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SURFACE CHARGE DENSITY ON DEVIATIONS FROM THE PRINCIPAL NORMAL LINE

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ABSTRACT

In this paper, we emphasize and make corrections on some matter asserted in an earlier work by Yuan Zhong Zhang. Zhang (Zhang, 1988) says that Enze's equation (Enze, 1986) of the phenomenon called "tip" effects is not correct. Here we correct the inconvenience of the equations in the matters mentioned above and the obtained results are easily presented numerically by a table.

Keywords: Surface Charge Density, Baundary Value Problem

YÜZEY NORMALÝNDEN OLAN SAPMALARA GÖRE YÜZEY YÜK YOÐUNLUÐU

ÖZET

Bu makalede daha evvel Yuan Zhong Zhang tarafýndan ileri sürülen bazý meseleler üzerinde durulmuþ ve bazý düzeltmeler yapýlmýþtýr. Zhang, Enze'nin "uç" etkisi denkleminin doðru olmadýðýný söyler. Bu çalýþmada denklemde söz konusu olan uygunsuzluklar düzeltilmiþtir. Bulunan sonuçlar tablo ile verilen nümerik deðerlerde kolayca görülmektedir.

Anahtar Kelimeler: Yüzey Yük Yoðunluðu, Sýnýr Deðer Problemi

1. INTRODUCTION

This paper deals with a differential equation for the electric and geometric features of an electrostatic field and make some corrections about the inconveniences on some points of this subject.

2. THEORY

The Laplacian of a scalar function V can be expressed in terms of curvilinear coordinates as,

$$\nabla^2 V = \frac{1}{h_1^2}\frac{\partial^2 V}{\partial u_1^2} + \frac{1}{h_2^2}\frac{\partial^2 V}{\partial u_2^2} + \frac{1}{h_3^2}\frac{\partial^2 V}{\partial u_3^2} \qquad (1)$$

where u_i, i=1,2,3, are curvilinear coordinates, x=x(u_i), y=y(u_i), z=z(u_i) are rectangular coordinates and h_i are scale factors (Spiegel, 1953).

For $h_1 = h_2 = h_3 = 1$, one finds a rectangular coordinates, for h_1=1, h_2=r(0), h_3 = r(0) Sinθ, 0⟨θ⟨$90^{0.}$ One obtains a spherical coordinates with radius r(0). To get a cylindrical coordinates one takes $h_1 = h_2 = 1, h_3 = \rho(0)$, where $\rho(0)$ is a cylindrical radius.

Enze's equations are:

$$k_x = (\frac{\partial^2 V}{\partial x^2}) / (\frac{\partial V}{\partial n}),$$

$$k_y = (\frac{\partial^2 V}{\partial y^2}) / (\frac{\partial V}{\partial n}),$$

$$\nabla^2 V = (k_x + k_y)\frac{\partial V}{\partial n} + \frac{\partial^2 V}{\partial n^2}$$

where n is normal line to the surface. Similarly, one may get

$$k_1 = (\frac{\partial^2 V}{\partial u_1{}^2})/(\frac{\partial V}{\partial u_3}),$$

$$k_2 = (\frac{\partial^2 V}{\partial u_2{}^2})/(\frac{\partial V}{\partial u_3}),$$

$$\nabla^2 V = (\frac{k_1}{h_1{}^2} + \frac{k_2}{h_2{}^2})\frac{\partial V}{\partial u_3} + \frac{1}{h_3{}^2}\frac{\partial^2 V}{\partial n^2} = 0 \qquad (2)$$

$$2C\frac{\partial V}{\partial u_3} + \frac{\partial^2 V}{\partial u_3{}^2} = 0$$

$u_3 = n$ where and so we get

$$2C\frac{\partial V}{\partial n} + \frac{\partial^2 V}{\partial n^2} = 0 \qquad (3)$$

where C corresponds to the Enze's average curvature $k = \frac{1}{2}(k_x + k_y)$ but we find the curvature differently as

$$2C = (\frac{k_1}{h_1{}^2} + \frac{k_2}{h_2{}^2})h_3{}^2. \qquad (4)$$

Mc Allister (1988) gives the equation of an homogeneous medium for the electric field intensity E as ε is dielectric constant.

$$\frac{dE}{dn} + 2CE + \frac{E}{\varepsilon}\frac{d\varepsilon}{dn} = 0 \qquad (5)$$

3. DISCUSSIONS AND RESULTS

As the electric field intensity $E = -\frac{dV}{du_3} = -\frac{dV}{dn}$ from eq.(3) we have a first-order ordinary differential equations:

$$\frac{dE}{dn} + 2CE = 0 \qquad (6)$$

which gives eq.(5) for dε=0 but here the expression related curvature is also given with eq.(4) which is a different form of Enze's curvature.

For example, in spherical coordinates, taking $G(0)=1/r(0)$ and $r=r(0)+\Delta r$, where $r(0)$ and Δr are the radius on the point of the surface of the conductor and the increment of distance from the point of interest, respectively. One may get

$$2C = \frac{[r^2(0)+1]\mathrm{Sin}^2\theta}{r} = \frac{A}{r} = \frac{A}{r(0)+\Delta r}$$

$$\frac{2C}{A} = \frac{1}{r(0)+\Delta\theta} = \frac{G(0)}{1+G(0)\Delta r},$$

$$A = [r^2(0)+1]\mathrm{Sin}^2\theta \qquad (7)$$

which implies McAllister's curvature. Here r is dynamic radius. Solution of eq.(6) for C in eq.(7) gives the normalised electric field intensity,

$$E_1 = \frac{E}{E(0)} = \frac{\varepsilon_1}{\varepsilon_2}(\frac{r(0)}{r(0)+\Delta r})^A$$

where E and E(0) are the field intensities at the near and on the surface of the conductor; ε_1 and ε_2 are the dielectric constant, respectively.

For $\varepsilon_1 = \varepsilon_2$, getting r(0)=1/G(0)=10, r=0.01 one obtains the following values table (E2 and E3 are McAllister's and Enze's results, respectively):

Table 1. Resulsts for some angles:

Angles (degrees)	E1 present study	E2 $[1-2G(0)\Delta r]$	E3 $[\mathrm{Exp}(-2G(0)\Delta r)]$
2	0.9998771	0.9980020	0.9980019
4	0.9995089	0.9980020	0.9980019
6	0.9988975	0.9980020	0.9980019
8	0.9980464	0.9980020	0.9980019

With respect to the above table, Enze's and McAllister's results do not give the effects of the different deviations from the principal normal line (z axis), by being constant, although the direction of the principal normal line of the surface of the conductor is, in general, different from that of the equipotential surface conductor. Whereas, my present results give the effects of the deviations from the normal line, by varying values.

Hence, the present approach involves a more general and sensitive results.

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