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Reliability based robust design optimization based on sensitivity and elasticity factors analysis

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ABSTRACT

In this paper, a Reliability Based Robust Design Optimization (RBRDO) based on sensitivity and elasticity factors analysis is presented. In the first step, a reliability assessment is performed using the First-and Second Order Reliability Method (FORM)/ (SORM), and Monte Carlo Simulation. Furthermore, FORM method is used for reliability elasticity factors assessment, which can be carried out to determine the most influential parameters, these factors can be help to reduce the size of design variables vector in RBRDO process. The main objective of the RBRDO is to improve both reliability and design of a cylindrical gear pair under uncertainties. This approach is achieved by integration of two objectives which minimize the variance and mean values of performance function. To solve this problem a decoupled approach of Sequential Optimization and Reliability Assessment (SORA) method is implemented. The results obtained shown that a desired reliability with a robust design is progressively achieved.

1 Introduction

Gears are used in almost all mechanical devices and they can do several important works, but more importantly, they provide a gear reduction. This is vital to ensure sufficient power and to get a sufficient torque.

In the area of the optimum design of gears, several research studies are developed in regard to analysis and deterministic design optimization of gear boxes [1-3]. Furthermore, the advanced optimization technique, Genetic Algorithm (GA) is used to find the optimal combination of design parameters for minimum weight of gears [4-5].

In contrast, very little work has been done in the field of probabilistic optimization design of gears; although reliability based design methods are becoming quite well mastered and are much applied to obtain robust designs of gear. In reality, uncertainties are inherent in gearing [6-7], such as those in random dimensions (tolerances), random clearances, random deformations of structural components, gear materials and heat treatment, working conditions, etc. The ignorance or mistreatment of the uncertainties during a mechanism design may result in significant kinematic and dynamic errors and are

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therefore found to have critical effect on the final rating of gears. Huang and Sen proposed in [8] a practical analytical method to determine the kinematic accuracy reliability of gear mechanisms and an analysis of the kinematic accuracy reliability of the gear mechanism, the accuracy and efficiency of the proposed method were analyzed. Zhang and Qiaoling employed techniques from perturbation method to present a useful practical program that can be used to obtain the reliability-based design parameters of gear pairs accurately and quickly [9]. Furthermore, based on reliability design theory, Yang and Zhang used the Edgeworth series method and the sensitivity analysis method to study the reliability sensitivity of the cylindrical gear pair with non-Gaussian random parameters [10]. These later are extensively discussed and a numerical method for reliability sensitivity design is proposed. The variation regularities of reliability sensitivity are obtained and the effects of design parameters on reliability of cylindrical gear pair are discussed. Based on the computation method of reliability in limit state theory an analysis of sensible random factors that influence gear reliability is analyzed in [11]. Recently, Shi and Yang [12] studied the limit state function contact strength of gear tooth surface obtained by response surface method. In this work, Monte Carlo reliability sensitivity analysis method is used to study the probability sensitivity of each random variable to the limit state function. As a result, sensitivity information of the reliability of the contact strength of the tooth surface with various random factors in the thermo elastic coupling condition is obtained.

Taking account uncertainties is an indispensable condition for optimal and robust design of gears; Gautham and Gupta [13] proposed a robust design method of gears with material and load uncertainties, which eliminate the safety factor and reliability factor used in AGMA based design procedure. The method is illustrated by an example of automotive gear design. Du and Chen [14] integrate (SORA) method in reliability and robust design assessment for a speed reducer, Chen [15] propose an optimal shifting vector (OSV) approach for efficient probabilistic design to enhance the efficiency of RBDO for the same example of speed reducer.

As can be concluded from previous works and other published researches, that different methods of reliability based design optimization have been used to optimize gear design, other studies have presented a gear reliability sensitivity analysis, and very little works have been made a robust design for gears model.

The present paper, prepared in this context, aims to perform a (RBRDO) based on elasticity factors analysis for a cylindrical gear pair. This analysis is done for three modes of failure occurring generally during the operation of gear pair, namely tooth pinion/wheel bending stresses and gear pair contact stress, considering uncertainties on all gear ISO 6336 parameters except the transmission ratio.

It should be noted that elasticity with respect to overage of each variables give an information about sizing to keep, but elasticity to standard deviations guides for quality control to avoid dispersion. This elasticity analysis leads to specify the random parameters that have the highest elasticity factors, only these variables have been chosen to perform variations in their statistics to improve reliability and get a more robust design.

Despite its important in reliability improvement, elasticity have not be made for gear studies, so we employed FORM method to made an elasticity analysis, and identify the parameters that play more important role than others in terms of reliability in practice. Furthermore, based on these results, a (RBRDO) using decoupled method (SORA) is done to calculate the optimum values of these critical parameters to achieve a desired reliability and robust design. The main idea of SORA is to reformulate the RBRDO problem into a sequence of deterministic optimization and reliability assessment; the reliability assessment is performed by an inverse MPP search algorithm based on performance measure approach (PMA) developed in [16].

The study consists of four parts. The first part covers a reliability calculation with respect of three failure scenarios that are based on tooth pinion/ wheel root stresses and tooth surface contact stress using the gear calculation ISO 6336 procedure. All variables are considered as random except the transmission ratio is taken a deterministic one. All distribution parameters are assumed to be known. The results of this part are then compared with others coming from different references [9-10].

The second part consists of an assessment of reliability elasticity factors with respect of each random variable parameters (mean, standard deviation) for both failure modes (the bending stresses of the pinion, and the Hertzian tooth contact stress), whose system failure probability is estimated by approximated methods: FORM/SORM and Monte Carlo simulation method, both methods are implemented in the reliability engineer Software (PHIMECA). Results of this section are then used to improve reliability and robustness of gear pair. In the third part, (RBRDO) is employed to optimize only the variable statistics that have higher elasticity factors to reduce the size of design variable vector. To solve this problem SORA is adopted, this optimization process is carried out and performed using MATLAB 2017a command ‘‘fmincon’’ with active set method.

Finally, the confirmation of results will be tested by the calculation of reliability sensitivities with respect to the input random variables means of gear pair.

2 Reliability based robust design optimization steps

Reliability Based Robust Design is a robustness that can be translated directly into a design criterion, a robust design is one that is least sensitive to the change in the statistics of input random variables (such as the mean, standard deviation and type of distribution) within acceptable range of cost [17].The main steps of Reliability Based Robust Design Optimization (RBRDO) are presented in Fig.1.

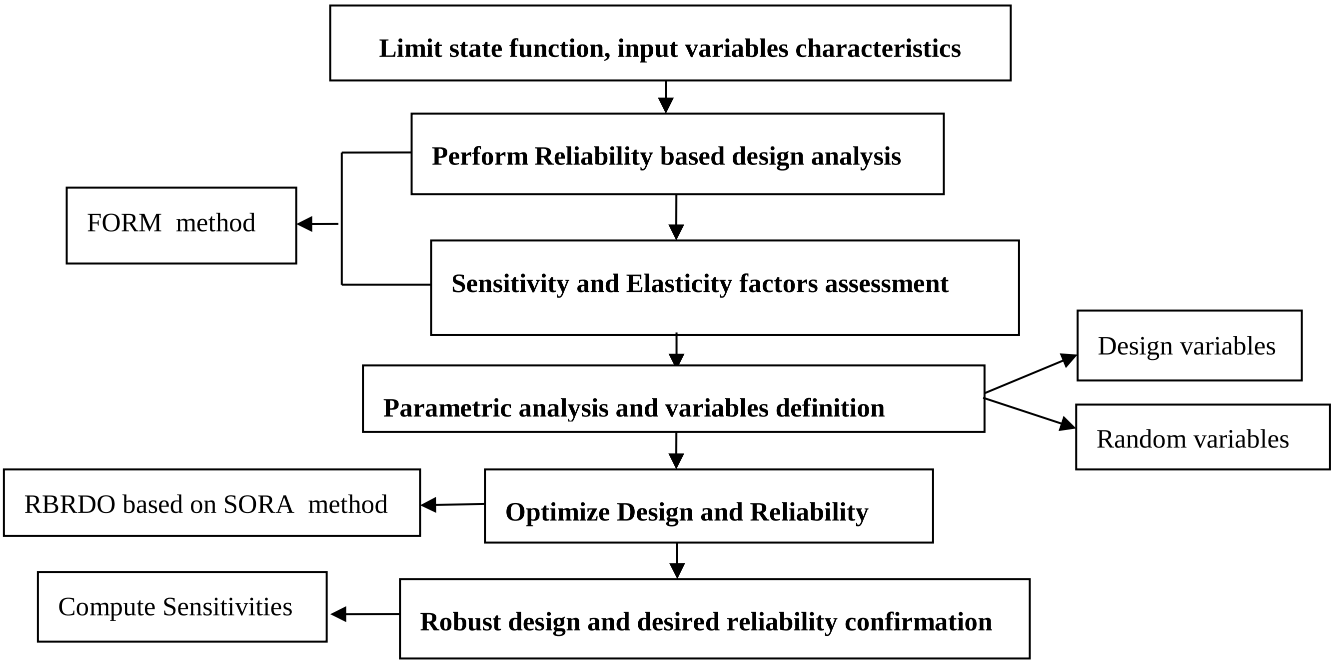

Fig.1- Flowchart of the proposed approach for the RBRDO based on reliability elasticity factors results

2.1 Reliability based design

Thus structural reliability analysis requires a definition of failure function G(x) and failure criterion. Generally, the main objective of reliability based design is to finding the most probable point MPP, when the MPP search is an iterative optimization problem presented by equation (1).

$$\begin{cases} \min \beta = \|u\| \\ \text{under the constraint } H(u_i) = 0 \end{cases} \quad (1)$$

The calculation of reliability index β for each failure scenarios $G_i(x)$ using approximation methods: FORM has been described in [18-19] and SORM by Breitung formula [20-21]. The principle of β calculation consists in determining the distance between the origin of the standard space and the limit state $H(u_i) = 0$

The reliability index is defined by equation (2):

$$\beta = \frac{\mu_G}{\sigma_G} \quad (2)$$

Where μ_G and σ_G represent the mean and the standard deviation of G respectively.

$$\mu_G = G(U^*) - \sum_{i=1}^{n} \frac{\partial G(U^*)}{\partial x_i} \sigma_{x_i} . u_i^* \quad (3)$$

$$\sigma_G = \sqrt{\sum_{i=1}^{n} \left(\frac{\partial G(U^*)}{\partial x_i} \sigma_{x_i}\right)^2} \quad (4)$$

For a normal distribution of x_i, transform these variables into their standardized forms by equation (5)

$$u_i = \frac{x_i - \mu_{xi}}{\sigma_{xi}} \tag{5}$$

The point $U^* = (u_1^*, u_2^*, u_3^* \ldots \ldots \ldots u_i^*)$ on G (U) =0 is the design point in U-space.

The failure probability is given by equation (6):

$$P_f = \text{Prob}[G(X) \leq 0] \tag{6}$$

However, the reliability is defined as (7):

$$R = 1 - P_f \tag{7}$$

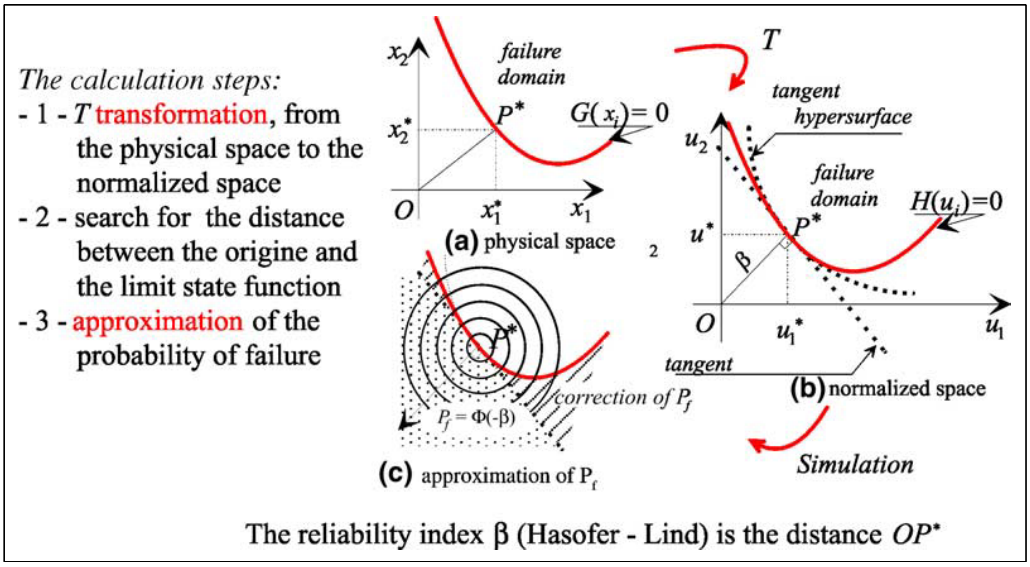

Fig.2 - Principle of β and P_f calculation using approximation methods [22]

2.2 Sensitivity and elasticity analysis

A reliability sensitivity analysis with respect to each random variable parameters should be done. An expression based on FORM method to evaluate sensitivity factors S_{pi} associated with a parameter pi (the mean value or the standard deviation of a stochastic variable or a constant in the failure function) around the most probable point MPP.by equation(8):

$$S_{pi} = \frac{\partial P_f}{\partial p_i} = -\frac{\partial R}{\partial p_i} = -\emptyset(\beta)\frac{\partial \beta}{\partial p_i} \tag{8}$$

Where $\emptyset(.)$ is the standardized gaussian density function.

However, it is not possible to compare directly the influence of these parameters between them without having firstly made a standardization of their sensitivities, so elasticity parameters should be introduced first. It is possible to define elasticity factors E_{pi} of R by (9):

$$E_{pi} = \frac{\partial R}{\partial p_i}\frac{p_i}{R} \tag{9}$$

2.3 Parametric analysis

From results of elasticity analysis, the most influential parameters are chosen, and then this analysis makes it possible to propose several tracks which can be explored to improve the design, this results help to determine the design variables vector and the random variables vector in the RBRDO process.

2.4 Reliability Based Robust Design Optimization formulation

A strategy aims to improve quality and reliability can be proposed. However, varying means values, or decreasing standard deviation is essentially an economic analysis. It is necessary to determine the optimum values for means and standard deviations of critical variables which are considered as design variables.

The objective of RBRDO is to improve both reliability and design under uncertainties; this approach is achieved by the integration of bi-objective optimization which minimize the variance and the mean value of performance function. This problem is formulated as:

$$\begin{cases} \text{minimize} \quad f1(\mathrm{x}) = [\mu_G] \\ \text{minimize}\, f2(\mathrm{x}) = [\sigma_G^2] \\ \text{Subject to} \quad \mathrm{Prob}[\mathrm{G}(\mathrm{d},\mathrm{X}) \geq 0] \geq \mathrm{R_T} \\ \mathrm{d}_k^l \leq \mathrm{d}_k \leq \mathrm{d}_k^u \end{cases} \tag{10}$$

The bi-objective optimization can be converted into a single objective as equation (11):

$$\begin{cases} minimize \quad f(x) = [w \times \mu_G + (1-w) \times \sigma_G^2] \\ subject\ to \quad Prob[G(d,X) \geq 0] \geq R_T \\ d_k^l \leq d_k \leq d_k^u \end{cases} \tag{11}$$

$\mathrm{f}(.)$ is the objective function, where w is the weighting factor. The weight can be varied in interval $0 \leq \mathrm{w} \leq 1$ to generate Pareto solutions, $\mathrm{G}(\mathrm{d},\mathrm{x})$ is the probabilistic constraint,

X is the vector for random variables,

R_T is the desired reliability,

$\mathrm{d_k}$ k=1, 2.......n are the design variables.

Many algorithms that can solve the above optimization are presented in literatures, in this case SORA method is applied to determine the optimal values of the critical parameters while checking a target reliability R_T.

2.4.1 Sequential optimization and reliability assessment (SORA)

The SORA method employs a decoupled strategy with a series of cycles of deterministic optimization and reliability assessment. In each cycle, optimization and reliability assessment are decoupled from each other; the reliability assessment is only conducted after the deterministic optimization to verify constraint feasibility. The key to this method is to shift the boundaries of violated constraints to the feasible direction based on the reliability information obtained in the previous cycle. The design is quickly improved from cycle to cycle and the computational efficiency is improved significantly [14]. The RBRDO problem can be written, according to SORA method, as:

$$\begin{cases} \min \mathrm{f}(\mathrm{d}^k, {\mu_x}^k) \\ \mathrm{G}(\mathrm{d}^k, \mu_x^k - s_i^{k+1}, P_{MPP}^{k+1}) \\ \mathrm{d}_k^l \leq \mathrm{d}_k \leq \mathrm{d}_k^u\, , \mu_k^l \leq \mu_k \leq \mu_k^u \end{cases} \tag{12}$$

The constraint boundary is shifted towards the feasible region using the shifting vector s_i^{k+1}

where: $s_i^{k+1} = \mu_x^k - x_{iIMPP}^K$. x_{iIMPP}^K designates the inverse MPP found by the reliability assessment loop,

For this a Performance Measure Approach (PMA) is used to involve the inverse reliability problem. The performance measure and the MPP u^{*T} corresponding to the desired reliability index (β^T) is determined after solving the inverse reliability problem defined as:

$$\begin{cases} \min \mathrm{G}(\mathrm{U}) \\ \text{subject to: } \|u\| = \beta^T \end{cases} \tag{13}$$

The new MPP obtained in reliability assessment phase equation (13) is used in the next deterministic optimization phase; this optimization is coded with MATLAB R2017a F_{mincon} function and used to minimize the objective function taking into account the lower and the upper bounds of the design variables and the design constraint.

These values coincide with the optimum values for the most important parameters and determine the best combination of manufacturing cost and standard deviation (accuracy).

3 Reliability based design of a gear pair

To carry out a reliability study, three types of models have to be defined: a physical, a probabilistic and a performance models. This procedure will be described in detail as follows.

3.1 Presentation of the physical model

Three scenarios of failures that may occur during the operation of gears lead to three corresponding physical models: the bending stresses of the pinion and gear, and the Hertzian tooth contact stress.

Assumed bending stress σ_F is the maximum tensile stress at the surface in the tooth root (Standard, ISO 6336-3:2006) [23], and it may be calculated by the equation (14):

$$\sigma_F = \frac{F_t}{bm_n} Y_F Y_S Y_\beta Y_\varepsilon K_A K_V K_{F\alpha} K_{F\beta} \tag{14}$$

σ_F : is the tooth root stress,

σ_{FP} : is the permissible bending stress.

The dedendum bending fatigue strength σ_{FP} is defined as (15):

$$\sigma_{FP} = \sigma_{Flim} Y_{ST} Y_{NT} Y_{\delta relT} Y_{RrelT} Y_X \tag{15}$$

Where: σ_{Flim} is the nominal bending stress.

To ensure the resistance to contact pressure (pitting), many parameters have been added to the basic formula demonstrated by Hertz (Standard, ISO 6336-2:2006) [24]. The evaluation of the contact stress is carried by the standard equation (16):

$$\sigma_H = Z_H Z_E Z_\beta Z_\varepsilon \sqrt{\frac{F_t}{d_1 b} \frac{u \pm 1}{u}} \sqrt{K_A K_V K_{H\beta} K_{H\alpha}} \tag{16}$$

σ_H is the calculated contact stress,

b is the width of the contact face,

σ_{HP}: is the permissible contact stress.

Gear limit contact stress 14 can be calculated by (17):

$$\sigma_{HP} = \sigma_{Hlim} Z_{NT} Z_L Z_V Z_R Z_W Z_X \tag{17}$$

σ_{Hlim} is the allowable contact stress.

3.2 Presentation of the probabilistic model

All variables have a standard normal distribution of the input data. Every mean value and standard deviation of random variables is obtained from bibliographic sources [9-10]. The random variables statistics are as given in Table 1.

Table1- Random variables statistics

Random variables of Bending stresses	*Symbol*	Mean values and standard deviations		Random variables of Contact stress	*Symbol*	Mean values and standard deviations
		Pinion	Gear			
Normal module	m_n	*N*(4 ,0.02)	*N*(4, 0.02)	Pinion pitch diameter	d_1	*N(148.75, 0.74375) mm*
Active face width	b	*N(200, 1) mm*	*N(200, 1)mm*	Work hardening factor	Z_W	*N(1, 0.033)*
Rated tangential tooth force at transverse pitch	F_t	*N(34644, 519.66)N*	*N(34644, 519.66)N*	Rated tangential tooth force at transverse pitch	F_t	*N(34644, 519.66)N*
Experimental gear bending fatigue strength	σ_{Flim}	*N(310, 62)N/mm²*	*N(310, 62) N/mm²*	Experimental flank contact fatigue strength	σ_{Hlim}	*N(1300, 156) N/mm²*
Tooth form factor	Y_F	*N(2.36, 0.07788)*	*N(2.14, 0.07062)*	Contact ratio factor	Z_ε	*N(0.81, 0.00405)*
stress concentration factor	Y_S	*N(1.75, 0.05775)*	*N(1.94, 0.06402)*	Nodal area factor	Z_H	*N(2.32, 0.0116)*
Contact ratio factor	Y_ε	*N(0.715, 0.003575)*	*N(0.715, 0.003575)*	Elastic factor	Z_E	*N(189.8, 9.49)* $\sqrt{N/mm^2}$*)*
Helix angle factor	Y_β	*N(0.8, 0.004)*	*N(0.8, 0.004)*	Helix angle factor	Z_β	*N(0.957, 0.004785)*
Stress correction factor	Y_{ST}	*N(2.1 , 0.0693)*	*N(2.1, 0.0693)*	Lubrication factor	Z_L	*N(0.92, 0.03036)*
Life factor for tooth root stress	Y_{NT}	*N(1 , 0.033)*	*N(1, 0.033)*	Life factor	Z_{NT}	*N(1, 0.033)*
Relative sensitivity factor	$Y_{\delta relT}$	*N(0.99, 0.03267)*	*N(1.01, 0.03333)*	Tooth fineness factor	Z_R	*N(1.03, 0.03399)*
Relative surface condition factor	Y_{RrelT}	*N(1.065, 0.035145)*	*N(1.065, 0.035145)*	Velocity factor	Z_V	*N(1.04, 0.03432)*
Size factor	Y_X	*N(1, 0.033)*	*N(1, 0.033)*	Size factor	Z_X	*N(1, 0.033)*
Work condition factor	K_A	*N(1, 0.033)*	*N(1, 0.033)*	Work condition factor	K_A	*N(1, 0.033)*
Dynamic load factor	K_V	*N(1.484, 0.1613)*	*N(1.484,0.1613)*	Dynamic load factor	K_V	*N(1.484, 0.1613)*
Longitudinal load distribution factor	$K_{F\alpha}$	*N(1.16, 0.03828)*	*N(1.16, 0.03828)*	Transverse load distribution factor	$K_{H\beta}$	*N(1.68, 0.05544)*
Transverse load distribution factor	$K_{F\beta}$	*N(1.603, 0.052899)*	*N(1.603, 0.052899)*	Longitudinal load distribution factor	$K_{H\alpha}$	*N(1.16, 0.03828)*

3.3 Presentation of the performance model

The first performance model is defined as the tooth root stress for the pinion, presented by equation (18)

$$G_1(x) = \sigma_{FP1} - \sigma_{F1} \tag{18}$$

The second failure scenario is presented by the limit state function of the bending stress for the gear, presented by :

$$G_2(x) = \sigma_{FP2} - \sigma_{F2} \tag{19}$$

The third failure scenario is given by the equation of the contact stress for a gear pair: equation (20)

$$G_3(x) = \sigma_{HP} - \sigma_H \tag{20}$$

4 Results validation and discussion

Data of table 1 are inserted in the performance model in order to determine the reliability and the reliability index for each failure scenarios$G_1(x)$, $G_2(x)$, and $G_3(x)$ using approximation methods: FORM has been described in [18-19] and SORM by Breitung formula [20-21], and simulation method: MCS has been described in [25]. The reliability results for each scenario$G_1(x)$, $G_2(x)$, and $G_3(x)$ are illustrated in table 2, 3 and 4 respectively. Also, results from published sources are shown such as those of perturbation method [9], and Edgeworth Series methods (R_E) and MCS[a] from [10]. In addition, the error of each method (ε_R) relatively to the MCS results as well as values of convergence criterion and appeals number of limit state are also presented.

4.1 Reliability based on pinion and wheel dedendum Bending Stress/Strength

Several significant results are presented below (Tables 2 and 3). The results show that FORM and SORM are in concord with the reference method MCS. However, it is seen that SORM method is the most accurate in this example (ε_R = 0.001%) with only 6 appeals to the limit state.

Furthermore, Table 2 shows that SORM approximation predict a reliability and reliability index more accurately than FORM, what may signify the nonlinearity of the response, and that SORM has correctly taken the curvatures of these limit states. On the other hand, the results obtained by the perturbation method and the Edgeworth series (published results) are somewhat different from the MCS results showing an order of error ε_R = 0.1% and ε_R = 0.09 % respectively.

Table 2 - Pinion bending fatigue reliability

Pinion	**MCS**	**FORM**	**SORM**	**Perturbation method**	**MCS [a]**	**RE Edgeworth**
Reliability R	0.99734	0.99742	0.99733	0.9963	0.9974	0.99644
Error ε_R (%)	-	0.008	0.001	0.104	0.006	0.090
Reliability index β	2.7867	2.7964	2.7860	2.754	2.6754	2.6754
Convergence criterion	Cov=0.00612	$\left\lvert\frac{G(x^*)}{G(x_0)}\right\rvert=6.08\times10^{-8}$	-	-	-	
Appeals number	10^7	5	6	-	-	-

Table 3 - Gear bending fatigue reliability

Gear	**MCS**	**FORM**	**SORM**	**Perturbation method**	**MCS [a]**	**RE Edgeworth**
Reliability R	0.99760	0.99767	0.99759	0.9966	0.9974	0.99644
Error ε_R (%)	-	0.007	0.001	0.1	0.02	0.090
Reliability index β	2.8206	2.8294	2.8190	2.705	2.6754	2.6754
Convergence criterion	Cov=0.00645	$\left\lvert\frac{G(x^*)}{G(x_0)}\right\rvert=6.38\times10^{-8}$	-	-	-	
Appeals number	10^7	5	6	-	-	-

4.2 Reliability based on flank contact stress strength

The same study as above is performed but considering the flank contact stress strength. For the contact stress, the results are shown in table 4, it is found that the reliability and the reliability index values calculated by approximation methods (FORM and SORM) are very close to that calculated by the MCS with ε_R= 0.003% and ε_R= 0.0002% respectively.

In terms of convergence and proximity, FORM and SORM used in this calculation are satisfactory; besides the ratio$\left\lvert\frac{G(x^*)}{G(x_0)}\right\rvert$ that is close to zero which may express the proximity of the last point of failure most likely estimated with the limit state surface. In addition, the MCS results are produced with high accuracy in terms of their associated coefficient of variation $cov_R < 0.05$. As a conclusion, and according to the above results, it can be said that reliability index calculation shows that SORM method produces a more accurate values of reliability and β with more economical cost than the MCS technique and more precise than FORM method due to the nonlinearity of the state function. So the results confirm the robustness of SORM method for treating reliability analyzes in similar cases where limit states functions are nonlinear.

Table 4 - Contact gear pair reliability

Gear	MCS	FORM	SORM	Perturbation method	MCS [a]	RE Edgeworth
Reliability R	0.999462	0.999496	0.999464	0.9985	0.9994	0.9989
Error ε_R (%)	-	0.003	0.0002	0.096	0.006	0.05
Reliability index β	3.2699	3.2880	3.2709	2.9685		2.9685
Convergence criterion	Cov=0.0136	$\left\|\frac{G(x^*)}{G(x_0)}\right\|$=4.56×10^{-7}	-	-	-	
Appeals number	10^7	5	5	-	-	-

4.3 ***Reliability*** elasticity ***analysis***

From the design standpoint, the elasticity to averages provides information about dimensions to retain whereas elasticity to standard deviation can direct quality control. For this reason, it is very essential to calculate the elasticity of the failure probability with respect to the random variables parameters. Thus, approximated FORM method can be used simply to calculate reliability elasticities for each random parameter with respect to means and standard deviations (equation 9).

However, the elasticity factors with respect to the means and their standard deviations, for the cases of bending and contact stress models, are visualized respectively in the figures 2 and 3.

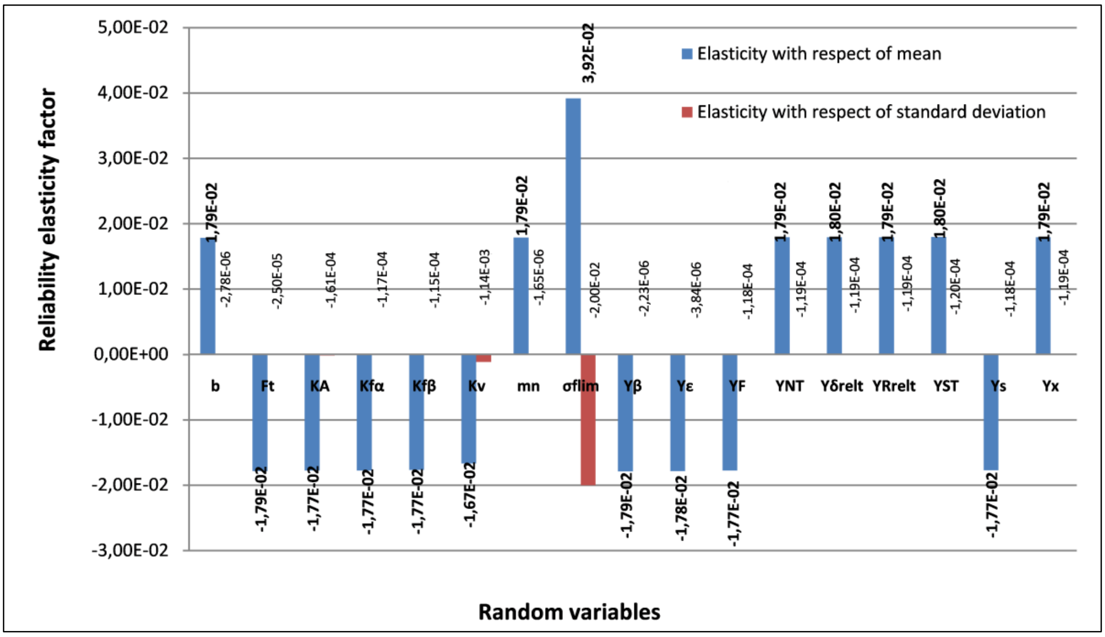

Fig.3 - Reliability elasticity for pinion bending stress G_1

From figure 3, it appears that reliability elasticity with respect to the average value of each variable have the predominance impact on the failure probability especially in cases G_1. In addition, all variables have almost the same elasticity factor value with a positive or negative sign, except the elasticity to the mean of σ_{Flim} which is the main important factor, that is to say, the dispersion of the variable σ_{Flim} is the most influencing on the failure probability. In addition, it can be noted that this value is positive; hence it is beneficial to the probability of failure, so that R is increased by (3.92×10^{-2}%) when the σ_{Flim} increased by 1%. The same beneficial effect for the variables Y_{NT}, Y_{ST}, $Y_{\delta relT}$, Y_{RrelT}, Y_X, b and m_n is also noted in

Figure2, which is in good agreement with the results reported by Yang [9] who studied the same example but by analyzing sensitivity. On the other hand, elasticity results relative to variables standards deviations, except $E_{\sigma_{\sigma Flim}}$ that is the highest ($E_{\sigma_{\sigma Flim}}$ = -2.0×10^{-2}), the other elasticity factors are all too small. This is explained by the non-sensitivity of R to the standard deviation of each parameter, thus the model is tolerant of uncertainty except for the σ_{Flim} standard deviation. As a result, σ_{Flim} must be controlled very closely.

The results of reliability elasticity analysis for the Hertzian contact stress failure scenario are shown in figure 4.

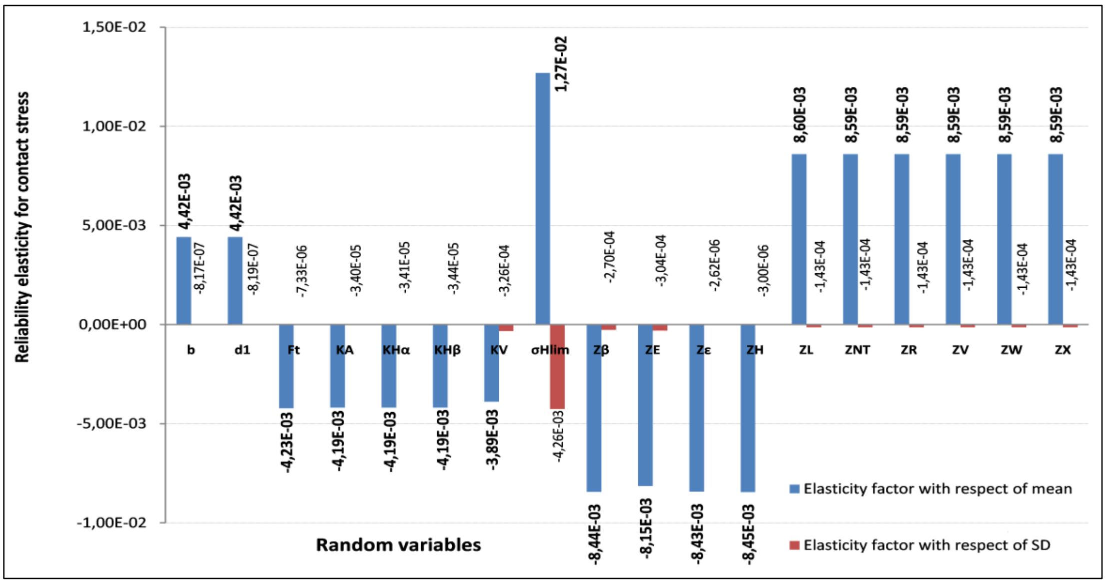

Fig. 4 - Reliability Elasticity for contact stress G_3

The results in fig 4 show that the most influential parameter is σ_{Hlim} . However, the influence of other factors is relatively low compared to σ_{Hlim} . The interpretation of elasticity histogram for contact stress model show that 1% increase in average variable σ_{Hlim} that would decrease nearly (1.27×10^{-2} %) of R. The results demonstrate again that a quality control of gear material by reducing the standard deviation of the variable σ_{Hlim} is necessary due to its great elasticity factor (4.26×10-3). This elasticity analysis leads us to specify the variables that will undergo changes to improve reliability.

4.4 Parametric analysis

The results of elasticity values relatively to means and standard deviations are used to see on what parameters it is interesting to play in order to get a more reliable gear. In the case study, σ_{Flim} and σ_{Hlim} with their standard deviations have the highest elasticity factors, for that they have been chosen as decision variables in the RBRDO process, in this process SORA method is used to optimize the current design variables to obtain robust design.

4.5 Optimize Design and Reliability

A reliability based robust design optimization of gear pair can be described by a bi-objective optimization, in which the objectives functions $\mathrm{f}_1(\mathrm{x})$ and $\mathrm{f}_2(\mathrm{x})$ are taken as the minimum of variances and means values of each limit stat functions $\mathrm{G}_1(\mathrm{x})$ and $\mathrm{G}_3(\mathrm{x})$:

$$\mathrm{f}_1(\mathrm{x}) = \left[\mathrm{w}.\,\mu_{\mathrm{G}_1} + (1-\mathrm{w})\sigma^2_{\mathrm{G}_1}\right] \tag{21}$$

The design parameters are: $100 \le \sigma_{\mathrm{Flim}} \le 450,\ 25 \le \sigma_{\sigma\mathrm{Flim}} \le 75$, with initial values $[\sigma_{\mathrm{Flim0}},\ \sigma_{\sigma\mathrm{Flim0}}] = [310, 40]$.

$$\mathrm{f}_1(\mathrm{x}) = \mathrm{w}(2.2589\sigma_{\mathrm{Flim}} - 283.7756) + (1-\mathrm{w})[5.102471088225 \times \sigma_{\sigma\mathrm{Flim}}{}^2 + (0.027782955075385 \times \sigma_{\mathrm{Flim}}{}^2)] \tag{22}$$

$$\mathrm{f}_2(\mathrm{x}) = \left[\mathrm{w}.\,\mu_{\mathrm{G}_3} + (1-\mathrm{w})\sigma^2_{\mathrm{G}_3}\right] \tag{23}$$

The design parameters are:

$600 \leq \sigma_{\mathrm{Hlim}} \leq 1500$, $100 \leq \sigma_{\sigma\mathrm{Hlim}} \leq 200$ with initial values $[\sigma_{\mathrm{Hlim0}}, \sigma_{\sigma\mathrm{Hlim0}}]$ = [1300, 156].

$$f_2(x) = w(0.985504\sigma_{\mathrm{Hlim}} - 700.9265850309833) + (1-w)((0.9712 \times \sigma_{\sigma\mathrm{Hlim}}{}^2) + (0.0063\sigma_{\mathrm{Hlim}}{}^2) + 3151.2) \quad (24)$$

The materials correspond to the limits values to σ_{Flim} and σ_{Hlim} in the RBRDO process are :

$[\sigma_{\mathrm{Flim}}, \sigma_{\mathrm{Hlim}}]_{\min} = [100\ \mathrm{N/mm^2}, 600\mathrm{N/mm^2}]$ correspond to " flame or induction hardened wrought and cast steel" quality ML, hardness HV=[485-615].

$[\sigma_{\mathrm{Flim}}, \sigma_{\mathrm{Hlim}}]_{\max} = [450\ \mathrm{N/mm^2}, 1500\mathrm{N/mm^2}]$ Correspond to «case hardened wrought steel» quality MQ, hardness HV=[660-800].

The material which correspond to $\sigma_{\mathrm{Flim}} = 310\ \mathrm{N/mm^2}$ and σ_{Hlim}=1300 $\mathrm{N/mm^2}$ is a hardened wrought steels grade ML (stands for the minimum requirement), with a surface hardness from 600 to 800 HV (Vickers hardness).

However, numerical values for allowable stress numbers (bending and contact) of materials used in gears are given in ISO 6336 part 5: [26] with specification of requirements for material quality and heat treatment.

From equation (12) $G(d^k, \mu_x^k - s_i^{k+1}, P_{MPP}^{k+1})$ the probabilistic constraints are G_1 and G_3; there are no random parameters P.

The weight coefficients w are selected for two sets of values: w=0; w=1, using these sets for each objective function, four optimum designs will be obtained.

The design variables evolution, objective function values and statistics of G_1 and G_3 after designing by RBRDO based on SORA method for target reliability R_T = 99.99% which corresponds to β=3.73; are calculated and presented in the following tables.

Table 6 - Design 1 for objective function $f_1(x)$ case w =0

cycles	σ_{Flim}	$\sigma_{\sigma\mathrm{Flim}}$	$f_1(.)$	G1	σ_{G1}
Original design	310	62	/	-114.8	213.5
1	125.63	25	3.627×10^3	-257.789	101.85
2	249,48	25	4.918×10^3	1.256	105.27
3	248.36	25	4.903×10^3	-0.0458	105.22
4	248.38	25	4.903×10^3	0.0016	105.22
5	248.38	25	4.903×10^3	-4.54×10^{-5}	105.22

Table 7- Design 2 for objective function $f_1(x)$ case w=1

cycles	σ_{Flim}	$\sigma_{\sigma\mathrm{Flim}}$	$f_1(.)$	G1	σ_{G1}
Original design	310	62	/	-114.8	213.50
1	125.63	40.27	0.0044	-360.89	149.50
2	294.25	40.03	380.90	5.525	146.70
3	291.23	40.05	374.08	-0.722	146.63
4	291.36	40.05	374.38	-0.722	146.63
5	291.36	40.05	374.38	-0.722	146.63

Table 8 - Design 3 for objective function $f_2(x)$ case w=0

cycles	σ_{Hlim}	$\sigma_{\sigma\mathrm{Hlim}}$	$f_2(.)$	G3	σ_{G1}
Original design	1300	156	/	-61.37	145.92
1	711,24	100	1.6050×10^4	-434.32	108.84
2	1215,98	100	2.2178×10^4	1.299	120.41
3	1214,37	100	2.2154×10^4	-0.346	120.34
4	1214,43	100	2.2155×10^4	-0.346	120.34
5	1214,43	100	2.2155×10^4	-0.346	120.34

Table 9 - Design 4 for objective function $f_2(x)$ case w=1

cycles	σ_{Hlim}	$\sigma_{\sigma Hlim}$	$f_2(.)$	G3	σ_{G1}
Original design	1300	156	/	-61.37	145.92
1	1300	156	/	-61.37	145.92
2	711,23667	155,71	2.0×10^{-8}	-593.674	150.10
3	1376,66	155.999	655.78	4.596	156.46
4	1370,47	155,999	649.68	-0.589	156.33
5	1370,71	155,999	649.91	-0.589	156.33

From the optimization results in tables 6, 7, 8 and 9 it is clear that the change in weight coefficients values leads to different design variables values, these four designs are drastically different. In case w=1, tables 7 and 9 only σ_{Flim} and σ_{Hlim} are optimized, there values after convergence of objectives functions are 291.36 N/mm^2 and 1370,71 N/mm^2 respectively.

Tables 6 and 8 (case w=0) show that, when standards deviations are considered as design parameters, there values reach their lower bound, with an optimal values of σ_{Flim} and σ_{Hlim} means equal to 248, 38 N/mm^2 and 1214,43 N/mm^2.

4.6 Confirmation the robustness of the design

Reliability sensitivity with respect of means should be performed for each random variables of G_1 and G_3. The results are illustrated in tables 10 and 11.

Table 10 - Reliability sensitivity with respect of means

Random variables for bending stress	Reliability Sensitivity factors for cov=20%	Reliability Sensitivity factors for w=0	Reliability Sensitivity factors for w =1
b	8.01×10^{-5}	5.55×10^{-6}	3.9×10^{-6}
F_t	-4.62×10^{-7}	-3.18×10^{-8}	-2.24×10^{-8}
K_A	-1.6×10^{-2}	-1.09×10^{-3}	-7.71×10^{-4}
$K_{F\alpha}$	-1.37×10^{-2}	-9.43×10^{-4}	-6.65×10^{-4}
$K_{F\beta}$	-9.93×10^{-3}	-6.82×10^{-4}	-4.81×10^{-4}
K_v	-1.01×10^{-2}	-7.48×10^{-4}	-5.25×10^{-4}
m_n	4.0×10^{-3}	2.78×10^{-4}	1.95×10^{-4}
σ_{Flim}	1.11×10^{-4}	1.25×10^{-5}	8.77×10^{-6}
Y_β	-2.0×10^{-2}	-1.38×10^{-3}	-9.74×10^{-4}
Y_ε	-2.24×10^{-2}	-1.55×10^{-3}	-1.01×10^{-3}
Y_F	-7.44×10^{-3}	-5.11×10^{-4}	-3.60×10^{-4}
Y_N	1.61×10^{-2}	1.32×10^{-3}	8.88×10^{-4}
$Y_{\delta relt}$	1.6×10^{-2}	2.3×10^{-3}	1.5×10^{-3}
Y_{Rrelt}	1.51×10^{-2}	2.2×10^{-3}	1.4×10^{-3}
Y_{st}	7.68×10^{-3}	1.11×10^{-3}	7.17×10^{-4}
Y_s	2.8×10^{-3}	5.62×10^{-4}	3.97×10^{-4}
Y_x	1.61×10^{-2}	2.34×10^{-3}	1.56×10^{-3}

Table 11- Reliability sensitivity with respect of means

Random variables for contact stress	Reliability Sensitivity factors for cov=12%	Reliability Sensitivity factors for w=1	Reliability Sensitivity factors for w=0
b	2,11×10^{-5}	4.61×10^{-6}	6.14×10^{-6}
d_1	2,83×10^{-5}	6.21×10^{-6}	8.27×10^{-6}
F_t	-1,22×10^{-7}	-2.66×10^{-8}	-3.54×10^{-8}
K_A	-4,19×10^{-3}	-9.22×10^{-4}	-1.23×10^{-3}
$K_{H\alpha}$	-3,61×10^{-3}	-7.89×10^{-4}	-1.05×10^{-3}
$K_{H\beta}$	-2,49×10^{-3}	-5.45×10^{-4}	-7.24×10^{-4}
K_V	-2,62×10^{-3}	-5.76×10^{-4}	-7.50×10^{-4}
σ_{Hlim}	9,73×10^{-6}	2.033×10^{-6}	2.58×10^{-6}
Z_β	-8,82×10^{-3}	-1.93×10^{-3}	-2.56×10^{-3}
Z_E	-4,27×10^{-5}	-9.41×10^{-6}	-1.24×10^{-5}
Z_ε	-1,04×10^{-2}	-2.28×10^{-3}	-3.03×10^{-3}
Z_H	-3,64×10^{-3}	-7.95×10^{-4}	-1.06×10^{-3}
Z_L	9,34×10^{-3}	2.08×10^{-3}	2.81×10^{-3}
Z_{NT}	8,59×10^{-3}	1.91×10^{-3}	2.58×10^{-3}
Z_R	8,34×10^{-3}	1.85×10^{-3}	2.47×10^{-3}
Z_v	8,26×10^{-3}	1.84×10^{-3}	2.48×10^{-3}
Z_w	8,59×10^{-3}	1.91×10^{-3}	2.58×10^{-3}
Z_x	8,59×10^{-3}	1.91×10^{-3}	2.58×10^{-3}

Tables 10 and 11, note that reliability sensitivities to means of each random variables (D_R/D_x) drops when designs parameters are optimized by RBRDO. As a result, models are less sensitive to design variables changes, so the tow designs are more robust and their reliability increase. These results emphasize the importance of quality control that should be made on gear material at design or manufacturing stage, where the series produced must ensure that the σ_{Flim} and σ_{Hlim} values remote as little as possible of their means. The causes of dispersion in material include especially, the preparation process may alter the material and its heterogeneity. Mayo in [27] said that these derivations may be caused by many factors including variations in material metallurgy and hardness such as cleanliness, residual stresses in material, micro structure, quality, the deformations generated by the heat treatment , poor machine tool condition , etc. The objective is therefore to minimize the deviations by a strict control of these sources of uncertainty and taking appropriate actions during design and manufacture.

5 Conclusion

The results of Reliability Based Robust Design Optimization of gear pair is performed with very promising results. Firstly, the search of system reliability, in relation to the dreaded event, was done by different ways, namely: Monte Carlo simulations and FORM/SORM methods. However, the MCS method, as in usual, is used to validate the precision of the applied approximation methods. Particularly, in the case study, SORM is considered more efficient in terms of the ratio between computation time and precision of the response due to the non linearity of the limit state functions. This study also shows that the mode of failure binds to the bending stress for the pinion and the wheel, and is slightly more dominant than the contact stress for the studied pair of gears.

Furthermore, FORM is used to obtain measurements of sensitivity and elasticity with respect to mean and standard deviations of each random variable parameters, these results can help to selects the parameters to focus on during the design optimization; then, reduce the size of optimization problem.

In this research MCS, FORM and SORM approximation are used for the search of system reliability. On the other hand the idea of SORA method is to formulate the RBRDO problem into a design deterministic optimization before a reliability assessment based on an efficient inverse MPP search algorithm (PMA), so this reliability analysis starts from the optimum design point to move this design point as quickly as possible to its optimum.

Finely, using SORA method, the optimum values of means and standard deviations that have higher elasticity factors was determined. The confirmation phase results have shown that the reliability sensitivity coefficients are reduced because of the role of uncertainties is reduced during the robust optimization procedure.

This study has shown that a robust design with a desired reliability can be achieved quickly and simply by using elasticity factors results, demonstrating the importance of elasticity analysis in the designing and manufacturing process.

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