

EVALUATING THE APPROPRIATENESS OF TASKS AND THE ELABORATION OF MULTIPLE SOLUTIONS TO OCCASION FOURTH-GRADERS' MATHEMATICAL CREATIVE THINKING

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Abstract

Creativity is not only for gifted students, but also for regular ones. This case study was aimed to analyze the appropriateness of tasks and the elaboration of multiple solutions to occasion fourth-graders' mathematical creative thinking through a documentary multiple-solution counting task in a figurative setting. The data came from the written report of 48 fourth graders in two classes in Taiwan, China. The appropriateness of creativity was reflected in the appropriateness of tasks and solutions, particularly suitable for complex problem solving. Elaboration was detail-dependent, and visualization was beneficial to the analysis of elaboration. The regular students who had just entered the fourth grade could show their creative thinking through different angles (horizontally or vertically) and starting points (holistic or partial), but with slightly more partial and horizontal than holistic and longitudinal, more adaptation than transformation. These fourth-grade students have had the basic mathematical creative thinking capability of adaptation, combination, change, rearrangement, extension or going back by using counting, combining, adding and reducing, overlapping, moving, and diagonal division strategies.

Keywords: *creative thinking, mathematical creativity, multiple-solution task, primary school students*

Introduction

Innovation consciousness and creative ability are not only the embodiment of global enhancement of national quality, but also the pursuit of reforming teachers' instruction and the development of students' literacy in the 21st century. Creativity manifested in the flexibility, novelty and openness of thinking, is a kind of high-level cognitive ability that needs to go beyond the traditional educational purpose. The traditional school education's standardized tests, teachers' direct instruction and students' rote learning become a stumbling block to the cultivation of students' creativity in school education (Akgul & Kahveci, 2017; Kim, 2011; Ma & Rapee, 2015). Since 2015, the Centre for Educational Research and Innovation of OECD has been leading a study on the framework of creative thinking in the primary and secondary schools to support cultivating students' creative ability (OECD, 2021). Taiwan's Ministry of Education (2001) pointed out that "the abilities to think creatively and problem-solving are basic abilities for future citizens of the world" at the beginning of the White Paper on Creativity Education. Primary and secondary schools in Taiwan have been carrying out various teaching model reforms based on mathematics classrooms since then, such as inquiry teaching, conjecturing teaching and so on, aiming at cultivating regular students' innovative thinking and problem-solving ability.

The cultivation and research of school mathematical creativity have received more and more attention from different perspectives for the last twenty years: The importance and feasibility of developing students' mathematical creativity in school education (e.g. Haylock, 1987; Silver, 1997; Sriraman, 2004); the effect of creative mathematical thinking cultivation in primary and secondary schools (e.g. Aizikovitch-Udi & Amit, 2011; Apino & Retnawati, 2017; Leikin, 2014; Schoevers et al., 2019; Tubb, et al., 2020); the multiple-solution tasks (MST for short) used to investigate the performance or potentials of school children in fluency, flexibility and novelty of creativity (e.g. Handayani et al., 2020; Kwon et al., 2006; Leikin & Lev, 2007; Levav-Waynberg & Leikin, 2012; Leikin, 2014; Sadak et al., 2022); The relationship between students' mathematical creativity and higher-order thinking ability, intelligence, insight and professional knowledge (e.g. Assmus & Fritzlär, 2022; Kahveci & Akgul, 2019; Leikin, 2013; Leikin, 2016); The interaction between cognitive process of problem solving and emotional state on creativity (Cai & Leikin, 2020; DeBellis & Goldin, 2006; Gilat & Amit, 2014; Kozlowski et al., 2019), etc.

Examination of the school mathematical creativity literature showed the patterns that studies mostly focused on the experiment of teachers' instructional interventions and few on the performance and characteristics of students' creativity under regular instruction context. More quantitative studies on the exploration of students' mathematical creativity in terms of fluency, flexibility and novelty (originality) through problem-solving, problem-posing or model-eliciting activities and less qualitative/quantitative studies on the appropriateness and elaboration of creativity and "seldom talks about students' creative thinking process of problem solving" (DeBellis & Goldin, 2006; Kozlowski et al., 2019). Most research studies supposed the quality of creativity by the quality of the multiple solutions to the task but "few studies actually measured the task-embedded potential for tasks to promote creativity" (Levenson et al., 2018, p.5). As results, there may be three reasons for this: Firstly, from the cognitive point of view, mathematical creativity is an abstract existence, which is related to the individuals' establishment of subtle ideas in their minds (Nadjafikhah & Yaftian, 2013). Secondly, compared with the visible creative outcomes (such as solutions), the creative thinking process is more implicit, while the elaboration and appropriateness depend on details of the situation, so it is difficult to achieve the quantitative indicators advocated by technical rationality in data collection and analysis. Third, tasks are often only a tool for research purpose, it is easy to ignore the function and value of the task in assessing or cultivating creativity. In fact, the task design should be appropriate because the task is a context which determines the cognitive level of how students think and understand the learning content (Doyle, 1988; Vale & Pimentel, 2011). For these reasons, the aim of this study is to analyze the appropriateness of tasks and the elaboration of multiple solutions according to the regular students' performance in the process of creative problem solving by means of visualization. The research questions are as follows:

- A. What are the characteristics of an appropriate task for evaluating regular students' creative thinking?
- B. How do the fourth graders perform the elaboration of the multiple-solution task?
- C. What are the characteristics of the fourth graders' mathematical creative thinking?

Previous Research on Creative Thinking in School Mathematics

Creativity generally refers to novel and valuable ideas or products (Guilford, 1967), and similar expressions are creative thinking or creative activity. School creativity is relative creativity, as "an ability to generate new mathematical ideas, processes, or products that are new to the students but may not necessarily new to the rest of the world, by discerning and selecting acceptable mathematical patterns and models." (Bicer, 2021, p.253) The work of a student trying to solve a geometric or algebraic problem differs only in degree and level

from that of a mathematician, and the essence between them is similar (Nadjafikhah & Yaftian, 2013). Relative creativity has changed the traditional concept that creativity was exclusive to gifted students, focusing on the flexible ability of regular students to engage in creative thinking (OECD, 2021).

Mathematical Creativity and Multiple-Solution Task

Creativity thinking is a synonym for creativity (Ervynck, 1991), which includes divergent thinking and convergent thinking. So far, most studies on evaluating creativity by means of creative thinking have focused on measuring the cognitive process of divergent thinking (Schoevers et al., 2019). Divergent thinking requires the ability to ask multiple questions or come up with multiple solutions. Convergent thinking requires the integration of various ideas to form a certain pattern or structure, which is high-level abstraction that is generally beyond the students' ability. Creativity is formed in the cognitive activities of discovering problems, trying to solve, rediscovering and re-solving them. The level of creativity is determined through the evaluation of the process and the results of the activity performance (Silver, 1997).

Creativity is based on the individual's prior knowledge and experience. The more knowledge an individual possesses, the more they can grasp and understand the relationship between knowledge, and the more likely they will generate creative ideas (OECD, 2021). However, the amount of knowledge is counterproductive to creativity, because on the one hand, individuals need to have sufficient knowledge in a field to produce creative ideas, and on the other hand, the existing knowledge may limit the way of thinking about, thus being unable to surpass the cognition of predecessors (Sternberg, 2006). A certain degree of relevant knowledge is a prerequisite for creative thinking, but the existing knowledge and cognitive inertia may also hinder the emergence of creative thinking.

Studies have shown that a task conducive to measuring and cultivating creativity not only needs to involve the necessary knowledge but also needs to be challenged (Leikin, 2014; Vale et al., 2012). A challenging task should: be able to use existing knowledge but require higher levels of cognitive activity; be located in the students' zone of proximal development with high cognitive needs; different representations or tools can be used, allowing students to have a variety of problem-solving methods and strategies. MSTs (have only one correct final answer but can be solved in different approaches) is an important form of challenging task, which is often used as a tool to measure or cultivate mathematical creative thinking. However, many studies focused on describing the kinds of tasks or the reasons for choosing them when assessing creativity, few studies discussed whether the tasks were appropriate to promote creativity. If a task mostly uses known routine or ready-made schema, the task can only be considered "exercise" (Vale et al., 2018).

Indicators and Visualization of Mathematical Creative Thinking

Because creative thinking is abstract and cannot be "seen" directly, researchers or teachers often need to take the creative performance or products as explicit indicators of creative thinking to make it visible by means of various transformations. By synthesizing the views of Torrance (2018), Haylock (1987, 1997) and Vale et al. (2018), this study holds the opinion that the indicators of school mathematics creativity can be constructed from appropriateness, elaboration, fluency, flexibility, novelty, abstractness, and openness. When these indicators are observed, the mathematical creative thinking is happening (Kozłowski et al., 2019).

Common Quantitative Indicators of Mathematical Creative Thinking

Torrance's Test of Creative Thinking (TTCT) is the most widely used and the most classic in the research on mathematical creativity, among which the most common quantitative indicators are fluency, flexibility, and novelty (originality). The fluency, flexibility and novelty of creativity are usually transformed into the total number of correct responses, the number of different kinds of categories/strategies/methods and the degree to which they are based on novel or insightful, allowing creativity to be quantified. Because these three quantitative indicators are concise and clear, they are favored by many quantitative researchers. However, researchers in mathematics education often pay attention to the creativity results of countable problem-solving solutions, they neglect to explore the specific situation of creativity (Schindler et al., 2018), resulting in some inappropriate solutions also being evaluated for creativity (Levav-Waynberg & Leikin, 2012). The statistical conditions of the three indicators have not received due attention.

Other Indicators of Mathematical Creativity

In addition to the three most commonly used indicators of creativity, a small number of studies have involved appropriateness, elaboration, abstractness and openness, which are considered to be a rather more complex procedure.

Elaboration, as the last and recently introduced quantitative indicator of mathematical creativity (Imai, 2010), generally refers to students' carefully thinking about specific aspects of the situation in problems (Vale et al., 2018), providing detailed explanations and justifications behind a solution path (Kozłowski et al., 2019), and related to the ability to describe, develop or expand ideas and details (Leikin, 2009), usually quantified by the extending or improving of methods (Haylock, 1987) or the number of new ideas added (Supriatin & Boeriswati, 2019; Vale et al., 2018). However, due to the difficulty in determining the details of creativity in mathematical tasks (Schindler et al., 2018), elaboration is not measured in the majority of studies.

The abstractness of creativity is proposed to be based on the abstractness of thinking (Haavold, 2018), which is the comprehensive process of themes and the organization process of abstract thinking (Bart et al., 2017). It is used to measure the degree to which themes surpass concrete ones.

Openness includes the openness of tasks and the openness of mind. The openness of tasks is relative to closeness, emphasizing multiple solutions rather than the only one correct answer. The openness of mind refers to maintaining an open mental state to resist premature closure of thinking (Bart et al., 2017). Openness is the ability to consider all kinds of information in the process of information processing (Kim, 2006), making thinking jump as much as possible to generate more ideas (Bart et al., 2017).

Appropriateness includes three aspects: First, the solution should meet the basic requirements and the constraints of the task (Vale et al., 2018). Novel ideas can be considered creative only if they are judged by explicit mathematical criteria for their appropriateness to the problem context (Haylock, 1987). Second, the task itself is appropriate, reflecting the internal consistency of test answers (Levine & Rubin, 1979). Third, there is both connection and difference between appropriateness and correctness. Correctness means the answer is flawless and accurate, appropriateness can refer not only to correct answers, also the answer may have flaws but the process is reasonable or understandable (Schindler et al., 2018). Appropriateness is particularly suitable for assessing mathematical creativity in complex problem-solving tasks (Leikin, 2013; Levav-Waynberg & Leikin, 2012; Schindler et al., 2018). Therefore, this paper uses appropriateness rather than correctness as a criterion of creativity.

Visualization

In order to analyze the complex and specific thinking process behind mathematical performance, various symbolic representation systems must be considered (Duval, 2006). Mathematics educators and researchers agree that visualization has great potential to enhance understanding in different areas of mathematics (Vale et al., 2018). Visualization as a visual perception thinking strategy, has the following characteristics: it helps students, teachers or researchers to understand and explore the complex and specific thinking process behind the problem (Gray et al., 1999); symbols (such as charts, formulas, words, graphic, figures, drawing, etc.) can be used as visual means to represent information in a situation (Arcavi, 2003). Symbols are abstract in nature, which can exist either as a single entity in mind or as a connected form of internal action for communication of listening, speaking, seeing, and writing (OECD, 2021). With the help of symbolic representations, this paper regards visualization as a visual perception thinking strategy to explore the regular education students' mathematical creativity.

Research Methodology

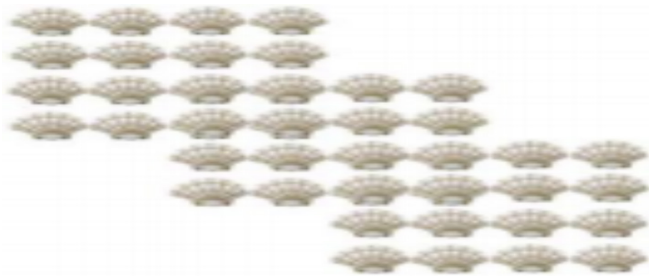
As mentioned above, many scholars used multiple-solution task to study the fluency, flexibility and novelty of mathematical creative products and a few did research on elaboration, abstractness, openness and appropriateness of creative thinking process of problem solving. This study used some visual tools to focus on the appropriateness and elaboration of creative thinking process and outcomes of problem solving by qualitative analysis. Due to space limitations, the openness and abstractness of thinking would not be discussed.

Test Task

The task was a visual counting pattern task adapted from the literature of Vale et al. (2018), which required students to transform visual patterns into arithmetic expressions to complete the calculation of 40 shells through the observation skills such as recognition, deconstruction, and reorganization in geometrical figure. Students' creativity was reflected in the different innovation of observation and figural patterns (Vale & Pimentel, 2011). In order to make the creative thinking process explicit, the participants were required to write corresponding thoughts on the paper in addition to draw out some symbols in the provided shell task diagram (see table 1).

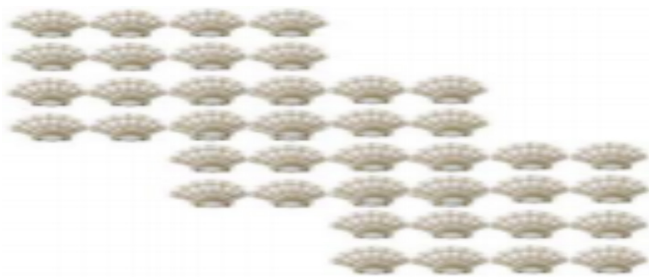
Table 1
Test Task

A second grader went to the beach to pick up shells. She organized the shells she picked up as the figure showed. Can you find out some ways to help her count them? Please discover as many methods as you can. Draw some symbols in the figure, and write down your thoughts, then list the formula for calculation.



My first formula is:

Here's what I think:



My second formula is:

I also think in this way:

.....
(The answer space is for 7 solutions, which can be increased)

Participants and Settings

In order to test the appropriateness of the task, 55 students in two third grades at one public school and 48 students in two fourth grades at another public school took the test in January 2022 and October 2021 respectively in Taiwan, China. All of the participants took the paper test in 20-25 minutes of a full 45-minute lesson in the afternoon. The two public schools were chosen because they were easily accessible, and the math teachers granted parents' permission for the study. Furthermore, the two classes have been implementing inquiry-oriented mathematics teaching reforms based on direct instruction since Grade One. After their mathematical teacher explained the specific task and told explicitly to find as many solutions as possible, the participants worked on the task independently. From the test results, some third-grade students used strategies of counting and combination, rarely used moving, and no one used strategies of adding and reducing or overlapping. Therefore, the data of the study was the written test reports of 48 fourth graders. Of the 48 participants were 26 boys and 22 girls.

The public primary school where the fourth grade is located is in Xinzhu, a newly-industrialized city in the northwest of Taiwan. The primary school has inclusive education, with about 1-2 special children in each class. The school unanimously pays attention to the reform of teachers' direct teaching and promotes inquiry-based mathematics teaching for more than ten years. One math teacher in the test class has been teaching for 23 years and is participating in

a university professor's conjecturing instruction experiment for about 8 years, the other math teacher has been teaching for about 6 years and was not a member of the experiment team but has been implementing inquiry-oriented mathematics teaching reforms based on direct instruction.

The mathematical content of the task combines multiplication within 100 and the area of rectangles and squares. Textbooks in Taiwan arrange these contents in the second term of Grade Two and the first term of Grade Three. Therefore, children of Grade Three and Four have just acquired the basic knowledge and skills to solve the shell task.

Data Analysis

In order to find out the performance and characteristics of the fourth-grade pupils' appropriateness and elaborations of creative thinking potentials, coding was performed according to which strategies and schemes were used. The participants' diagrams, words expressions, arithmetic formula were coded over whether these expressions coordinated each other. So, descriptive analysis, which consisted of two codes (strategies and schemes) was carried out in data analysis. The first step was to sort out the strategies and schemes of the multiple solutions by the author and a master student who had just studied the mathematical creativity framework under the guidance of an expert professor. The second step was to count the total number of problem-solving schemes and strategies and calculate the percentage (short for P) of each strategy in the total, then divided the strategies and schemes into conventional ($P > 40\%$), unconventional ($15\% < P < 40\%$) and innovative ($P < 15\%$) according to the percentage (Levav-Waynberg & Leikin, 2012). The third step was to descriptively analyze the appropriateness and elaboration of the solutions. The last step summarized the general performance and characteristics of the participants' creative thinking.

The author and the master student did the coding separately and then both of them reviewed the encodings three times in different time periods to ensure the reliability of the strategies and schemes. After the paper was completed, the author submitted it to the teachers who participated in the research for joint verification to ensure the reliability of the analysis results.

Research Results

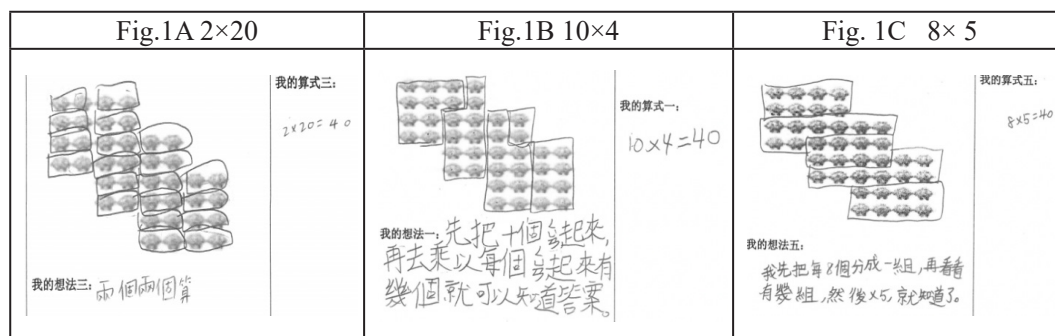
The task required seeing different patterns to count the number of shells. Students needed to observe the arrangement in different ways and translate them into arithmetic or geometric expressions. According to the results, some third and fourth grade students could reorganize or decompose the given figure into the area of rectangles and squares or/and the length of line segments for calculation, which showed that a visual counting pattern task was appropriate for the cultivation and assessment of creativity of the third and fourth graders in primary schools. However, there were subtle differences in the degree of task challenge. This kind of task seemed to be more appropriate for the fourth-grade students' independent inquiry than the third-grade students, so the data came from the written report of 48 fourth graders. The following firstly presented the types of multiple-solution strategies and patterns, then qualitatively analyzed the appropriateness and elaboration of creative performance of the participants.

Types of Multiple-Solution Strategies

By analyzing the solutions provided by the participants, it was found that there were six kinds of strategies such as counting, combining, overlapping, moving, adding and reducing, and diagonal division.

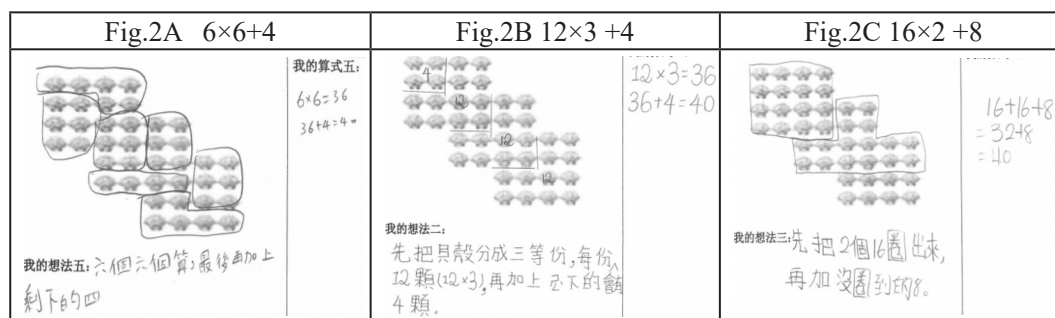
Counting: Use the same unit to count. The results showed two cases: there was no remainder and there was a remainder. There was no remainder in the same unit such as counting one by one, or five by five, etc. The statements described by the participants were “first divide some numbers into a group, see how many groups there are, and then multiply them”, “first circle, and then multiply the number of each circle”, “divide them into several parts first, then count the number of parts”. The formula was $A \times B = 40$ (Figure 1).

Figure 1
Counting Without Remainder



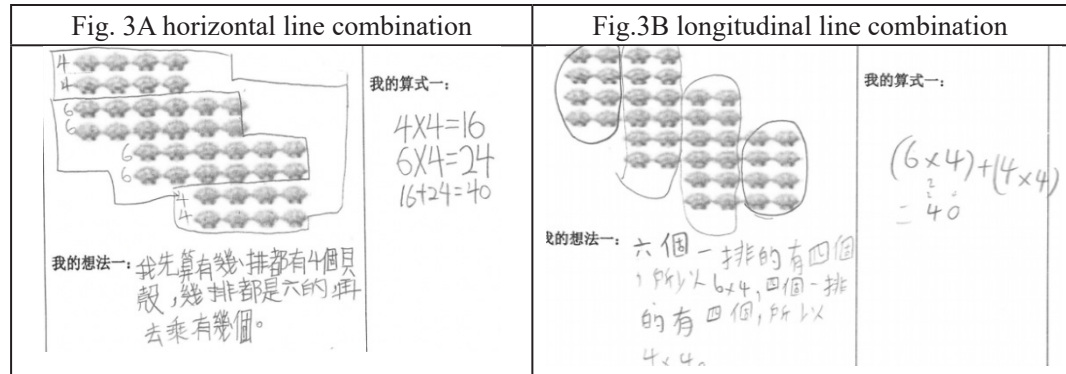
When using the same unit but not counting out, you need to add the remaining number. The quantity of units chosen vary from student to student, such as 3, 6, 9, 12, 16 and so on. The statements including “divide into several equal parts, count how many parts each, and then add the rest”, “circle several equal parts, and add the ones that are not circled”. The formula was $A \times B + C = 40$ (Figure 2).

Figure 2
Counting With Remainder



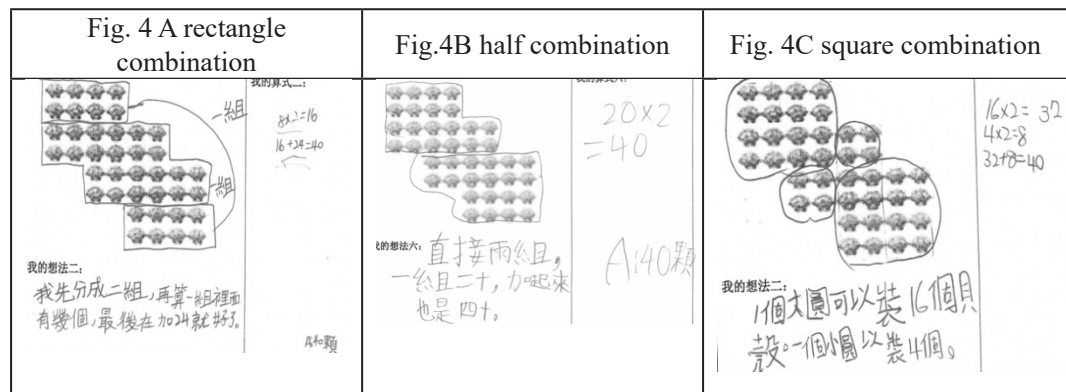
Line combination. Combining from the same number of a single row or column was named the line combination because a row or a column was similar to a line. The statements were “first look at a row as a group, then calculate how many shells in each group, and add them all up”, “first count how many rows have 4 shells, and how many rows have 6 shells, then multiply and add together”. The formula was $4 \times 4 + 6 \times 4 = 40$ (Figure 3).

Figure 3
Line Combination



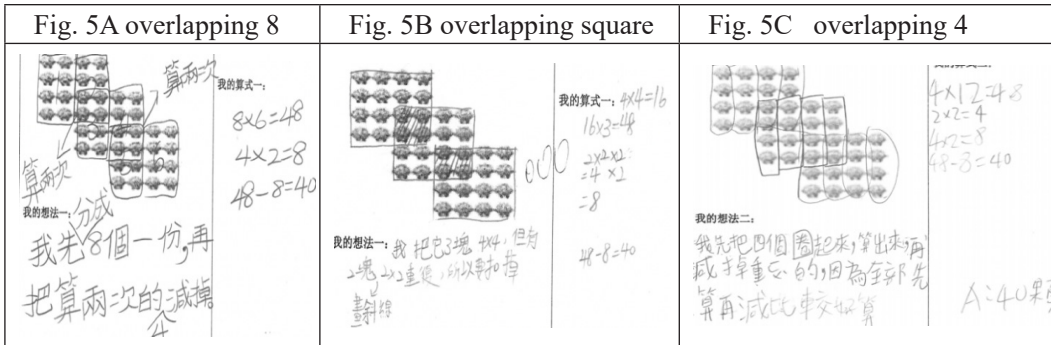
Area combination. Finding out the rectangle, square or L-shape hidden in the arrangement and calculating. The area combinations were 16 and 4, 8 and 12, etc. The statements were “divide into four parts, add each part”, “first divide all into two parts, and then multiply by 20 to get the answer” (Figure 4).

Figure 4
Area Combination



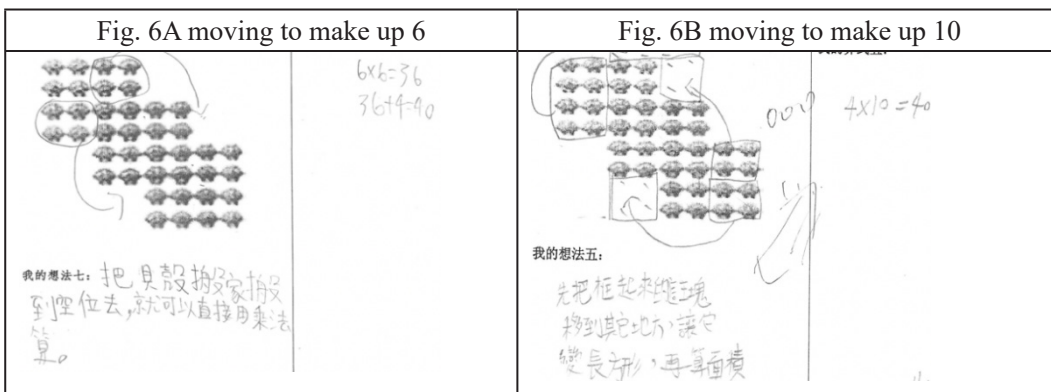
Overlapping: Thinking of the original shape with some overlapping regular figures or quantities. The statements such as “divide into several parts first, and then minus the number that counted two times”, “there are three parts of 4 by 4, but 2 pieces of 2 by 2 are repeated, so they should be minus” (Figure 5).

Figure 5
Overlapping



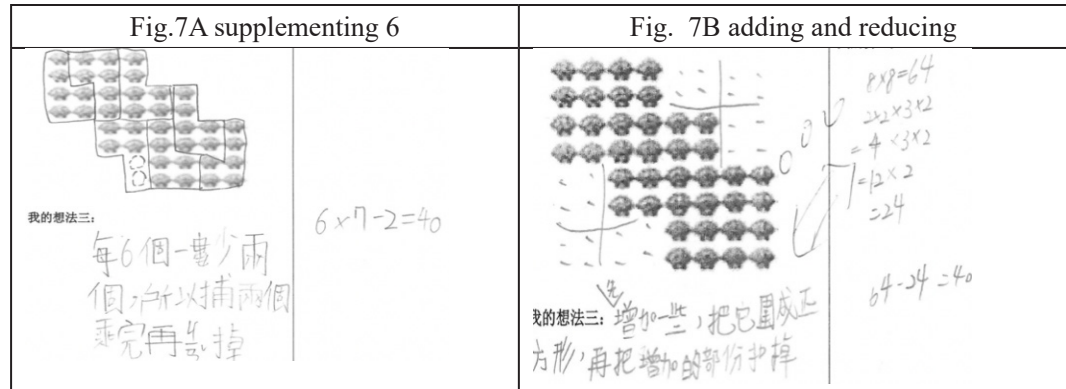
Moving: Forming a new regular shape by moving or changing the original partial arrangement. The statements were “move the framed part to another place first, make it a rectangle, and then calculate the area”, “move the shell to the vacant space and multiply it directly” (Figure 6).

Figure 6
Moving



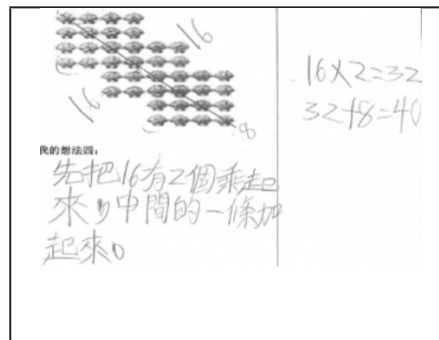
Adding and reducing: Supplementing the original shapes to form a regular pattern and then minus the added. The statements included “fill up the vacant space first, then subtract the filled”, “add some to form it into a square, and then minus the added” (Figure 7).

Figure 7
Adding and Reducing



Diagonal division: Dividing the diagram into two equal parts diagonally. The statement was “multiply 2 of 16 and add the number of the middle” (Figure 8).

Figure 8
Diagonal Division



Appropriateness Analyses

If there are no constraints, the first graders can calculate the result 40 by counting. However, the diagram gives the constraint of each row and column as 4, 4, 6, 6, 6, 6, 4, 4. There might be no difference in correctness in solutions, but there were differences in appropriateness. Based on the arrangement of the given diagram, this study considered that the most appropriate units of counting for the task were 1, 2, 4, 8, 20. Though 5 and 10 are conventional counting strategies and counting without remainder, they were not appropriate for this task. In addition, the strategy of “A×B+C” used by the participants was inappropriate, either. (For a detailed analysis, see the comparison of elaboration in the next section). Thus, there were 24 types of appropriate schemes to the task. Counting 1, 2, 4, 8, and 20 were the conventional schemes (P >40%) (Figure 9), Combination was the unconventional schemes (15% <P<40%) (Figure 10). Overlapping, moving, adding and reducing and the diagonal division were the innovative schemes (P<15%) (Figure 11).

Figure 9
Conventional Strategies (Counting)

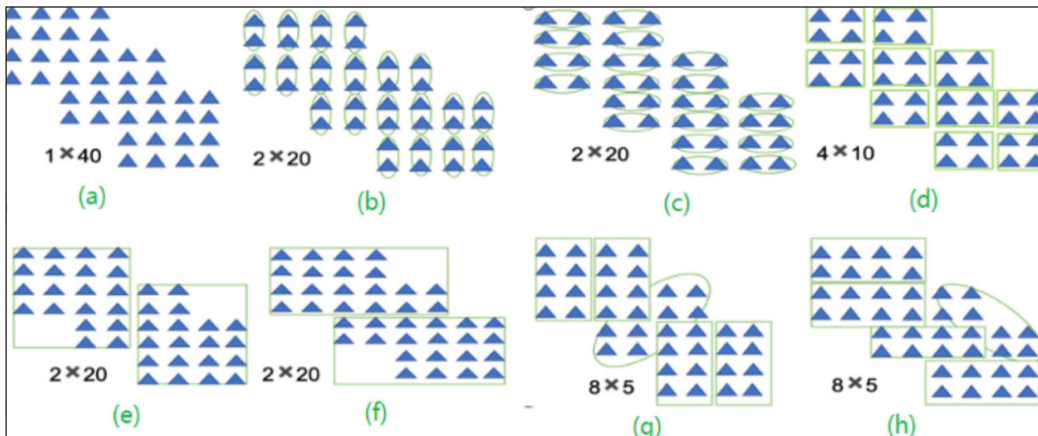


Figure 10
Unconventional Strategies (Combination)

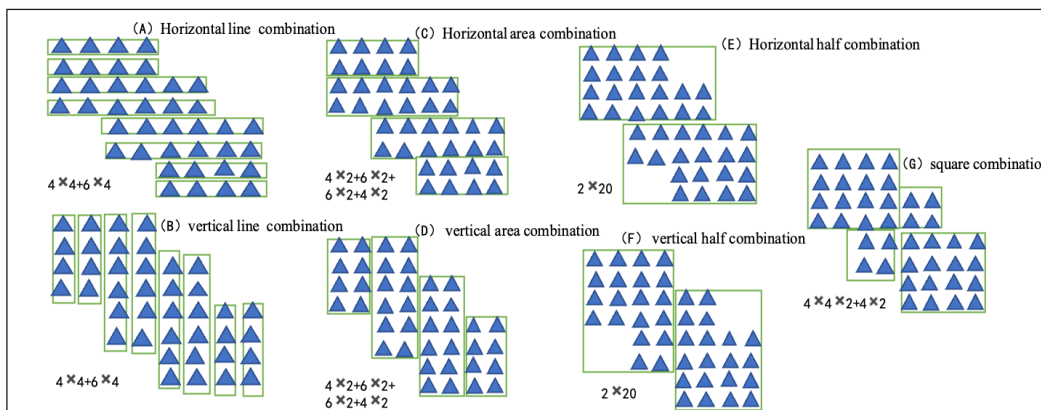
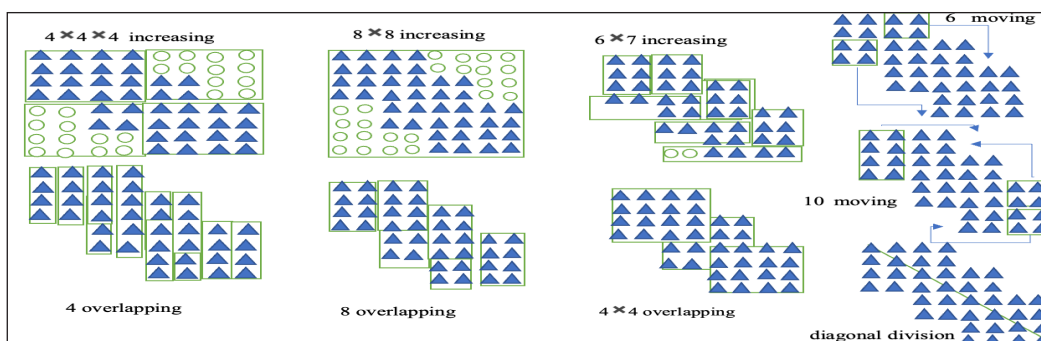


Figure 11
Innovative Strategies



It should be noted that “2 by 2” (Figure 9b, 9c), “20 by20 “(Figure 9e, 9f) and “divide into two halves” (Figure 10e,10f) could be expressed by the same formula 2×20 . But with the help of the word expressions, they had different thinking processes and belonged to different types of schemes. On the whole, the participants provided 257 solutions to the task, and 193

appropriate solutions accounted for 75% of the total. Among them, counting accounted for 61% which was conventional; combination accounted for 26%, which was unconventional. Adding and reducing (5%), overlapping (4%), moving (3%), and diagonal division (1%) were innovative schemes (Table 2).

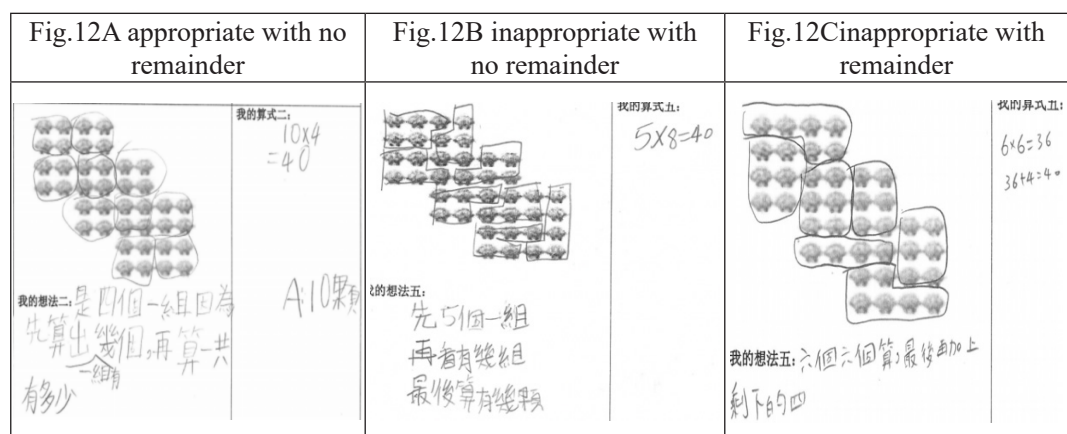
Table 2
Multiple Strategies and Schemes for the Task

Strate- gy	Counting			Combination			Overlapping	Moving	Increasing Decreasing	Diago- nal		
	no remainder	remainder	line	line	face	face						
Scheme	1, 2, 4, 8, 20	5, 10	3, 6, 12 ...	4/6	4 × 2/ 6 × 2	4 × 4/ 2 × 2	2 × 20	4 8 12	6 10	4 × 4 8 × 8 6 × 7	16 × 2 + 8	
Total	122	38	26	9	9	16	16	1 1 5	3 2	1 6 1	1	
Percent	61%	cull		26%			4%		3%		5%	1%
Types	Conventional			Unconventional				Innovative				

Elaboration Analyses

Elaboration of Counting. As the above is mentioned, if there are no constraints to the task, the result 40 can be calculated in any way. However, the task is restricted by the arrangement of diagram. When answering “how to count”, it needs to identify from partial or holistic, horizontally or vertically to discover the patterns. The counting methods used by participants were quite diverse. Figure 12 took 4, 5 and 6 as an example to analyze the appropriateness and inappropriateness of counting. From the point of view of creative thinking and its delicacy, using “ $A \times B = 40$ ” required higher thinking abilities than using “ $A \times B + C = 40$ ”. From the visualization shown in figure 12, the cluttered regular arrangement had a greater cognitive load than the clear regular arrangement. Although counting five or ten is a conventional strategy, it is not appropriate in this task. The students needed to break away from the stereotypical solutions and were required to think flexibly.

Figure 12
Elaboration of Counting



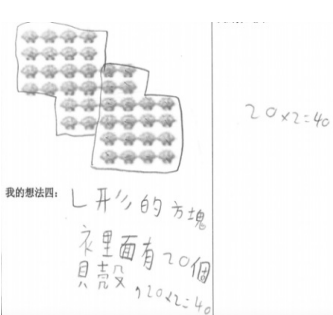
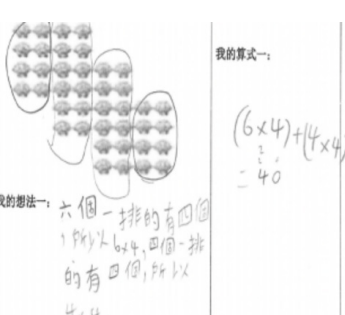
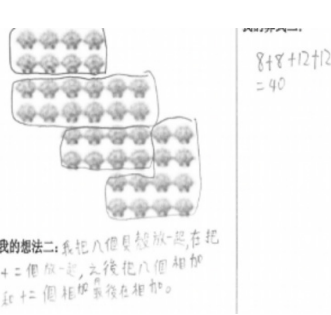
Elaboration of combination. Combination needs to identify the line segments,

rectangles, squares or L-shapes hidden in the arrangement horizontally or vertically. The observation of “rows” or “blocks” lied in the difference between one-dimensional and two-dimensional thinking. In addition to the difference between horizontal and vertical, line and area, the combination also showed the difference of creativity in graphic representation, calculation formula and thought expressions. Figure 13A showed that the pupil saw not only the 20 shells in the area, but also the L-shape graph, while Figure 13B and C were both rectangular combinations, but only through the formula and thoughts expressions could be found that one student used “one-dimensional” linear strategy and the other used the two-dimensional area strategy. The differences in the details of the illustration in Figure 13C (there were many such cases) also indicated that some students were lacking in flexibility in thinking, or some students were accustomed to focusing on “partial”, “holistic view” still needed to be further developed.

In terms of formula, there were differences between addition and multiplication, stepwise and comprehensive. As far as the fourth graders are concerned, the multiplication formula is more abstract than the addition formula, the comprehensive formula is more in line with the age of the fourth-grade students.

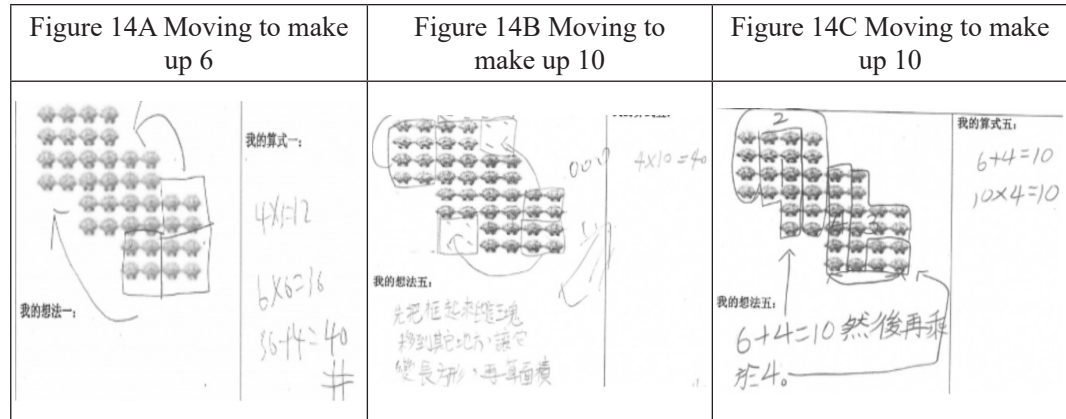
Neither counting nor combination changed the layout of the original diagram, which was equivalent to the adaptation.

Figure 13
Elaboration of Combination

Fig.13A longitudinal half combination	Fig.13B longitudinal line combination	Fig.13C horizontal planar combination
 <p>我的想法四：L开少的方块 壳里面有20个贝壳，$20 \times 2 = 40$</p>	 <p>我的想法一：六个一排的有四个 1排有6个，4个一排的有4个，所以 $4 \times 4 = 16$ 我的算式一： $(6 \times 4) + (4 \times 4) = 40$</p>	 <p>我的想法二：我把八个贝壳放一边，在把 + 二个贝壳，之后把八个贝壳 + 二个贝壳放在另一边。 $8 + 8 + 12 + 12 = 40$</p>

Elaboration of novelty. In the conventional scheme, it was not appropriate to count 6 or 10. But if the original arrangement and quantity were changed to form a new regular arrangement, it showed flexibility and novelty of thinking. Moving broke the limitation of the original diagram by “taking away” without changing the quantity (Figure 14). However, Figure 14B was more elaborate than Figure 14C in the recognition of sub-graphs.

Figure 14



Overlapping did not change the constraints of the original diagram, but it needed to reorganize the position and quantity of the “identified subgraphs”. Seeing three squares overlapping 4×4 was much more elaborate than seeing two (or one) squares. Overlapping 4 or 8 required a higher level of cognitive skills than simply counting 4 or 8.

Adding and reducing transformed the original shape and quantity into regular graphs and quantities through reconstruction and reorganization. Though diagonal division belonged to the combination, it became an innovative scheme because of its uniqueness.

Different from conventional and unconventional schemes, the innovative schemes needed reconstruction or deconstruction instead of adaption to the original conditions, which required students to break through the traditional practiced solutions and the constraints of given diagram by manifesting new patterns of sub-graph.

Comparison of Elaboration

Counting and combination. It was impossible to judge the difference of elaboration of thinking only from the formula such as 20×2 , but we could know the difference with the help of written words expressions or other symbols. We could see the same two parts from a global perspective in Figure 15A and B, their difference was horizontal or vertical. Figure 15A and C had the same illustration, but one adopted the counting method of “20 by 20” from partial, and the other was the combination of “cutting in half” from overall. The elaboration of thinking was reflected in the difference in observation angle (horizontal or vertical) and thinking perspective (holistic or partial).

Figure 15

Comparison between Counting and Combination

Fig. 15A Half vertical combination	Fig.15B half horizontal combination	Fig.15C Counting 20 by 20

Elaboration among Counting, Adding and Reducing, Overlapping and Moving.

Counting 6 was inappropriate, but if we broke the mindset of conventional method and had the ability to visually distinguish the pattern between the original figure and each sub-figure, the innovative strategies such as adding and reducing, moving, overlapping might appear (Figure 16). This situation also happened in counting 8 and overlapping 8, counting 10 and moving 10, combining $4 \times 4 \times 2 + 2 \times 2 \times 2$ and overlapping $4 \times 4 \times 3$. However, finding a new pattern is not easy, it needs the ability of reorganization, deconstruction, or reconstruction. The low proportion of innovative strategies showed that the ability of students' thinking flexibility still has room for improvement.

Figure 16
Elaboration among Conventional Method and Innovative Strategies

Fig.16A counting 6	Fig.16B supplementing 6	Fig.16C moving 6

In this study, counting was the basic conventional strategy and the foundation for innovative strategies. When conventional strategies couldn't solve the task well, the flexibility of thinking provided the possibility for creativity.

Discussion

The purpose of this research is to qualitatively analyze the patterns and strategies of regular fourth graders in solving a multiple-solution counting task in figure setting, so as to examine the appropriateness and elaboration of creative thinking process and outcomes of 9-10-year-old pupils in public schools. The multiple strategies were based on adapting or/and changing the given figure into the line segments or/and the area of rectangles and squares for calculation. It is difficult for the primary school students who learn the concept of number primarily through counting to associate the quantity of objects with areas of rectangles and squares or the line segments of equal length/ quantity by using horizontal and vertical relationship or the idea of adaptation or/and rearrangement. In this research, the conventional counting (61%) and unconventional combination (26%) strategies characterized by adaptation were mastered by some participants, innovative strategies (13%) such as overlapping, moving, adding and reducing, and diagonal division characterized by changing or rearrangement also appeared in a few students. On the whole, the fourth-grade participants provided 24 conventional, unconventional and innovative schemes and 6 kinds of strategies including counting, combining, overlapping, moving, adding and reducing, and diagonal division, mostly consistent with those found by Lu and Hou (2014) for the fifth and sixth-grade gifted students in Taiwan. Although the percentage of innovative schemes was slightly lower than Lu and Hou (2014), but more kinds of innovative schemes than Vale et al. (2018) and Lu and Hou (2014). The participants were equipping the creative strategies of adaptation, combination, change, expansion, and rearrangement.

Tasks are another important factor for creativity. A multiple-solution task is often used as a tool for creativity research, but the researchers are easily ignored the task's function and value for assessing or cultivating creativity. First of all, the task should be appropriate because it is a context which determines the cognitive level of how students think and understand the problem content. What is an appropriate task for creativity? "It seems appropriate to use a content area that not only has high relevance for mathematics and special potentials for creativity, but also requires only a little knowledge and is easily accessible" (Assmus & Fritzlar, 2022, p. 5). In this research, the appropriateness of creativity was reflected in the appropriateness of tasks and the appropriateness of solutions. An appropriate task not only requires a little prior knowledge, but also should be challenging to some extent. The appropriateness of a solution which should be reflected in both meeting the task requirements and constraints, can not only refer to the correct answers, but also refer to the incorrect answers that are reasonable or understandable, and those solutions that are correct but do not meet the constraints may also be inappropriate. These findings extended the interpretation of appropriateness by Schindler et al. (2018).

In addition, visualization can help focus on the elaboration of thinking process, it is a cognitive strategy. When an arithmetic problem is combined with visual means, abstract algebra becomes a visual geometric problem. In this study, the fourth-grade students were skilled in diagram representation, but there was a gap between diagrams to represent thinking outcomes and the words to describe thinking process. The use of diagrams and formulate calculations emphasized the results after thinking, while writing out thoughts externalized the implicit thinking process. Written word expressions can not only make one's own thinking visible, but also can clarify one's thinking with the help of verbal (oral or written) carding, which has a meta-cognitive function. Moreover, diagrams, words, and formulas belong to different ways of representation. In this study, although the participants' written word expressions were generally good, there was a small number of students whose written language clarity was a little poor, and some students did not even have the idea of writing, which might be related to the fact that math teachers usually focus on the calculation procedures and accuracy while neglecting explanatory analysis when giving lectures or organizing tasks (Vale, et al., 2012). Therefore, it is recommended that math teachers should not only pay attention to basic skills such as

listening and speaking and the correctness of answers but should also ask students “what do you think” and “write them in words” to strengthen the meta cognitive thinking ability.

The elaboration in this research, was reflected in many different aspects, such as meticulous and defective expressions of diagrams or words, the mutual transformation between diagrams and numerical expressions, stepwise and comprehensive formulas, multiplication and addition, etc. Through visualization, it was found that the participants used horizontal observation more than vertical observation, partial starting more than global starting, addition and step-by-step calculations more than multiplication and comprehensive calculations. The word expressions were more about “how to calculate” than “how to see” and “how to think”. These elaborate performances provide a development direction for the cultivation of creativity in the future.

At last, the study revealed some results in terms of terminology use. Some participants employed their own terms according to their life experiences. In this research, the students explained the addition with “increase” or “supplement”, the subtraction with “deduct”. They used “moving house” to mean moving, “blocks” for the description of area. Similar to this study, “there are other studies in which students create their own concepts for that subject area” (Çankaya et al., 2022, p. 750). Therefore, language and terminology are crucial components and data on student' learning and thinking.

Conclusions and Implications

When creative thinking through a multiple solution counting task in figure settings was examined at middle graders in a primary school, it had been found that:

- The fourth-year students in regular mathematical education could reorganize or decompose the given figure into the area of rectangles (squares) or /and the line segments for calculation by way of adaptation and/or change.
- Some 9-10-year-old students have had the basic creative thinking capability of adaptation, combination, change, rearrangement, extension and going back. They can use conventional, unconventional, and innovative patterns including counting, combining, adding or reducing, overlapping, moving, and diagonal division to solve multiple solution counting tasks in figure setting.
- The students who had just entered the fourth grade could show their creative thinking through different angles (horizontally or vertically) and starting points (holistic or partial) or using a variety of comprehensive strategies at the same time by observation, analysis, synthesis and imagination according to the given figures and numerical arrangements, but they employed slightly more partial and horizontal than holistic and longitudinal, more adaptation than transformation when thinking flexibly.

These findings offered a possible way for teachers to think about the status of the creative thinking potential of the third and fourth-grade students and of the way how to cultivate the creative thinking ability of middle and upper-grade students.

- Pay attention to design appropriate creativity-directed tasks. Since students' learning tasks are mainly designed by teachers purposefully, multiple solution tasks or/and open-ended tasks are generally considered effective tools to cultivate and evaluate creativity. However, creativity has not been commonly implemented in mathematics classrooms, teachers need to have the sense and ability to convert routine mathematical tasks into creativity-directed tasks. When teachers offer creativity-directed tasks to encourage students to modify their cognitive framing by employing their thinking flexibility and novelty, it gives the opportunities for students to be creative in mathematics classrooms and criticizes the limitations of

traditional closed-ended problem-solving.

- Cultivate students' multiple-angle thinking skills. From the middle grades on, teachers may strengthen the systematic thinking strategies of partial and holistic, horizontal and vertical, and improve students' creative ability through alternative thinking such as replacement, combination, adaptation, change, expansion, removal and rearrangement.
- Using visualization as an effective perception thinking tool and cognitive strategy. Math teachers should not only pay attention to basic skills such as listening and speaking and the correctness of answers but should also use some visual tools such as drawing and writing to strengthen their mathematical meta cognitive thinking abilities.

This study contributed to our understanding of the performance and characteristics of creative thinking of 9 to 10-year-old students in public primary schools under the regular instruction context and added some new knowledge on the indicators of mathematical creative thinking, especially appropriateness and elaboration.

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