



Applications of Tulip Motif in Turkish Art with Geometer's Sketchpad Program

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Abstract

In the 21st century we live in, technology is developing rapidly. Inevitably, the technologies used in almost every area of daily life will also reflect in the field of education. Educational technologies, which enable students to grasp abstract concepts more easily and facilitate the teaching process for teachers, have increased their impact in schools. The effects of dynamic geometry software on course success, attractiveness, and concretization in mind have been the subject of different studies. Dynamic geometry software that can visualize algebraic expressions with graphics creates an interdisciplinary working environment with its drawing features. Thanks to Geometer's Sketchpad (GSP), one of this software, students can dynamically create very different patterns and shapes. Students can realize higher-level cognitive learning thanks to the relationships and inferences they make on these shapes. These and similar patterns that emerged thanks to GSP can increase students' awareness in different fields by combining different disciplines such as history, mathematics and art. In this study, the drawing stages of the tulip motif, which we come across in important architectural works in the Ottoman and Anatolian Seljuk history, which have been the subject of ornament art, are shown via GSP using both the transformation geometry and functions.

Keywords: Geometer's Sketchpad, transformation geometry, function graph, tech-assisted math, tulip motif.

1. Introduction

The development of computer technology affects educational technologies as well as different fields. New developments in technology, computers and communication also led to changes in the understanding of teaching and have brought the use of new techniques and methods in today's teaching. In the Information Age we live in, new concepts and technologies such as computers, multimedia, sound, image, animation, Internet and developing Internet technologies have taken their place in education and training (Alakoç, 2003). Today, instructional technology is considered as the development and use of technology to facilitate and improve learning. With the use of educational technologies in learning environments, the tools that help develop the high-level thinking skills expected from a 21st century individual have been the subject of many discussions. In order to raise individuals who can solve real-life problems in our age, it is necessary to gain various skills to individuals first. These skills are reasoning based on knowledge, organizing data, generalizing, proving and problem-solving (Toluk, 2003). Educational technologies are preferred in terms of concretizing abstract concepts, attracting students' interest

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in the lesson and providing effective feedback. These advantages provided to the users of educational technologies provide convenience to the users in the fields of mathematics and geometry as well as in every discipline. Dynamic geometry software with ease of use and great interfaces in mathematics-geometry are more common in classrooms.

- Computer technologies facilitate the learning of mathematics and geometry.
- Interdisciplinary approaches such as mathematics, history and art attract students' attention and increase their motivation.
- Geometer's Sketchpad program is used to acquire transformation geometry topics.
- Different patterns are drawn with function graphs via Geometer's Sketchpad.

Mathematics, a system on its own with its language, consists of structures and relations. Mathematics is an abstract concept that includes the generalization processes formed by these structures and relations. It is also known that mathematics is difficult for students because it is difficult to acquire abstract concepts. For this reason, mathematics teaching methods are a subject that should be especially emphasized in our age. Teaching appropriate to the structure of mathematics helps students understand concepts and operations related to mathematics and establish the links between these concepts and functions (Alakoç, 2003). In recent years, the integration of technology into mathematics and geometry lessons aims to ensure students' active participation and students better understand the achievements of abstract mathematics and geometry. In mathematics education, creating learning environments that include exploratory educational activities in which students can actively participate positively affects innovative and creative thinking skills (Yani & Oikawa, 2019; Azizah et al., 2018). Skills such as creativity and critical thinking expected from 21st century individuals can be developed through interdisciplinary studies and effective use of technology.

With the development of educational technologies, different software has started to replace the education made by using only blackboard-chalk, pencil-paper-ruler in the field of geometry, especially in mathematics lessons in schools. Thanks to information and communication technologies, students can easily understand the background of the abstract concepts they learn through multimedia in mathematics and geometry lessons. Different technological products can help students model the concepts in primary and secondary school mathematics and geometry course outcomes and contribute to the problem-solving stages. In particular, dynamic geometry software has become an important tool for its users due to its ability to visualize abstract concepts. There are studies in which learners contribute positively to their success with Computer-assisted learning with increased functionality via dynamic geometry software (Bedir, 2005; Sulak, 2002; Aktümen & Kaçar, 2003). GeoGebra, Cabri Geometry, Geometer's Sketchpad are the most preferred dynamic geometry software. Dynamic geometry software is very useful because it allows the creation of geometric structures in two and three dimensions and the movement of these structures (Karaarslan, Boz & Yıldırım, 2013). Geometer's Sketchpad (GSP), one of this software, provides a student-centered interdisciplinary work environment by enabling geometric shapes to move, creating different shapes, measurements and patterns (Vatansever, 2007). Geometer's Sketchpad software can be used at all stages of learning, from primary to higher education. The suitability of using Sketchpad software on interactive boards is very useful in the classroom environment. It can be used for ratio-proportion activities, algebra and graphic drawings, modeling of rational numbers, applications on the number axis, and geometry patterns in primary education. In the secondary education stage, GSP software can be easily used in transformation geometry and function graphs (Karaarslan, Boz & Yıldırım, 2013). Transformation geometry includes coating and decorative arts in different branches such as mathematics, geometry, visual arts, and technology design. Transformation geometry, which provides interdisciplinary work, lays the groundwork for developing high-level thinking skills such

as creative thinking skills and critical thinking. Combining the studies created using creativity with educational technologies, GSP can facilitate their learning by attracting attention to subjects such as transformation geometry and functions.

The interdisciplinary approach can be defined as bringing together traditional subject areas in a meaningful way, especially around common concepts. The interdisciplinary teaching process helps both learn the knowledge and skills of certain disciplines and use them in a meaningful way. Although the interdisciplinary approach in teaching is not a new understanding, it has become an approach that has been emphasized in recent years. One of the most important purposes of interdisciplinary teaching is to provide students with a versatile way of thinking. The ability of the student to integrate knowledge in different fields in critical and creative thinking and decision-making processes gains great importance today.

Altun (2005) defined mathematics, saying, "Mathematics is a way of thinking, an art, a study of structures and relationships." Via GSP, which allows learning by experiencing the relationships, harmony and different aspects of geometric shapes, an interdisciplinary application can be made with traditional patterns and motifs. In this study, the "tulip" motif, one of the traditional Turkish patterns, was applied using GSP software.

The tulip, which heralds the spring by blooming with colorful flowers, symbolizes life and fertility for the Turks. It is known that the homeland of the tulip, which is a bulbous and perennial ornamental plant, is Central Asia and spread to Anatolia from there (Altuntaş, 2019). tulip motifs are frequently seen in works from the Seljuk Empire. Tulip lived its heyday in the Ottoman Empire between the 16th and 18th centuries. During the reign of Ahmet III (1673-1736), its use as an ornamental plant and ornamental motif peaked, and it was called the "Tulip Age" between 1718-1730 (ILAV, 2021). In this period, newly built gardens, palaces and pavilions were equipped with tulips, and the love for tulips increased a lot. Spreading from Anatolia to Europe, the tulip attracted attention worldwide and became influential in the fusion of different cultures.

One of the reasons why the Ottomans so accepted the tulip is that it is written in Arabic letters as (ل ل ه) and it is the same as the letters in the word Allah (الله). When the tulip is written in Arabic letters and read backward, the emblem of the Ottoman state (ل ه ل), the Crescent, that is, the Moon appears (İLAV, 2021).

The tulip, which is also included in mythology, was formed by being frozen by a lightning strike on the dew on a leaf. The burn scar from the lightning is hidden in the blackness of his belly. The tulip, which is widely used in poems, is likened to the lover's cheek, the glass of wine, the wound, etc. (Hakverdioglu, 2008).

In history, tulip took place in many works of art such as Sufism, garden culture, wood carving, tile, ceramics, architecture, poetry, literature, weaving culture (Altuntas, 2019). When the literature is reviewed, it is seen that tulip motifs are utilized in Central Asian Turkish Art (Kuru, 1997). Drawings of tulip motifs on the door decorations of Ibrahim, I Madrasa in Samarkand in Kuru's work draw attention. Besides its historical meaning, the tulip, which maintains its value in today's modern lines, has a great place in our culture. In our daily life, a tulip pattern can be seen almost everywhere. Examples of tulip motifs in different areas are given in images 1, 2, 3, 4.

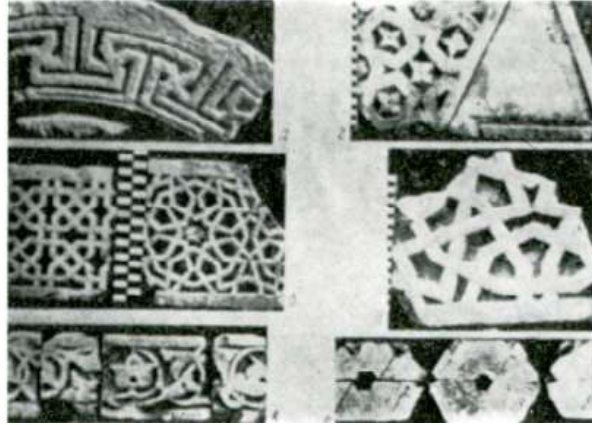


Image 1. Tulip motifs on the door decorations of Ibrahim I Madrasa in Samarkand

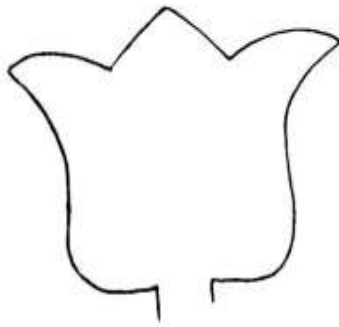


Image 2. Tulip motif drawing in Kuru's essay



Image 3. An example of "tulip motif" in handicrafts



Image 4. An example of Turkish Airline's tulip motif in advertisements

2.Theoretical framework

The National Council of Teachers of Mathematics (NCTM) (2000) emphasizes the importance and necessity of technology in helping students acquire mathematical knowledge and skills from kindergarten to 12th grade. The use of technology in mathematics teaching enriches the teaching environment by presenting multiple representations (numerical, algebraic and graphic) (Erbaş, 2005). With the help of technological tools, students can form their hypotheses with the data they have while solving problems, test their hypotheses and generalize their results to similar situations (Erbaş, 2005; Güveli & Baki, 2000). The use of technology in mathematics lessons makes the lesson interesting and keeps the concepts in mind (Baki & Özpinar, 2007) and helps students move away from memorization (Güveli & Baki, 2000). Dynamic geometry software can visualize drag tools and abstract geometry concepts (Sinclair & Yurita, 2008). In addition, this software provides users with a visualized basis for the proof of theorems by establishing connections between them in the digitization of geometric shapes (Bintaş & Smart 2008). Çalışkan, Gökçe, and Önal (2019) stated that dynamic geometry software would be beneficial in the lesson designs prepared by the teachers by the achievements and target behaviors in their classrooms. This software is very valuable in providing an environment where students can generalize by visualizing mathematical symbols (Güven, 2002). Cabri, GeoGebra, Geometer's Sketchpad programs are frequently preferred among dynamic geometry software in terms of ease of use.

When interdisciplinary studies on combining two different fields are examined, some studies are encountered which examine students' attitudes and indicate that their attitudes towards these disciplines have improved positively (Lou & Chen, 2013; Güder & Gürbüz, 2018; Ürey, Çepni & Kaymakçı, 2015). Another example of interdisciplinary studies is the combination of mathematics and art. Throughout history, geometric shapes have different religious and philosophical meanings. Dönmez (2020) states the frequently used meanings of some geometric shapes as follows.

“The hexagon, which is a shape close to a circle, is the sky symbol. The square shape represents the earth and matter. If the rectangle is used horizontally, it gives a balancing effect, and when used vertically, it gives a caring effect. All these shapes are essentially basic geometric figures that can be derived from the circle.”

Mathematics is intertwined with art both in its discipline and in many artistic applications such as painting, sculpture, music, architecture, and decorative art. (Adam, and

İşleyen, 2005). In teaching mathematics, students' discovery of mathematics in nature and structures and seeing the aesthetic dimension of mathematics will enable them to develop positive attitudes towards mathematics and learn better (Atasay & Erdogan, 2017.) When the literature was reviewed, it was seen that artistic works created with the GSP program were included (Bodner, 2003; Majewski & Wang, 2009; Marani, 1997; Bodner, 2004.) In these studies, the drawing feature of the GSP program was generally used. Studies in which traditional handicrafts combine mathematics and art by using GSP draw attention. For example, banana stem carving, one of Thai traditional handicrafts, is designed with the help of mathematical functions using GSP (Sangwaranatee & Suraprap, 2010).

Drawing the tulip motif, which has such an important place in our history, with Geometer's Sketchpad program can increase students' awareness of technology, history and art relations. In this study, from past to present; the tulip motif, which has been the subject of many branches of art such as mysticism, garden culture, wood carving, tile, ceramics, architecture, poetry, literature, and weaving culture, is drawn in two different ways with the help of GSP.

3. Methodology

In this section, the tulip motif is created in GSP both with the drawing feature of the program and with the help of function graphics.

3.1 *Drawing of tulip motif in GSP using transformation geometry*

The Iznik tile model of the tulip motif given in Image 5 is discussed. Iznik tiles are important in reflecting the characteristics of Roman, Byzantine, Seljuk and Ottoman art in history. Iznik tile art brought a new breath to tile art by adding flower motifs to classical tile patterns in the 16th century. The most beautiful examples of tiles produced in Iznik can be seen in structures such as Süleymaniye Mosque, Kanuni Tomb, Hürrem Tomb, Rüstem Pasha Mosque, Topkapı Palace, Şehzade Mustafa Tomb, Eyüp Sultan Mosque.



Image 5. Iznik tile model

Before determining the process steps in the drawing phase, each motif in the model to be drawn was examined, and motifs that repeated or continued each other were determined. When the Iznik tile model in Picture 5 is examined, it is noticed that there is a vertical axis of symmetry. When a line axis is drawn from top to bottom in the middle of this model, it is observed that the parts on the right and left sides are symmetrical to each other (Image 6).



Image 6. Vertical symmetry axis of Iznik tile model

In creating this model, the whole model was obtained by drawing half of the figure vertically using the GSP features. Then, the circle-like patterns in the four corners of the Iznik tile pattern given in Image 5 were examined. It has been observed that 90-degree rotations and translations form the shapes resembling circle patterns at the corners of the tile motif. For this reason, when starting the process steps, it was started to be implemented by using the circle drawing feature of the GSP so that the frame was a circle and the degrees to be used would resemble the shape aesthetically. The drawing stages of this model are shown below step by step, using the properties of GSP and transformation geometry.

- 1. Step:** To create the frame of the pattern, a full circle is drawn by selecting the “circle” tab from the left menu. To calculate the vertical axis of symmetry and the degrees of rotation, the drawing is started (Figure 1)

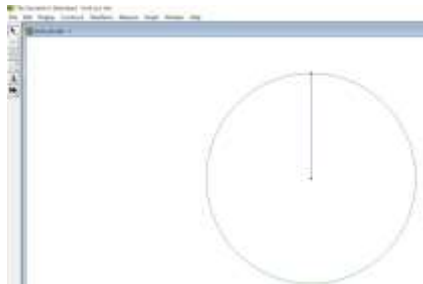


Figure 1. The window in GSP

The small circle-like motifs seen in Figure 3 are located at the four corners of the whole figure. To place these motifs in the figure, a total of 45-degree circle segments are created considering that 360 degrees are divided into four 90-degree parts as in Figure 2,

- determining the radii from the center, using the “Rotate” tool from the “Transform” menu, and rotating 22.5 degrees twice.
- By rotating the radius shown in red in Figure 2 by 22.5 degrees, the locations of the small circle patterns in Figure 3 are determined.



Figure 2. The initial stage of the tile pattern in the GSP Figure 3. $\frac{1}{4}$ visible part of the tile pattern

2. Step: The middle points of the circle segments created in Figure 2 are combined as in Figure 4, and the tile pattern is started to be created. The midpoint between the two radii is selected and joined. Thus, two symmetrical small circle motifs are created.



Figure 4. Beginning of symmetrical circle motifs

To complete the small motif given in Figure 3, the half-line segment in the middle is marked as the center, and its mirror image is taken through “Reflect” in the “Transform” menu as in Figure 5. A small circle is drawn in the middle. The reason for drawing the center circle is to assist the formation of the motif symmetrically.

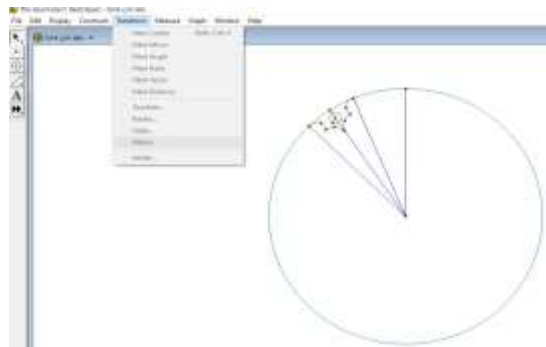


Figure 5. Drawing of the center circle

3. Step: To complete the small circle motifs in Figure 7, a second concentric circle is added around it; as in Figure 6, intertwined circles are drawn on the outer part of the formed slice. Thus, small circles in the tile pattern are created.

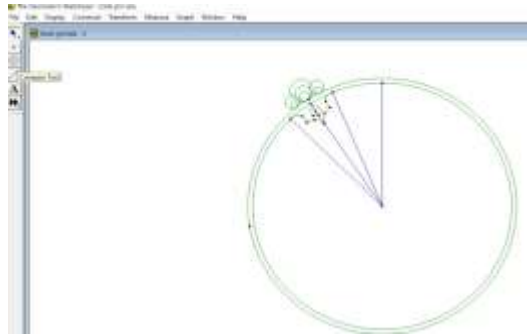


Figure 6. Formation of small circles in the tile pattern



Figure 7: 1/4 part of the tile pattern

4. Step: To proceed to color the Iznik tile pattern in Figure 7, the points in the pattern in Figure 6 are determined, and the first step of the coloring process is started.

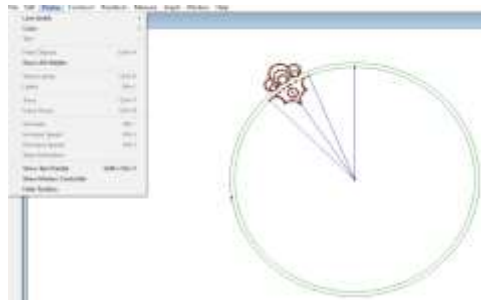


Figure 8. Determination of spots before painting

With the help of the points shown in Figure 8, the colorings are made according to the Iznik tile pattern from the “color” tab in the “Display” menu, as seen in Figure 9.

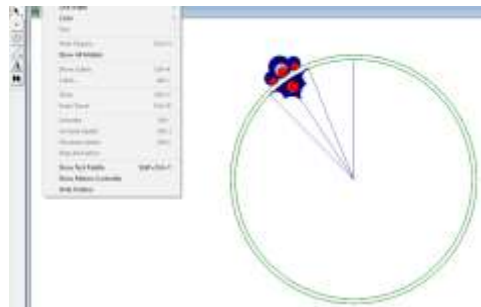


Figure 9. Beginning of the coloring process

5. Step: Since the small circle motifs created are located at the figure's diagonal corners; they are created symmetrically. The pattern in Figure 9 is selected and rotated 112.5-67.5-112.5 degrees, respectively, using the "Rotate" tool in the "Transforms" menu according to the circle's center. Thus, for the same motifs to be located at the four corners of the figure, reflection, translation and symmetry operations using 90' and 45' degree differences are applied using the "Rotate" and "Transform" features in GSP, and Figure 10 is obtained.

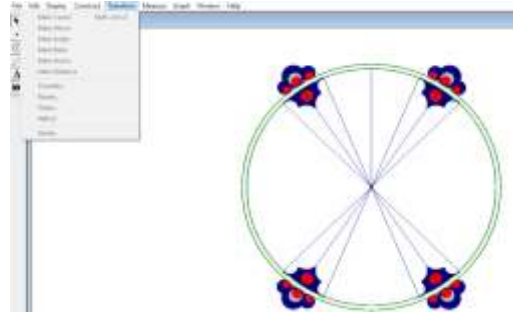


Figure 10. Mirroring and rotating colored parts

6. Step: The Iznik tile model given in Image 5 consists of two equal vertical parts. For this reason, the diameter of the circle is considered the axis of symmetry. For this reason, the motifs in the half of the pattern are drawn first, and then the pattern is completed by performing the reflection process. The points that will form the drawing of motifs such as tulips and leaves in the tile motif are determined and combined. The points determined by the "Arc Through 3 Points" tool in the "Construct" menu are combined, and Figure 11 is obtained.

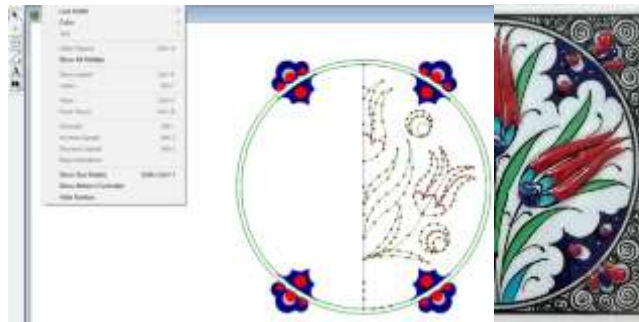


Figure 11. Drawing 1/2 of Iznik tile pattern; Figure 12. 1/2 part of Iznik tile pattern

7. Step: The part whose drawing is completed in Figure 11 is colored following the pattern in Figure 12. Figure 13 is obtained by hiding unnecessary lines and points with the "Hide Objects" tool in the "Display" menu.

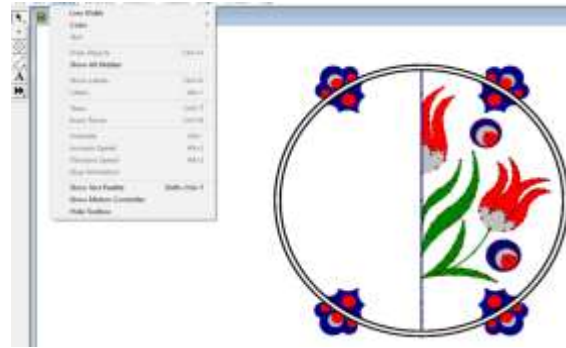


Figure 13. Coloring 1/2 part of Iznik tile pattern

8. Step: The circle's diameter is marked as the center, and the pattern is reflected on the opposite side with the "Reflect" tool in the "Transform" menu. Thus, the whole model is obtained by drawing only half of this model with the help of the drawing feature of the GSP (Figure 14).



Figure 14. Completed view of Iznik tile pattern

3.2 Drawing the tulip motif in GSP with the help of function graphs:

It is seen that the tulip motif is also included in cross-stitch works, which is one of the handicrafts, as in every field of art. Cross-stitch works, which can be found worldwide and exhibited in public museums, also have an important place in Turkish handicrafts, and it is inevitable to use the tulip motif as a cross-stitch. A tulip pattern was created with crosses embroidered on etamine, and cross-stitch work is given in Image 3. To draw the tulip motif, which has an important place throughout history, using functions in the GSP program, it is thought that the symmetry axis passes through the middle of the pattern, as in Image 7.

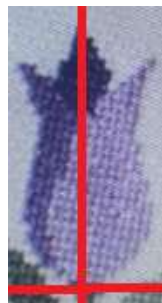


Image 7. The vertical symmetry axis of tulip pattern

Considering the symmetry axis of this tulip motif as the y-axis and the lower end as the origin, it can be said that the three functions given in Image 8 are needed. After taking the

parts of these functions in the specified intervals, the symmetry concerning the y-axis is taken, and the tulip motif is completed by following the steps below.

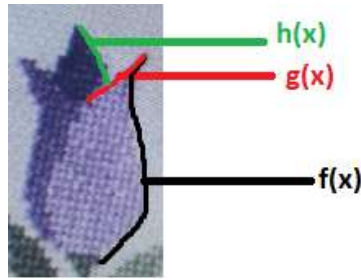


Image 8. Functions determined in tulip motif

1. Step: Figure 8 shows the parts of the tulip to be drawn. After deciding to plot $f(x)$ with a third-degree polynomial function, $g(x)$ with a sine function, and $h(x)$ with a linear function

$$f(x) = (x-1.5)(x-1.5)(x-1.5)$$

$$g(x) = 5 + \sin(x + 4.7)$$

Figure 16 is obtained by determining the $h(x) = -x + 7$ functions and writing them in the GSP program as in Figure 15.

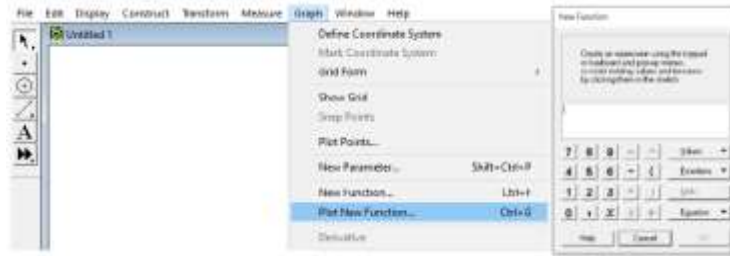


Figure 15. New function writing in GSP

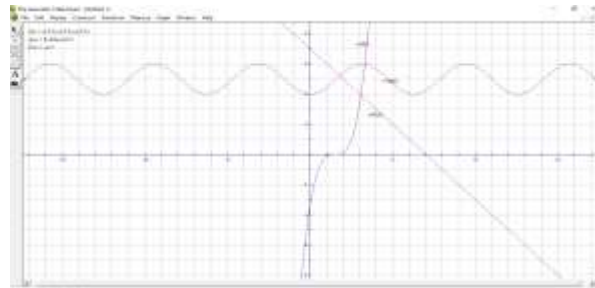


Figure 16. Display of desired function graphs in GSP

2. Step: To get the necessary part of the function graphs in Figure 16, the ranges are determined for the functions. The selected function is right-clicked, and the desired range is written to the Properties tab as in Figure 17. For functions $f(x)$ and $g(x)$, x is selected in the range of $[0, 3.32]$, for function $h(x)$, x is selected in the range of $[0, 1.8]$, and Figure 18 is formed.

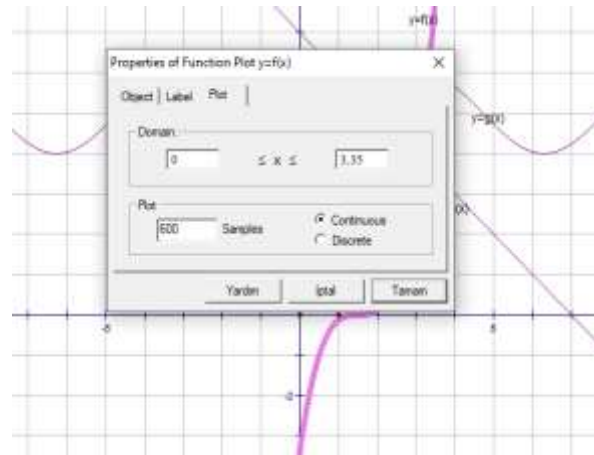


Figure 17. Determining the range of the function in GSP

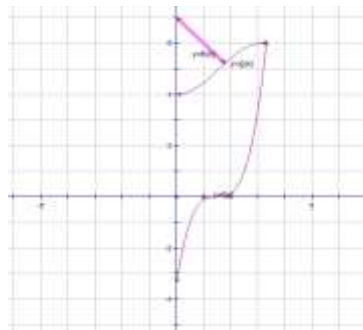


Figure 18. Ranged functions

3. Step: In the resulting form, points are taken on the functions. Then, the points are selected, and the areas created with “Ctrl+P” are right-clicked, and the desired color is painted with the “color” option, and Figure 19 is obtained.

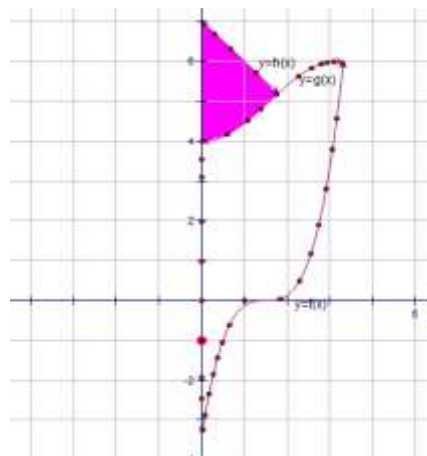


Figure 19. Selecting points on function in GSP

4. Step: After half of the motif is painted, the function names and points are hidden with the hide option by right-clicking, and Figure 20 is formed.

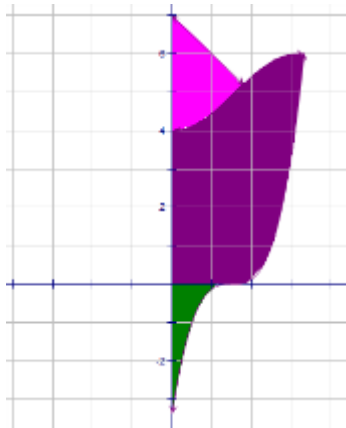


Figure 20. Creation of half of the tulip motif in GSP

5. Step: The painted areas are selected, the y-axis is double-clicked, and the half-formed motif is symmetrized to the y-axis from the transform menu, and Figure 21 is formed.

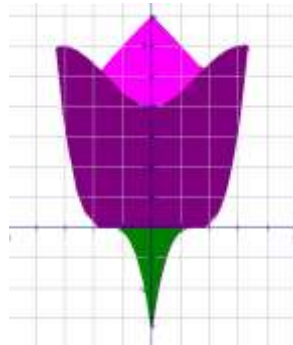


Figure 21. Tulip motif created in GSP

6. Step: Then, leaves are added to the tulip motif obtained in Figure 21 with the help of parabola functions. Similar steps are followed with the tulip motif to draw the leaves.

For leaves,

$$q(x) = \frac{1}{12}x^2 - 3 \text{ ve } r(x) = -\frac{1}{8}(x-3)(x-8)$$

Functions are determined. With the plot new function, Figure 22 is formed.

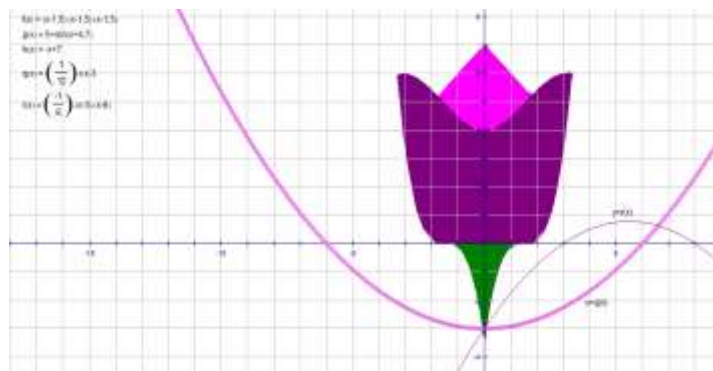


Figure 22. Creating leaves in GSP

7. Step: The intersection of functions $r(x)$ and $q(x)$ in the range of $x [0,6.6]$ is taken and figure 23 is obtained.

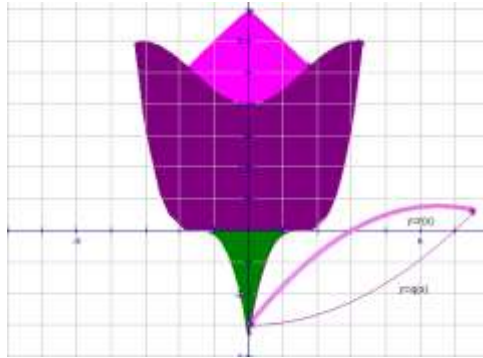


Figure 23. Display of leaves in the desired range

8. Step: Points are created around the leaf. The area is created with the selected points and painted green, and Figure 24 is formed.

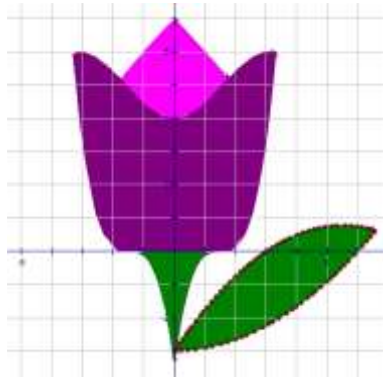


Figure 24. Leaf painted in GSP

9. Step: By selecting the painted leaf, its symmetry concerning the y-axis is taken with reflection from the transform menu, and figure 25 is formed.

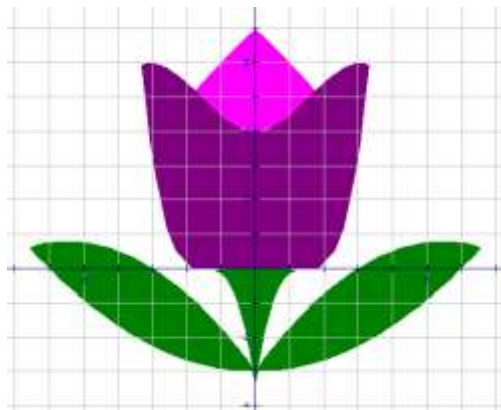


Figure 25. Double leaf tulip motif

10. Step: Then right-click and hide all lines on the plane. A pattern close to Image 3 is created using combinations of tulips in different colors, and Figure 26 is obtained.



Figure 26. Tulip pattern created in GSP

4. Conclusion and recommendations

From Central Asia to the present, tulip has an important place both as a flower and a motif in many art fields. The historical and artistic value of the tulip motif, which we encounter almost everywhere, will motivate secondary and high school students while gaining transformation geometry gains.

According to Bintaş and Akıllı (2008: iii):

“Dynamic geometry programs can be used to make teachers’ learning environments practical, as well as for teachers to create a constructivist teaching environment. These technologies help the student reach a higher cognitive level. It makes it easier for the student to establish relationships and make inferences on geometric shapes. Dynamic geometry programs enable students to reach assumptions and inferences on geometric shapes with the help of a drag-and-drop processor, thanks to a series of constructivist activities and guiding questions in learning environments. Students are encouraged to formulate theorems and form their deductions. Dynamic geometry software allows students to create various geometric shapes in the virtual environment, establish relationships between these shapes, and change this draft according to their wishes. The software also allows the desired measurements and comparisons to be made on this structure”.

According to İpek and Özmüş (2014), it was observed that teacher candidates had a positive attitude in the creation of Anatolian Patterns with GSP, and they could easily benefit from geometric concepts while creating the pattern or motif they chose.

Tulip motif drawing with the GSP program can increase students' motivation about transformation geometry and lead to easier learning. In addition, the fact that high school students can draw tulip motifs with the help of functions they are familiar with will show them that they can create different patterns or motifs with the help of the GSP program, which may increase their interest in function graphics.

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