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# Optimization Model of Credit Strategy of Commercial Bank

## ABSTRACT

Economic and mathematical model of credit management of commercial banks is developed. The model allows obtaining optimal strategies of variant calculations of this kind of banking services. The procedure of quantitative solution of this problem is substantiated to help to make optimal decisions in reference to allocation of assets.

**Key words:** bank, deposit, credit, loan portfolio, model and optimal management.

Successful functioning of the bank depends not only on workers' skills, their knowledge and experience, but also on the optimization of the whole process of making and implementation of management decisions.

Modeling of bank financial activity is very important and difficult task, as the bank is a system in which deterministic and random processes take place simultaneously and that are interrelated by very complicated factors. Furthermore, subjective management decisions are essential in bank activity. However, a bank interacts with the financial markets in the process of functioning, which are difficult to model. All these facts point at considerable complexity of creating an analytical model of bank financial activity, which can be used in practice.

K. Sealy, E. Baltensperger, H. Markowitz, M. Kleim, N. Murphy, N. Egorova, A. Smulov, and others drew on the problem of modeling of optimal loan portfolio. However, one of the most important tasks of commercial bank in the field of financial activity is to balance between profitability and risk and to find optimal combination in the form of trade-offs.

Based on the classic approach of modeling of credit portfolio, we model optimal structure of commercial bank loan portfolio.

The purpose of this study is the use of existing models and the development of applied economic and mathematical model for finding profitable credit strategies of commercial banks in Ukraine and abroad.

Commercial bank may carry out credit, investment and other active operations only within available financial resources. Deposits are principal source of financial resources of commercial bank; they determine the scope and range of profitable operations of the bank.

The main management instrument of credit activity is economic and mathematical methods and models.

The prerequisite of building economic and mathematical model of a bank loan portfolio is the following conditions:

- 1) the timing of the credit arrangement;
- 2) the calculation of annual and monthly crediting rates;
- 3) the determination of the conditions for loan repayment;
- 4) the determination of the home equity sum that can be used for the crediting process;
- 5) the determination of the risk magnitude as the probability of default of all types of credits and the calculation of default risk by periods;
- 6) the use of current percent of credit repayments in the crediting process;
- 7) the determination of the period, at the beginning of which the calculation of profits is carried out;
- 8) Taking into account the liquidity ratios (H4, H5 and H6) that regulate the credit activity of a commercial bank.

We denote the scope of granted credits through  $x_{it_i}$  ( $i=\overline{1, n}$ ;  $n=18$ ),  $t_i$  – a period of time – a month for the  $i$ -type credit  $t_i \in T_i$ ,  $T_i$  – the set of periods for the  $i$ -type credit, a quarter;  $(a_{it_i})$  – a matrix that shows the process of providing the  $i$ -type credit in the period  $t_i$  or the lack of it:

$$a_{it_i} = \begin{cases} 1, & \text{if } i\text{-type credit is given in period } t_i; \\ 0, & \text{if } i\text{-type credit is not given in period } t_i; \end{cases}$$

Highly liquid investments of customers – demand deposits – are used in order to provide short-term liquidity for lending short-term credits (up 1 month).

Bank funds and term deposits are used for lending other types of credits.

The bank's ability to pay its current obligations from the assets of the primary and secondary liquidity is determined by the current liquidity H5, which value for a given period for commercial banks is 0.4. At present, current obligations of the bank can be more than 2.5 times bigger than its current assets.

In order to organize the efficient crediting process with the help of rapid accumulation of funds, credits with the same expiration dates have different interest rates and different conditions of return. Some are traditionally paid, i.e. monthly interest, and the principle

amount of the credit is at the end of the term; the others are returned with monthly interest and part of the principle amount of the credit, which is defined as following. Principle amount of the credit is divided into equal parts for the whole credit term.

We accept that funds received from loan interest of the previous period, and refunded credits are used for credit arrangement in the next period.

We introduce the following notation:  $r_i$  – monthly interest rate for i-type credit;  $\tau_i$  – the period for which the i-type credit is granted;  $S_t$  – the amount of payments received from credits granted with special conditions in the period  $t$ ;  $SK$  bank stock;  $p_i$  – probability of default of i-type credit;  $u_{it}$  – the risk value of default of i-type credit in the period  $t$ , it is calculated as following:  $t=1, u_{i1} = p_i$ ;  $t=2, u_{i2} = p_i(1-p_i)$ ;  $t=3, u_{i3} = p_i(1-p_i)^2$ ;  $D_{jt}$  – the amount of available deposit funds of j-type in the period  $t$ ,  $m$  – the number of deposit types on lead time on investment;  $z_j$  – monthly percentage rate on deposits payment of j-type.

Taking into account the conditions described above and introduced notation, we form the economic and mathematical model of the problem.

The target function of the problem is the highest income yield at the beginning of the next period:

$$Z = \sum_{t_i=1}^{T_i} \sum_{i=1}^n r_i x_{it_i} - \sum_{t=1}^T \sum_{j=1}^m z_j D_{jt} \rightarrow \max, t_i \in T_i$$

Accomplishing the following conditions:

1 Balance conditions in the primary period of crediting process  $t = 1$ :

$$\sum_{i=1}^n x_{i1} = SK + \sum_{j=1}^m D_{j1}$$

2 Balance conditions in the secondary period of crediting process  $t = 2$ :

$$\sum_{i=1}^n x_{i2} + \sum_{j=1}^m z_j D_{j1} + D_1 \leq \sum_{j=1}^m D_{j2} + \sum_{i=1}^n r_i x_{i1} + S_1$$

$$S_1 = x_{1,1} + \frac{x_{1,3}}{\tau_3} + \frac{x_{1,5}}{\tau_5} + \frac{x_{1,7}}{\tau_7} + \frac{x_{1,9}}{\tau_9} + x_{10,1} + \frac{x_{12,1}}{\tau_{12}} + \frac{x_{14,1}}{\tau_{14}} + \frac{x_{16,1}}{\tau_{16}} + \frac{x_{18,1}}{\tau_{18}}$$

3 Balance conditions in the third period of crediting process  $t = 3$ :

$$\sum_{i=1}^n x_{i3} + \sum_{j=1}^m z_j D_{j2} + D_2 \leq \sum_{j=1}^m D_{j3} + \sum_{i=1}^n r_i x_{i2} + \sum_{i=2}^n r_i x_{i1} + S_2$$

$$S_2 = x_{1,2} + 2 \cdot \left( \frac{x_{1,3}}{\tau_3} + \frac{x_{1,5}}{\tau_5} + \frac{x_{1,7}}{\tau_7} + \frac{x_{1,9}}{\tau_9} \right) + x_{10,2} + 2 \cdot \left( \frac{x_{12,1}}{\tau_{12}} + \frac{x_{14,1}}{\tau_{14}} + \frac{x_{16,1}}{\tau_{16}} + \frac{x_{18,1}}{\tau_{18}} \right)$$

4 Restrictions for short-term and highly liquid credits:

$$x_{11} + x_{10,1} \geq D_{11}$$

$$x_{12} + x_{10,2} \geq D_{12}$$

$$x_{13} + x_{10,3} \geq D_{13}$$

5 Restrictions for the scope use of borrowed funds in the total amount of credits:

$$\frac{\sum_{j=1}^m \sum_{t=1}^3 D_{jt}}{\sum_{i=1}^n \sum_{t_i=1}^3 x_{it_i}} < 1;$$

6 Restrictions accounting the implementation of current liquidity standards by periods:

$$\sum_{i=1}^{18} x_{i1} \geq 0,4 \sum_{j=1}^5 D_{j1}$$

$$\sum_{i=1}^{18} x_{i2} \geq 0,4 \sum_{j=1}^5 D_{j2}$$

$$\sum_{i=1}^{18} x_{i3} \geq 0,4 \sum_{j=1}^5 D_{j3}$$

7 Restrictions for size of average risk of credit defaults:

$$\frac{\sum_{i=1}^n \sum_{t_i=1}^{T_i} u_{it} \cdot x_{it_i}}{\sum_{i=1}^n \sum_{t_i=1}^{T_i} x_{it_i}} \leq U^*$$

8 Restrictions that determine the marginal sum of volume of credit:

$$\sum_{i=1}^n \sum_{t_i=1}^{T_i} x_{it_i} \leq K_0$$

9 Restrictions for nonnegative variables:

$$x_{it_i} \geq 0, i = \overline{1, n}; t_i = \overline{1, T_i}$$

The given economic and mathematical model enables obtaining the optimal scheme of the crediting process, which is divided monthly, that allows us to trace the funds movement, calculate idle balances and direct them to acquire marketable instruments, provide short-term interbank credits and deposit in another bank.

Since the model is designed for one quarter (3 months), bankers can orientate themselves in the current situation, summarize their activities per quarter, and make adjustments in the subsequent period (it may be a change of rates and credit conditions, which is important in a competitive environment and economic instability). It should be noted that the optimization model is quite flexible. Additional restrictions can be added to it, with the help of which the current situation of credit and deposit activities is modeled. For example, these may be the restrictions accounting known volume of credits at the beginning of the crediting period, the current values, and necessary reserve funds. If the desirable amount of profit is noted in the target function, then we obtain necessary scheme of credit allocation for periods and types. In addition, if necessary, you can change the amount of the deposit incomes.

We use software application package EXEL to obtain numerical solutions of constructed model.

Optimization algorithm indicates the direction of the search; conjugate gradient method is selected, which is used to solve big optimization problems. During the solving of the problem with this search method, the large number of iterations is performed; this gives the possibility to obtain accurate results.

At the end of the period (at the beginning of the first month of the next quarter) maximum profit and the scheme of credit arrangement on volume, types and periods is obtained.

Quantitative analysis of different scenarios provides a selection of profitable option for the bank, the estimate of the probability of a bad situation and makes it possible to develop an adequate plan of action. Using simulation models one can evaluate and analyze alternative scenarios of profit and risk management. In the process of optimization modeling, banks can develop their own models that account the specificity of their activity the most closely or use already created ones.

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