USE OF DYNAMIC PROGRAMMING FOR SHORT-TERM HYDRO-THERMAL COORDINATION

Milan Kočović, *Zoran Miljanić*, *Ilija Vujošević*

Keywords: *Dynamic programming, Dijkstra algorithm, Hydro-thermal coordination*

Abstract: This paper presents the application of dynamic programming method in the hydro-thermal coordination, which is based on finding the optimal trajectory of the use of water in reservoir storage with the limitation of initial and final storage volumes. Initial and final volumes of the reservoir present the input data in the optimization and which were obtained in short-term planning of water consumption. Also, this paper presents the modeling and the possible approximations of the input-output characteristic of steam and hydroelectric plants. For the purpose of hydro-thermal coordination, the program was implemented using MATLAB software and its graphical user GUI interface. The mentioned program includes hydro-thermal coordination that was made on the principle of dynamic programming using Dijkstra algorithm.

1. INTRODUCTION

The coordination of the system for production of electrical power with hydroelectric plants is usually more complex than scheduling of an all-thermal generation systems. The hydroelectric plants may be coupled both electrically and hydraulically (downstream multiple plants).

There are no two hydroelectric systems in the world that are alike. There are many reasons for this fact/ the natural differences in the watersheds, reservoirs storage as well as so many different restrictions on the operation of the system, such as: controlling floods,

* Milan Kočović, Spec. Sci. works at the CGES AD in Podgorica, Montenegro and is MSc student at the Faculty of Electrical Engineering in Podgorica, Montenegro. (phone: 382-68-814259; e-mail: kocovicmilan@gmail.com, milan.kocovic@cges.me).

** Doc. dr Zoran Miljanić is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: zormi@ac.me).

*** Professor emeritus Ilija Vujošević is with the Faculty of Electrical Engineering, University of Montenegro, Montenegro (e-mail: ilijav@ac.me).

regular discharge of water for irrigation, limited water release because of navigability of river at all time or availability of recreation center downstream of reservoir storage. Sudden changes, with high-volume releases of water, may induce high waves with potentially damaging effect for downstream object [1].

The goal of hydro-thermal coordination is to use the amount of water that is given through short-term scheduling of water consumption in the best possible way, which means to obtain the minimum production costs of thermal plants [1].

The coordination of the operations of hydroelectric plants involves, naturally, the scheduling of water releases.

The long-range hydro-scheduling problem involves the long-range forecasting of water availability that is based on meteorological and statistical analyse.

Short-range hydro-scheduling involves scheduling of amount of water that can be used for generation in short-term (1 day to 1 week).

The coordination of hydro - scheduling must be in coordination with water inflow in order to keep the amount of water within an specified range [1].

2. HYDROELECTRIC PLANT MODEL

The amount of energy that is available in a unit of stored water is equal to the product of the weight of water and the height of the water fall. From that standpoint, it can be concluded that a small amount of water and a big drop produce energy which can be produced with a big amount of water and small drop. For example, 1kWh of energy can be obtained in the following way:

$$1kWh=24.45m^3 \cdot 15m,$$
$$1kWh=15m^3 \cdot 24.45m.$$

The amount of energy that is available in the unit quantity of water is not equivalent to electrical energy obtained via that quantity and that fall and it must be reduced by a certain value, caused by energy losses in the energy conversion.

Power provided by the water is proportional to the flow of water in m^3/s and the coefficient of energy conversion, which takes into account the net head (gross head - flow losses into the penstock) and the coefficient of efficiency of the turbine-generator.

Flow losses typically is the amount of about 5% of the gross head, while in the case of hydro power plants with low pressure and a case of long penstocks, these losses can be much higher. Also, a factor of the efficiency of the turbine is load, and efficiency for smaller loads may fall below 70%.

Of course, it should take into account the existence of loss in the conversion of mechanical energy of the turbine into electrical energy of the generator[1].

Typical performance characteristic of hydropower plants is characteristic of incremental water rate $\delta[m^3/MWh]$, that is shown on Fig.1. for three hydroplants.

The rise in incremental water rate with increasing unit output results primarily from the increased hydraulic losses with the increased flow. With the increasing number of hydro units, the hydraulic losses are being increased, primarily due to rise in afterbay which produces decreasing in net and gross head [1].

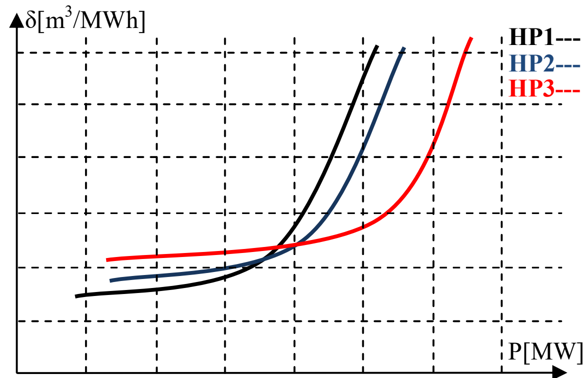


Fig.1. Incremental water rate versus output power

Fig.2. is showing the basic hydroelectric plant characteristics that represent the dependence of water flow q[m^3/h] and output power P[MW].

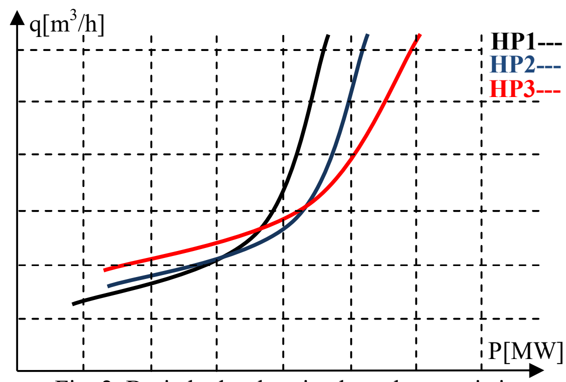


Fig. 2. Basic hydroelectric plant characteristics

This basic energy characteristic of hydroplant has such a form that it can be approximated by a quadratic equation, namely. It can be written in the following form:

$$q = A + B \cdot P + C \cdot P^2, \tag{1}$$

where *A*, *B* and *C* are coefficients of quadratic equation which are calculated on the principle of achieving the minimum squared deviations between the actual value of the flow q_i in the point P_i and the approximated value of the flow q_i in the same power P_i [2].

In case it is known *n* points (P_i, q_i), then the principle of calculation of the coefficients *A*, *B* and *C* consists of the following steps:

$$J = \sum_{i=1}^{n} (A + B \cdot P_i + C \cdot P_i^2 - q(P_i))^2, \tag{2}$$

where is J sum of quadratic deviations between the actual and the approximated flow.

The goal is to find the minimum of the function J depending on A, B and C coefficients, as shown below [2].

$$\frac{\partial J}{\partial A} = \sum_{i=1}^{n} 2 \cdot (A + B \cdot P_i + C \cdot P_i^2 - q(P_i)) = 0 \tag{3}$$

$$\frac{\partial J}{\partial B} = \sum_{i=1}^{n} 2 \cdot P_i \cdot (A + B \cdot P_i + C \cdot P_i^2 - q(P_i)) = 0 \tag{4}$$

$$\frac{\partial J}{\partial C} = \sum_{i=1}^{n} 2 \cdot P_i \cdot (A + B \cdot P_i + C \cdot P_i^2 - q(P_i)) = 0 \tag{5}$$

$$n \cdot A + \left(\sum_{i=1}^{n} P_i\right) \cdot B + \left(\sum_{i=1}^{n} P_i^2\right) \cdot C = \sum_{i=1}^{n} q(P_i) \tag{6}$$

$$\left(\sum_{i=1}^{n} P_i\right) \cdot A + \left(\sum_{i=1}^{n} P_i^2\right) \cdot B + \left(\sum_{i=1}^{n} P_i^3\right) \cdot C = \sum_{i=1}^{n} P_i \cdot q(P_i) \tag{7}$$

$$\left(\sum_{i=1}^{n} P_i^2\right) \cdot A + \left(\sum_{i=1}^{n} P_i^3\right) \cdot B + \left(\sum_{i=1}^{n} P_i^4\right) \cdot C = \sum_{i=1}^{n} P_i^2 \cdot q(P_i) \tag{8}$$

Solving this system, namely finding the values of coefficients A, B, and C, it is obtained the approximate curve for the planning of generation of hydroplant. That curve is sufficiently precise for economical scheduling [2].

When planning production, in addition to the energy characteristic of hydroplant, it is necessary to have information of the minimum q_{min} and maximum q_{max} allowed water flow that also provides data of the minimum and maximum power of hydroelectric plant.

3. MODEL OF STEAM PLANT

A. Input-output characteristic of steam plant

Electrical energy of steam plant is produced by conversion of heat energy to mechanical than mechanical to electrical energy. The curve of heat consumption shows the consumption of heat energy per hour versus the output power of steam plant. More often used characteristic of steam plant is curve of consumption fuel, i.e. fuel cost curve. The total cost of operation includes the fuel cost and the cost of labor, supplies, and maintenance that are expressed as fixed percentage of the incoming fuel costs. Fig.3. shows typical example of input-output characteristic of steam plant. [3].

On Fig. 3. by points 1 to 5 are indicated the processes of opening the valve. In the vicinity of these points there are discontinuities as the product of increased losses in valves in that range [3].The curve of fuel cost can be defined by polynomial:

$$F = \alpha + \beta \cdot P + \gamma \cdot P^2 \tag{9}$$

The coefficients α, β i γ are calculated on principle of minimum squared deviation between real and approximated value of fuel cost, similar as finding coefficients A, B and C in previous chapter.

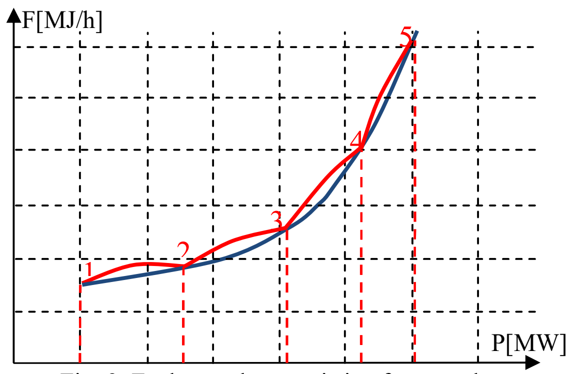


Fig. 3. Fuel cost characteristic of steam plant

B. Optimal operation of an all-thermal system

Typical operation of this group of system is operation of thermal generation units on same bus. Assume that α_i, β_i and γ_i are calculated for all generation units. The total fuel cost F for m thermal units that supply the demand P_D can be expresed as:

$$F = \sum_{i=1}^{m} \left(\alpha_i + \beta_i P_i + \gamma_i P_i^2\right) \tag{10}$$

If transmission losses are negligible, it can be written next equation:

$$P_D = \sum_{i=1}^{m} P_i \,, \tag{11}$$

P_D - power demand of the system and P_i - power that is produced by i steam plant.

The most used method for solving constrained minimization problems is technique of Lagrange multipliers. With constraint:

$$P_D - \sum_{i=1}^{m} P_i = 0, \tag{12}$$

it is obtained:

$$\hat{F} = F_T + \lambda \cdot \left[P_D - \sum_{i=1}^{m} P_i\right], \tag{13}$$

$$F_T = \sum_{i=1}^{m} F_i(P_i)\,, \tag{14}$$

and where λ is multiplier which is unknown at the beginning.

Lagranges method is based on the introduction of penalization for any deviation from the set limits, and the minimum is achieved by equating the first derivatives of the functions with 0.

$$\frac{\partial F_i}{\partial P_i} - \lambda = 0, \tag{15}$$

$$\lambda = \frac{\partial F_1}{\partial P_1} = \frac{\partial F_2}{\partial P_2} = \frac{\partial F_3}{\partial P_3} = \cdots = \frac{\partial F_m}{\partial P_m}. \tag{16}$$

It is concluded that each optimal operation unit should share the load such that their incremental costs λ are equal. For equation of fuel cost $F_i(P_i)$ for i units with Lagrange method it is obtained:

$$\beta_i + 2\gamma_i P_i - \lambda = 0\,. \tag{17}$$

From equation (17) it follows that:

$$P_i = \frac{\lambda - \beta_i}{2\gamma_i}, \tag{18}$$

$$P_D - \sum_{i=1}^{m} P_i = 0, \qquad P_D - \sum_{i=1}^{m}\left(\frac{\lambda}{2\gamma_i} - \frac{\beta_i}{2\gamma_i}\right) = 0, \tag{19}$$

$$\lambda = \frac{2P_D + \sum_{i=1}^{m}\frac{\beta_i}{\gamma_i}}{\sum_{i=1}^{m}\frac{1}{\gamma_i}}. \tag{20}$$

When value of incremental cost λ was calculated, distribution of power is calculated from equation (18)[2].

4. HYDRO-THERMAL SCHEDULING WITH DYNAMIC PROGRAMMING

If we consider that the input-output characteristic of the steam plant is given by the relation:

$$F = \alpha + \beta \cdot P_T + \gamma \cdot P_T^2, \tag{21}$$

while the characteristic of hydroplant is given by:

$$q = A + B \cdot P_H + C \cdot P_H^2, \tag{22}$$

for $P_H > P_{min}$ and $q=0$ for $P_H=0$.

For such interval of planning j, it can be written that volume of storage in the end of that interval is:

$$V_j = V_{j-1} + n_j\left(r_j - q_j - s_j\right), \tag{23}$$

where are: V_j - volume storage in the end of interval j, V_{j-1}- volume storage in the end of interval j-1, n_j - duration of interval j, r_j - inflow of interval j, q_j - flow of interval j, s_j - spillage of interval j.

If the volume storage on the start of interval j is $V_{j-1}=V_i$ and on the end of interval j $V_j=V_k$, and spillage is $s_j=0$, than the flow q_j is equal:

$$q_j = \frac{V_i - V_k}{n_j} + r_j \,. \tag{24}$$

For the obtained flow q_j it is calculated output power of hydroplant P_j, and when its power P_j was reduced for the power of demand P_D, it is obtained the remaining power that is needed to produce by steam plant. That remaining power is optimized on the basis of the principle that is that is known as equal incremental cost-loading principle. That principle is given in chapter III.B. If it is obtained that the optimized power of an steam plant is less than the allowed minimal power, than its obtained power is equated with minimal power. After that, process of steam optimization is run again. An analogous procedure is carried out for the case when constraint of maximal power was reached.

The problem of hydro-thermal coordination with dynamic programming is based on finding optimal trajectory of water release whereby the total fuel cost of steam plant must be minimized.

If we consider i as denoted the volume states on the start of interval j, and with k as denoted volume states in the end of interval j. Further, let with $TC_k(j)$ is denoted the total cost from the start of scheduling period to the end of period j for the reservoir storage state V_k, and with $PC(i, j\text{-}1;\ k, j)$ is denoted product cost of the thermal system in period j from an initial volume V_i to volume V_k of end of period [1]. Further, algorithm of dynamic programming is given with next equations:

$$TC_k(0) = 0, \tag{25}$$

$$TC_k(j) = \min[TC_k(j-1) + PC(i, j-1; k, j)]. \tag{26}$$

Therewith, the trajectory must be chosen so that the constraints of initial and final volume state are satisfied, such as it is shown on Fig. 4. [1].

The value of volume V_i and V_k can be limited, because of the possible limits of volume states or because of some other non-technical constraints [1].

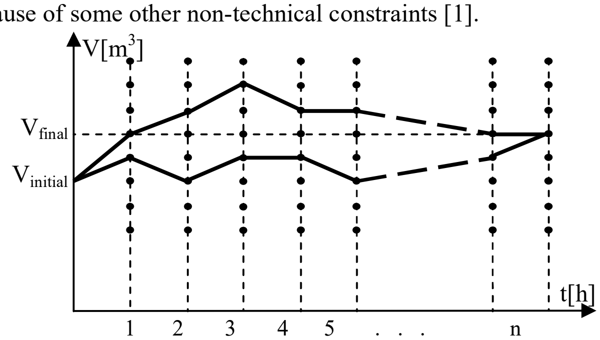


Fig. 4. Possible trajectories of water release

5. THE SOFTWARE IMPLEMENTATION OF DYNAMIC PROGRAMMING FOR HYDRO-THERMAL COORDINATION

A. Review of program of hydro-thermal coordination

The program of hydro-thermal coordination, that is realized using MATLAB and GUI interface, is shown on Fig. 5. The program was implemented for coordination of production from a hydroelectric plant and more steam plants.

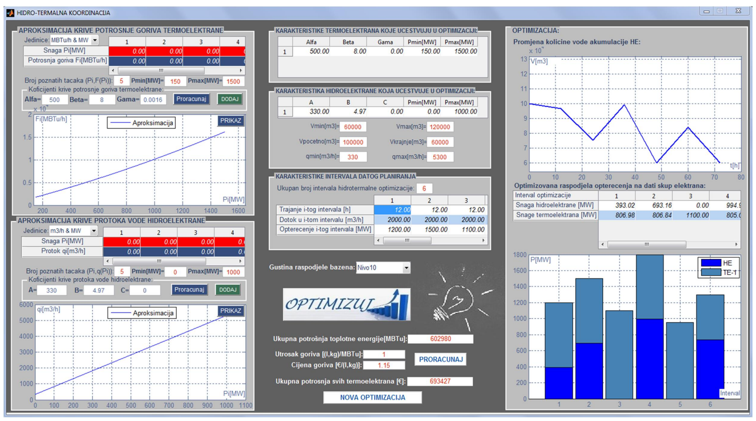


Fig.5. Display of program of hydro-thermal coordination

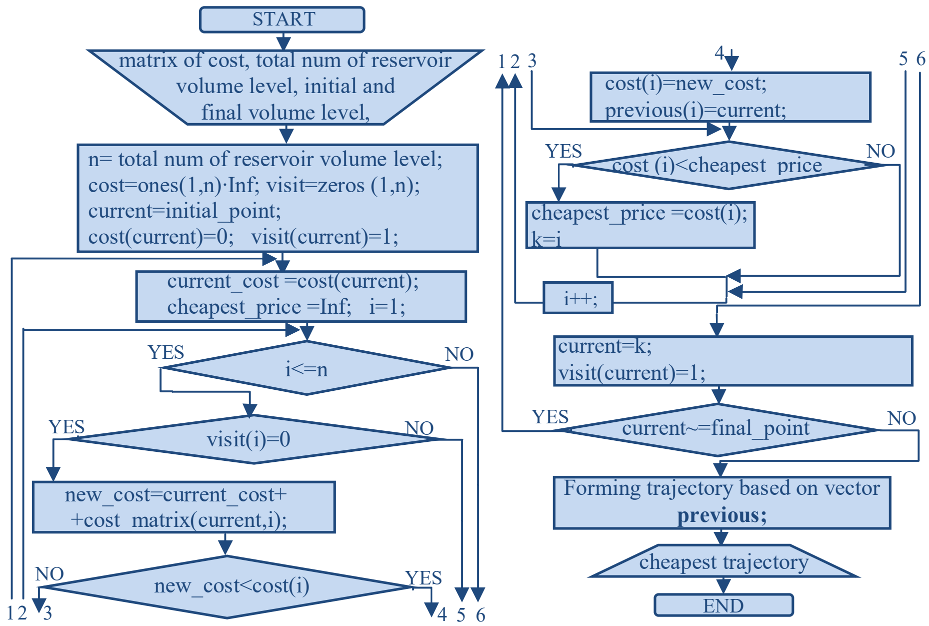
Entering basic data is performed via the graphical user interface of interactive objects. Entering parameters that describing the fuel cost curve of steam plant is carried out by directly entering the coefficients α, β and γ or by entering the table of known fuel consumption, on the basis of which it is calculated the aforementioned coefficients. Analogously it is performed input/calculation of *A*, *B* and *C* coefficients of hydropower characteristic. Besides the power consumption curve coefficients, program requires data of the maximum and minimum power that can be provided by plant. Hydropower plants, for description, need more inputs, unlike steam power plants, which are: minimum and maximum allowed flow rates, minimum and maximum level of the volume of water in the reservoir storage, the initial amount of water in the reservoir and the ultimate level of storage capacity that is needed to accomplish by proper use of water for a given period of planning. Output data of optimization are shown in tabular and graphical. The table shows the optimized load distribution on a given set of plants and also that load distribution is shown graphically. Also, the graphic is displayed trajectory of volume change of reservoir storage.

The level of precision optimization is adjusted by changing the level of density distribution of the reservoir storage. Of course, the higher the density distribution of reservoir results the greater precision of optimization.

The optimization process begins by forming a matrix of path costs between the individual points of the reservoir volume and by forming vector of redistribution of load between the plants for given path of the changes in the volume reservoir, i.e. it is being formed three-dimensional vector.

B. Use of Dijkstra's algorithm to determine the cheapest trajectory of storage volume changes

After matrix of costs were formed, the proces of finding the optimal trajectory is being started. That proces of optimization is realized by using Dijkstra's algorithm that is modifed and adapted for this problem of optimization. Dijkstra's algorithm is executed over a matrix of distance between nodes, but for this problem that matrix was replaced with matrix of cost. The algorithm that was used for the optimization process is shown below.



C. Use of the realized program of concrete example of hydro-thermal coordination

In addition, it is given an example for hydro-thermal coordination that is taken of reference [1], which was realized by gradient principle for short-term optimization:

HP: $q[m^3]=407049+6130.4\cdot P_H$, $0MW \le P_H \le 1000MW$.

SP: $F=527.53+8.44\cdot P_T+0.00169\cdot P_T^2 [MJ/h]$,

$150MW \le P_T \le 1500MW$, Fuel cost: 1.09 [*R/MJ*].

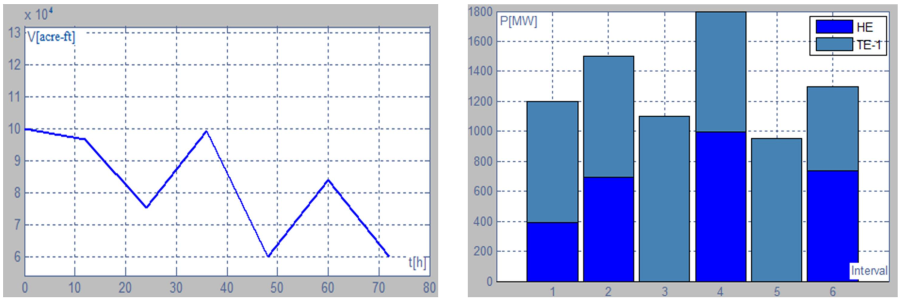
Demand:

1th day: 24:00-12:00 1200 MW, 12:00-24:00 1500 MW
2th day: 24:00-12:00 1100 MW, 12:00-24:00 1800 MW
3th day: 24:00-12:00 950 MW, 12:00-24:00 1300 MW

Hydro-accumulation:

123348183m^3 (100000 *acre-ft*) - at the beginning, 74008910m^3 (60000 *acre-ft*) - at the end,
74008910m^3 (60000 *acre-ft*)$\le V \le$148017820m^3 (120000 *acre-ft*),
2466963.68m^3 (2000 *acre-ft/h*) - inflow.

The results of hydro-thermal coordination which were calculated with implemented program, that was defined before, are shown on Fig.6.



a) b)

Fig.6. Results of hydro-thermal coordination: a) Deviation of accumulation volume amount, b) Optimized load distribution of power plants

The optimization of previous example with the realized program produces the total fuel cost of 693428R for the whole period of planing, while the total fuel cost, which is obtained by gradient principle, is 709877R.

6. CONCLUSION

Efficient and economical use of existing power facilities are some of the basic requirements of the power system. Except optimization with dynamic programming today there are more methods for optimization, namely coordination of electricity production, like: method of Lagrange multiplier, gradient method, etc. The advantages of these methods, in the case with more hydro plants that are included in hydro-thermal coordination, are simpler programming and faster calculation than method of dynamic programming. The advantage of dynamic programming is that there are no iterations and tests of convergence in its execution, which are weak points of previously mentioned methods and can cause that calculation doesn't converge. So, the motivation for using of Dijkstra's algorithm for dynamic optimization is that in case of real request it can be always expected an acceptable solution into practice.

REFERENCES

[1] Allen J. Wood, Bruce F. Wollenberg, Gerald B. Sheblé ,"Power Generation, Operation, and Control", december 2013.
[2] Mohamed E. El-Hawary, "Electrical Power System-Design and Analysis", 1994. god.
[3] Prof. dr Ilija Vujosevic, "Eksploatacija i planiranje elektroenergetskih sistema", Lectures, Faculty of electrical engineering in Podgorica, unpublished.
[4] Prof. dr Slobodan Djukanovic, "Programiranje I", Lectures, Faculty of electrical engineering in Podgorica, unpublished.
[5] "Dijkstra's Algorithm", Melissa Yan, MIT, Massachusett, Available: http://math.mit.edu/~rothvoss/18.304.3PM/Presentations/1-Melissa.pdf, [Accessed: January , 2017].