vehicle located at a single depot and a virtual system spoiler (evil-entity, demon) who aims at failing one link in the system in order to give maximum possible damage (cost) to the operator. The term 'non-cooperative' implies that the operator is not aware of which link is going to be failed by the evil-entity and conversely the evil-entity has no idea about which links are going to be chosen by the route planner to transport the hazardous material required by the customers.

The mixed-strategy Nash equilibrium (Nash, 1951) is a good tool to obtain the expected routing cost at equilibrium with respect to the link usage and failure probabilities. This equilibrium routing cost highlights the outcome of a mental game in which one of the players is a rational route planner with an extremely pessimistic view about link costs. At the mixed strategy Nash equilibrium, expected trip costs are both minimised with respect to link choice probabilities and maximised regarding link failure probabilities. The game is a Nash-game due to the fact that no player is superior or dominant to the other and no player knows the next action to be taken by the other player.

The routing problem that is specifically addressed in this research is related to shipment of hazardous materials from a depot to some customers through a set of vehicles located at a depot. More specifically, it is assumed that the probability of any vehicle involving in an incident during the shipment is quite low, but the resulting consequence is extremely high. One of the main shortcomings of this problem is that availability and accuracy of the data related to probability of the occurrence of incidents is always questionable. Game theory illustrates how a route planner should determine his routing strategies when the incident probabilities are not known. The extremely pessimistic (hence, risk averse) nature of the planner, on the other hand, ensures the best way of escaping from the risk, as the possible consequences are extremely high.

4. MODELLING CONSIDERATIONS OF RISK AVERSE ROUTING OF HAZARDOUS MATERIALS

To understand the modelling framework of this research it is important to clarify the underlying concept of risk-averseness. In the context of this research, risk-averseness means that the route planner pursues a set of vehicle routes as to minimise his maximum expected loss on the assumption that unknown link failure probabilities will govern the routing of the vehicles in such a way that they are least favourable to him. Hence, in this way the planner can determine his best routes regardless of the failure probabilities of the links. As long as the operator sticks to his equilibrium link choice probabilities, the total expected cost of the system never goes above his equilibrium expected loss regardless of the link failure probabilities. At this juncture it should be mentioned that, these link failure probabilities do not represent the actual failure probabilities (the probabilities of occurrence of incidents, which are unknown), rather they characterise the worst case scenario. Thus, the link failure probabilities of the solution indicate the relative importance of the links as they depict the case where the consequences of a failure are expected to be worst. As the data regarding the actual failure probabilities of the links are rare, the failure probabilities are quite small and unknown for each link. However, one of the main characteristics of the route planner is to be risk averse. Accordingly, he chooses his best set of routes on the basis of the fact that only one link is failed at each iteration, which yields the worst expected consequence. This implies that multiple failures are not considered and, hence, the total link failure probabilities are equal to 1.

The equilibrium solution obtained analysing game-like nature of the problem gives the minimised and maximised expected system cost in terms of link choice probabilities and link failure probabilities, respectively. If either the route operator or the virtual evil-entity deviates from his equilibrium probabilities, this will result in giving a chance to the other side to increase his expected benefit. The optimum assignment of the vehicles to the routes yields a mixed strategy Nash Equilibrium of a non-coperative two player, zero-sum game between a route planner seeking the most reliable (least cost) set of vehicle routes and a virtual network tester with the ability to fail a link. Furthermore, this equilibrium point is known as risk-averse equilibrium as it is based on worst-case link failure probabilities.

The objective function of this problem can be formulised as follows:

$$\text{Min}\,[\text{Max} \sum_{\text{ij}} p_i\ (\text{c}^{\text{f}}_{\text{ij}} q_j + \text{c}^{\text{n}}_{\text{ij}}(1-q_j)\ _{\text{i}})] \qquad (2)$$

subject to

$$\sum_{\text{j}} q_j = 1$$

$$p_i \geq 0 \quad \forall \text{all} \quad \text{links}\, i$$

$$q_j \geq 0 \quad \forall \text{all} \quad \text{scenarios}\, j$$

where;

p_i is the link-usage probability

$\text{c}^{\text{f}}_{\text{ij}}$ is the travel cost on link *i* under scenario *j* when link *i* is failed

q_{j} is the probability of scenario *j*

$\text{c}^{\text{n}}_{\text{ij}}$ is the travel cost on link *i* under scenario *j* when link *i* operates normally

The objective function is maximized with respect to probability of scenarios and minimized with respect to link-usage probabilities.

This mental game starts with the assumption that the operator determines his best routes by considering that each link in the system has the same failure probability. The best routes at this stage are determined through Savings Algorithm. This heuristic algorithm is one of the earliest and perhaps the most widely known algorithm for VRP. It should be pointed out that there is always a possibility that there could be single-customer routes for some individual customers. The saving values for this algorithm are based on the expected costs, calculated by using link usage and failure probabilities, rather than the actual costs.

The vehicle routes obtained from Savings algorithm are improved by using Tabu Search, a metastrategy for guiding known local-search heuristics to overcome local optimality. It consists of exploring the search space by identifying neighbourhood of a given solution, which contains so-called transformed solutions that can be reached in a single iteration. The tabu search algorithm used in the model is inspired from the algorithm developed by (Barbarosoglu and Ozgur, 1999).

A feasible solution for the problem that we described above is represented by $S=(S_1,S_2,S_3,\ldots\ldots S_m)$, where, S_i defines uniquely the subset of customers of $V=(1,2,3,\ldots,k)$ routed by a vehicle i from the set of vehicles $M=(1,2,3,\ldots\ldots,m)$ such that,

$$\bigcup_{i=1}^{m} S_i = V$$

$$S_{i_1} \cap S_{i_2} = \Phi \quad \forall \;\; i_1, \;\; i_2 \;\; \in M$$

$$\sum_{k\in S_i} d_k \le \rho_k \quad \forall \;\; i \in M$$

$$C(S) = \sum_{i\in M} C(S_i)$$

In the light of above explanations and formulations, the steps of the solution procedure to obtain both link usage and scenario probabilities can be set out as below;

- Step1. Initialise q_j for all scenarios j and $\tau \leftarrow 0$
- Step2. Set expected link costs to $\sum_{ij} \mathbf{c}_{ij}^{n}(1-q_j)+\mathbf{c}_{ij}^{f} q_j$ for all links i.
- Step3. Find the initial least expected cost routes using Savings Algorithm and then employ the Tabu Search to improve the initial routes.
- Step4. Determine the auxiliary link usage probabilities $p_i^{aux} \leftarrow 1$ if link i is on the set of best routes, $p_i^{aux} \leftarrow 0$ otherwise.
- Step5. Update link usage probabilities: $p_i^{\tau+1} = p_i^{\tau} + \frac{1}{\tau+1}(p_i^{aux} - p_i^{\tau})$ for all links i
- Step6. Find the scenario j which maximises $\sum_i p_i^{\tau+1} * c_i$, the link cost c_i is equal to c_{ij}^{f} if link i is failed for scenario j, c_{ij}^{n} otherwise.
- Step7. Get the auxiliary scenario probabilities. $q_j^{aux} \leftarrow 1$, for all other scenarios, $q_l^{aux} \leftarrow 0$
- Step8. Update the link failure probabilities: $q_j^{\tau+1} = q_j^{\tau} + \frac{1}{\tau+1}(q_j^{aux} - q_j^{\tau})$ for all scenarios j.
- Step9. $\tau \leftarrow \tau+1$ and go back to Step 2 and repeat all the steps till a satisfactory convergence is obtained.

What this procedure basically provides is that both the route planner (system operator) and the system spoiler (evil-entity) can determine their best strategies in terms of link usage and failure probabilities, respectively, based on the accumulated history of the other player's choice of strategies.

It should be pointed out that the following assumptions prevail the whole solution algorithm described.

1. The links connecting depot and the customers are assumed to be a straight line. Although there may be many alternative paths available between the customers and depot, it is assumed that the shortest straight line between the customers and depot is the link, which is used for hazmat transportation.
2. The characteristics of each link are assumed to be homogenous within each specific link. Accordingly, the probability of occurrence of an incident is assumed to be the same along the link. Characteristics of a road segment including number of lanes, surface quality, traffic volume may affect the probability of occurrence of an incident while the vehicle traveling on that link.
3. Although the actual failure probabilities of the links are unknown, they are deemed to have quite low values (1 million per truck miles). The possible consequences of each failure, however, are extremely high. Hence, the route operator is thought to be risk-averse for deciding the vehicle routes through which hazmats are to be carried.
4. The normal and failed costs of the links are homogenous along the links.
5. The hazardous material is assumed to be carried in repeated shipments from depot to the customers.

In the light of explanations above, the following Figure 1 summarise the calculation steps of the main algorithm developed.

4. 1. Improving the Initial Solution Through Tabu Search

The computational results indicate that the initial solution obtained by using Saving Algorithm is not good enough. Hence, a tabu search procedure has been used to improve the initial solution. During this process, the feasible solutions are improved by obtaining their neighbourhoods**.** In order to intensify the solution improvement in each route locally we use a generation mechanism, which describes how a solution S can be altered to generate another neighbouring solution S'. In the model developed, the neighbourhood solution is generated through a mechanism known as *λ-interchange generation mechanism*, introduced by Osman (1993). Once the neighbouring solutions are obtained through this generation mechanism, it is well-known that these solutions can be improved through local moves based on exchanging edges (Caseau and Laburthe, 1999). *Insertion procedure* is employed to determine the best possible position of the customer to be inserted within the route. The *2–opt exchange procedure* first introduced by Lin (1965) is used to generate a so-called *2-optimal* (2-opt) tours. A tour is called 2-optimal if there is no possibility to shorten the tour by exchanging two links.

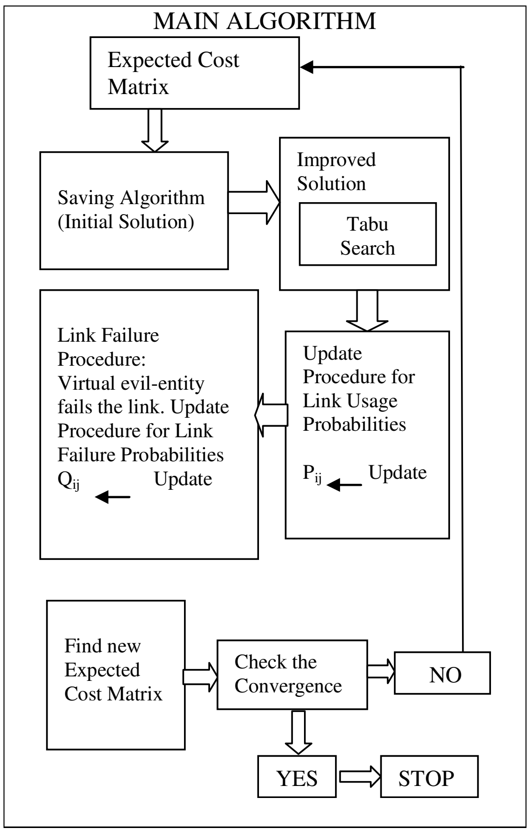

Figure 1. Calculation steps of proposed algorithm.

4. 2. Method of Successive Averages

Once the final solution set of routes is obtained, then the link usage and failure probabilities are updated based on the Method of Successive Averages (MSA). MSA is a flexible and stable solution algorithm. But it may also be costly in terms of the number of iterations needed. Bell and Iida (1997) provide a comprehensive description of this method, which is employed in the algorithm.

5. LINEAR PROGRAMMING FORMULATION

As stated by Equation 2, the objective function of the problem is the minimisation and maximisation of the total expected routing cost in terms of link usage probabilities and link failure probabilities, respectively. Equation 2 can be reformulated with respect to routes, rather than links, as;

$$\text{Max (Min } \mathbf{C} = \sum_{i,j} h_i \mathbf{c}_{ij} q_j \ h\text{)} \qquad (3)$$

subject to;

$$\sum_i h_i = 1$$
$$\sum_j q_j = 1$$
$h_i \geq 0$ for all routes i
$q_j \geq 0$ for all scenarios j

where;

$\mathbf{C}$ is the total expected routing cost
$\mathbf{c}_{ij}$ is the cost of route i under scenario j
h_i is the probability of route i is chosen
q_j is the probability of scenario j

The linear programming formulation of this maximin problem is given by (Bell, 2000) as follows:

Let $\mathbf{C}^*$ be the solution to the problem. Then;

$$\sum_i \mathbf{c}_{ij} h_i \leq \mathbf{C}^* \text{ for all scenarios } j$$

The following linear program is proposed in order to find out the optimal mixed routing strategy for vehicles carrying extremely dangerous hazardous materials.

Min M with respect to h_i subject to
$$\sum_i \mathbf{c}_{ij} h_i - M \leq 0 \text{ for all scenarios j} \qquad (4)$$
$$\sum_i h_i = 1$$
$h_i \geq 0$ for all routes i

In the same way, the optimal mixed link failure strategy for the evil entity is given by following linear program.

Min M^* with respect to q_j subject to

$$\sum_j \mathbf{c}_{ij} q_i - M^* \geq 0 \quad \text{for all scenarios } j \qquad (5)$$

$$\sum_j q_j = 1$$

$q_j \geq 0$ for all scenarios j

It should be pointed out that the route choice probabilities obtained from Equation. 4 and link failure probabilities obtained from Equation. 5 are dual variables. In other words, the dual variables of Equation. 4 with the constraints are equal to the scenario probabilities q_j of Equation.5 with constraints given. At the solution, M is equal to M^*. The solution of this linear programming problem is unique in M ,hence M^*, but not in general unique in h_i or q_j.

6. NUMERICAL CALCULATIONS

Some related problems are solved using the algorithmic model described to illustrate the effectiveness of the solution procedure. Since, there is no data available from the real world cases, the solution algorithm is tested using some hypothetical problems. As the solution process reflects a mental game between the route planner and virtual network tester, the obtained results should be regarded as the possible outcomes of this mental game.

The problems that are dealt with involve one depot and a number of customers with known demands. The demands are serviced by the vehicles with a fixed capacity located at the depot. The shipment costs of the links are represented by two matrices: normal cost matrix, and failed cost matrix. Normal cost matrix illustrates the case when the links operate normally. Failed cost matrix, however, indicates the case when an incident occurs on the link during the shipment. These failed costs are assumed to be catastrophic costs, i.e. the resulting cost to the community is higher than a predetermined acceptable level.

In order to illustrate the quick convergence of the expected trip cost, a problem involving 5 customers, one depot and one vehicle is tested using the proposed algorithm .The cost matrixes of the problem are given by Table 1 and Table 2 below.

As the minimum cost routes are obtained by enumerating all possible routes and selecting the one with minimum trip cost, the resulting routes are the absolute minimum cost tours. The Figure 2 below illustrates the convergence behaviour of the expected trip cost.

Table 1. Normal cost matrix.

	0	1	2	3	4
1	11				
2	8	17			
3	10	15	11		
4	13	10	17	15	
5	15	16	18	17	16

Table 2. Failed cost matrix.

	0	1	2	3	4	
1	20					
2	21	24				
3	20	33	25			
4	22	33	22	25		
5	24	30	27	30	27	5

As the minimum cost routes are obtained by enumerating all possible routes and selecting the one with minimum trip cost, the resulting routes are the absolute minimum cost tours. The Figure 2 below illustrates the convergence behaviour of the expected trip cost. The link usage and failure probabilities of this problem are illustrated by Figure 3 and Figure 4, respectively. In this problem, there are 15 link usage probabilities and also 15 link failure probabilities. Those whose usage and failure probabilities are bigger than zero are selected and their convergence behaviour is presented. The Figure 3 reveals the fact that the sum of the link usage probabilities taken over all links that enter or leave any node sums to 2. For example, the values of these probabilities for links 0-2, 2-3, 2-4, 2-5 sum to 2. On the other hand, the Figure 4 illustrates the fact that the value of the total link failure probabilities is 1. Convergence of link usage probabilities is slow. This is believed to be because of the nature of the non-uniqueness of these probabilities. The same equilibrium expected trip cost may be given by different set of link usage and link failure probabilities. Another reason could be related to the MSA procedure that we employed to update the link-related probabilities. As we mentioned previously, although MSA procedure is quite simple and effective, it may cause the convergence of the link usage and failure probabilities, expected trip cost as well to some extent, to be slow. Thereby, another solution method, rather than MSA, could result in a quicker convergence with regard to these probabilities. The following figure illustrate the convergence of the link failure probabilities of the given problem.

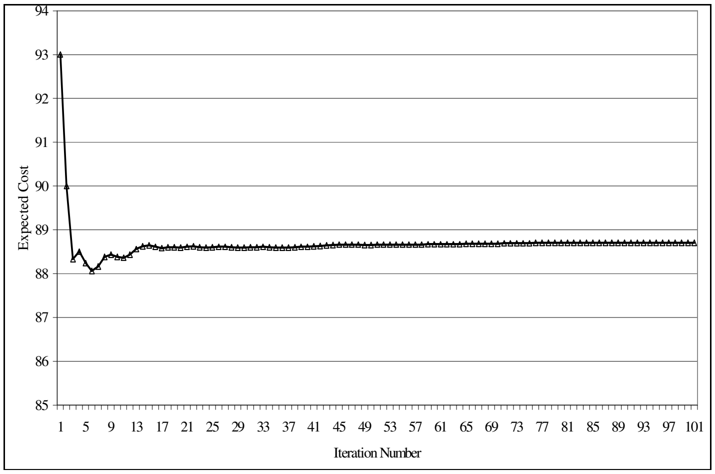

Figure 2. Convergence of total expected trip cost.

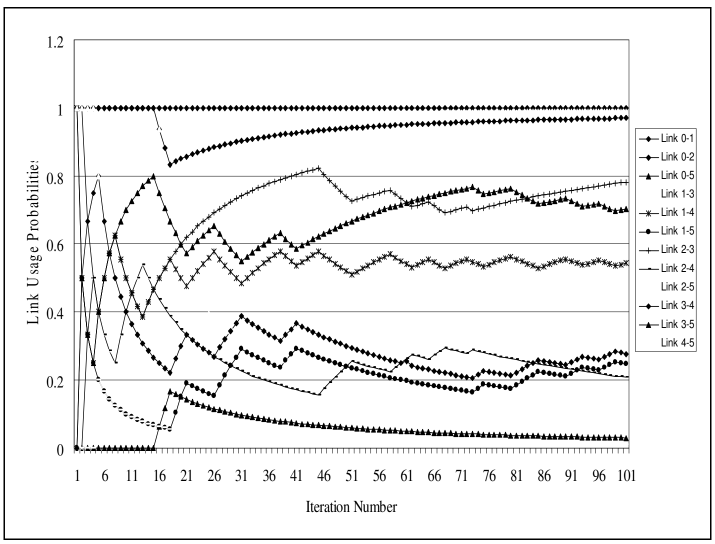

Figure 3. Convergence of link usage probabilities.

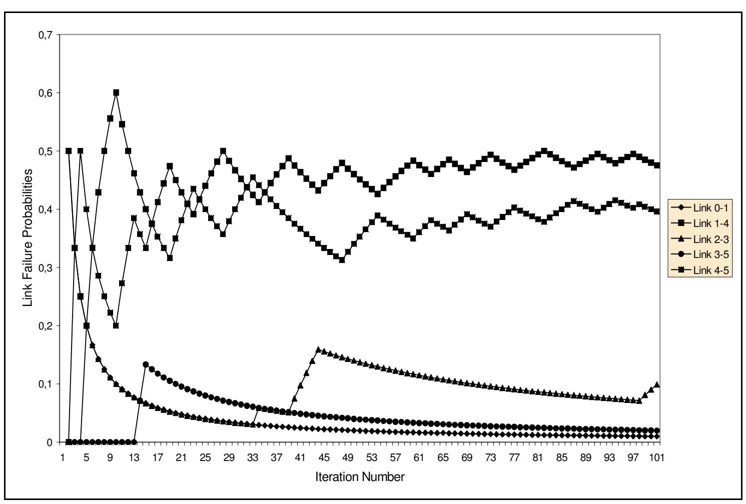

Figure 4. Convergence of link failure probabilities.

The above problem was also solved by using linear programming solution in order to determine the exact values for the link usage and link failure probabilities. It should , however, be pointed out that this method is suitable for only small size problems as it would be an extremely hard job to determine the all available actual routes. This is the main motivation for why heuristic approach is employed for the solution procedure of the developed algorithm.

This solution procedure is implemented to determine exact values for the link usage probabilities in the 5-customer problem, in which complete 120 different routes are available, in order to compare with the values obtained by the iterative MSA method and to establish whether there are multiple optima hence whether there are multiple optima.

In order to employ this method, let us represent the Equation 4 in detail for the problem with 120 different routes.

Min *M;*

$$CR_{1_1} h_1 + CR_{2_1} h_2 + ... + CR_{120_1} h_{120} \le M$$

$$CR_{1_2} h_1 + CR_{2_2} h_2 + ... + CR_{120_2} h_{120} \le M$$

$$CR_{1_3} h_1 + CR_{2_3} h_2 + ... + CR_{120_3} h_{120} \le M$$

$$CR_{1_{15}} h_1 + CR_{2_{15}} h_2 + ... + CR_{120_{15}} h_{120} \le M.. \qquad (6)$$

subject to

$$h_1 + h_2 + h_3 + h_4 + ... + h_{120} = 1$$

$$h_1, h_2, h_3, h_4, ..., h_{120} \ge 0 \qquad (7)$$

In order to solve this problem we employed E04MFF/E04MFA computer program. This program was designed to solve linear problems of the form;

$$\underset{x \in F^N}{\text{minimise}} \quad c^T x, \quad \text{subject to} \quad l \le \left\{ \begin{matrix} x \\ Ax \end{matrix} \right\} \le u$$

where c is n element vector and A is an m_L by n matrix.

The resulting route choice probabilities, obtained using this program through possible combination of different 120 routes, are given by the Table 3 below. To be more precise, first route (R_1) is, for example, 0-1-2-3-4-5-0 and the second route (R_2) is 0-1-3-2-4-5-0, and so on. Starting from the top left corner towards right, each cell represents these possible routes.

Table 3. Route usage probabilities: Linear programming.

0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.55	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.24	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.21	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

As can be seen from this table, there are only 3 different routes out of 120 possible routes to be used in order to make sure that the selected routes provide the operator with the minimum total expected routing cost regardless of the links to be failed by the evil entity which tries to maximise the total travel cost. Table 3 indicates that Route18, Route 34 and Route42 are used to deliver the required hazardous materials from the single depot to the customers. These routes consist of the following links;

Route18: 0-1-4-5-3-2-0 $h_{18} = 0.55$

Route34: 0-2-3-4-5-1-0 $h_{34} = 0.24$

Route42: 0-2-4-5-3-1-0 $h_{42} = 0.21$

The following Table 4. illustrates the comparison values for the link usage probabilities obtained from both MSA method and Linear Programming.

Table 4. Comparison of link usage probabilities.

	Linear Programming	Heuristic Solution
Link 0-1	1	0.9703
Link 0-2	1	1
Link 0-5	0	0.0297
Link 1-3	0.21	0.2376
Link 1-4	0.55	0.5446
Link 1-5	0.24	0.2475
Link 2-3	0.79	0.7822
Link 2-4	0.21	0.2079
Link 2-5	0	0.0099
Link 3-4	0.24	0.2772
Link 3-5	0.76	0.703
Link 4-5	1	0.9901

These values indicate that the slow convergence of the link usage probabilities, although the MSA method resulted in quite close to the optimum values after 100 iterations, is basically due to the non-uniqueness of the link usage probabilities. The total expected travel cost yielded by MSA method is 88.70 unit (See Figure 2). The Linear programming solution gives this value as 88.71 unit. As a result, we can state that the MSA method is an effective and reliable method for the solution the problem formulated in the model developed.

7. CONCLUSIONS

Convergence of link usage and link failure probabilities is slow. This is believed to be because of the nature of the non-uniqueness of these probabilities. The same equilibrium expected trip cost may be given by different set of link usage and link failure probabilities. Another reason could be related to the Method of Successive Averages (MSA) procedure that we employed to update the link-related probabilities. Although MSA procedure is quite simple and effective, it may cause the convergence of the link usage and failure probabilities, expected trip cost as well to some extent, to be slow. Thereby, another solution method, rather than MSA, could result in a quicker convergence with regard to these probabilities.

The obtained expected trip cost states that as long as the route operator sticks to his equilibrium link usage (choice) probabilities, this cost would be higher than this value under no scenario. In other words, whatever the link failure probabilities are, the total expected trip cost would not be higher than this expected trip cost if the link usage probabilities at this equilibrium point are in use. Thus, the choice of routes is based on the link failure probabilities. The corresponding equilibrium expected trip cost, hence, may be interpreted as the result of best routing strategy based on the worst set of link incident probabilities assuming that an incident occurs. This routing strategy clearly reflects the extremely pessimistic and hence risk-averse nature of the route planner.

The convergence behaviour of the expected trip cost is also dependent on the randomly selected customers during the tabu search for the improvement of the initial routing solution. If the customers selected randomly are those whose inclusion or deletion would result in best possible route improvement, then relatively better routes are obtained. This, thereby, would cause the convergence to be relatively quicker to those random customers whose insertion and deletion does not give such an improvement.

8. REFERENCES

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