ENHANCEMENT OF THE PREDICTION ACCURACY OF GREY SYSTEM MODEL USING A PARTICLE SWARM OPTIMIZED INITIAL CONDITION

Justice **OHENE-AKOTO**[1], Elvis **TWUMASI**[1], Emmanuel A. **FRIMPONG**[2]

[1] *Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, United States,* [2] *Department of Electrical and Electronic Engineering, Kwame Nkrumah University of Science and Technology Kumasi, Ghana*

justiceakoto8363@gmail.com, etwumasi.coe@knust.edu.gh, eafrimpong.soe@knust.edu.gh

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Abstract*: The grey system model has seen vast application in many fields due to its good accuracy in predicting systems with a limited dataset. In this paper, the prediction accuracy of the traditional grey system model is enhanced using the particle swarm optimization algorithm. The enhancement is mainly the use of PSO to predict an optimum initial condition value based on the input dataset to improve the prediction accuracy of the original grey model that uses the first data of the input dataset as its initial condition. The performance of the enhanced model was tested against the traditional grey model and another model that seeks to enhance the initial condition using the average-minimum-maximum absolute error and the mean absolute percentage error to prove its adaptability. Sample monotonic increasing and decreasing datasets, value of lost load and value of lost load per GDP datasets were used as testing data to prove the accuracy of the proposed model. The proposed model predicted optimum initial condition values of 18.9241, 5.9160, 5.0203 and 3120012789 that resulted in the lowest MAPE OF 0.1798%, 0.1799%, 2.1359% and 11.2813% for the monotonic increasing and decreasing, value of lost load and value of lost load per GDP datasets respectively. It was shown that the proposed model outperforms the traditional grey system model and an improved initial condition model in literature.*

1. INTRODUCTION

In recent years, the grey system model has gained much attention among researchers in performing short-term forecasts because of its efficiency in making accurate inferences from short-term data [1-3]. The effectiveness of this model has been verified through application to a wide range of real-life problems, including economics, science, and all aspects of engineering [3-7]. The grey system model can understand uncertainties that may arise in a real-life dataset and estimate the future without necessarily modeling each uncertainty. This model has the capability of recognizing, representing, manipulating, interpreting, and utilizing data and information that are vague and lack complete certainty.

Data with few historical variables are primarily challenging to predict [7]. Computational models, statistical models, and fuzzy models always yield high errors when using limited data for prediction because of their dependence on many datasets and the need for the data to satisfy certain assumptions [8-10]. Thus. making meaningful forecasts from imprecise and vague information challenges these models. However, data in real-life systems hardly follow a particular pattern. This puts the grey system model on a level that makes it very important in dealing with grey modeling, grey control, and grey programming [2].

Notwithstanding the advantages of the original grey system model compared to other models, the grey system model suffers from high prediction errors [5,11]. Therefore, many authors have, over the years, tried to overcome these challenges by improving the prediction accuracy of the traditional GM (1,1) by addressing the various parameters that hinder its accuracy in dealing with different data sets. The parameters that affect the prediction precision of the grey model found in the literature are the choice of initial condition value [12], the neglect of the first entry in the prediction [13], and the choice of adjacent neighbor weight [14]. Therefore, many authors have proposed different methodologies to address these challenges. Twumasi et al. [14] used the particle swarm optimization algorithm to choose the optimum adjacent neighbor weight based on the input data to improve the forecasting accuracy of the original grey model. The work in [15] ensured the data utilization efficiency by combining a data grouping technique with modification of the initial condition to establish an optimized grey model. The paper by Tan et al. [16] improved the initial condition for the grey model by using the weighted combination of the latest and oldest components of the original data. Mahdi and Mohamed in [12] have shown that the choice of initial condition value has a high weight in producing accurate forecasts with the grey system model. Their study provided a modified initial condition for the grey system model using a new approach to find the initial condition value. Though these models outperform the original grey model in their prediction, their adaptability on other datasets is low.

This paper seeks to use the particle swarm optimization algorithm to enhance the prediction performance of the traditional grey system model by choosing an optimum initial condition value based on the input dataset to produce accurate predictions. The rest of the paper are arranged in these sections. Section 2 describes the traditional grey system model. Section 3 elaborates the proposed approach in choosing the optimum initial condition and also describes the testing criterion and the type of dataset used. The results and analysis section in section 4 discusses the test results and conclusions are drawn in section 5.

2. GREY SYSTEM MODEL

2.1. The Grey System Model

The Grey system model was developed by Deng Ju-Long in the year 1982 [17]. The model was developed mainly to help improve prediction of limited datasets that come with high level of uncertainty. The model makes prediction by understanding the initial data, and then carry out mathematical modelling on this basis. The mathematical model for the original grey system model is constructed as follows:

Given a non-negative sequence of raw dataset $X^{(0)}(k)$:

$$X^{(0)}(k) = \left(x^{(0)}(1), x^{(0)}(2), \dots x^{(0)}(n)\right) \tag{1}$$

the accumulated generated sequence, $X^{(1)}(k)$, is given as

$$X^{(1)}(k) = \left(x^{(1)}(1), x^{(1)}(2), \dots x^{(1)}(n)\right) \tag{2}$$

where:

$$X^{(1)}(k) = \sum_{i=1}^{k} X^{(0)}(k), \; k = 2,3,\dots,n$$

The background value, $Z^{(1)}(k)$, is calculated using the equation:

$$Z^{(1)}(k) = \left[0.5X^{(1)}(k) + 0.5X^{(1)}(k-1)\right] \quad k = 1,2,3,\dots, n \tag{3}$$

The basic form of the first-order grey system model GM (1,1) is given by:

$$x^{(1)}(k) + hz^{(1)}(k) = q \tag{4}$$

where h is the development coefficient and q is the grey action quantity.

The whitenization equation 4 is given as:

$$\frac{dX^{(1)}}{dt} + hX^{(1)} = q \tag{5}$$

The coefficients h and q found using equation 6:

$$[h,q]^T = \left(B^T B\right)^{-1} B^T Y \tag{6}$$

where:

$$Y = \begin{bmatrix} X^{(0)}(1) \\ X^{(0)}(2) \\ . \\ . \\ . \\ X^{(0)}(n) \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ . & . \\ . & . \\ . & . \\ -z^{(1)}(n) & 1 \end{bmatrix}.$$

The time response function of the whitenization equation is given by:

$$\hat{X}^{(1)}(t) = \left(x^{(1)}(1) - \tfrac{q}{h}\right)e^{-ht} + \tfrac{q}{h} \tag{7}$$

Therefore, the predicted values of the sequence of the model can be found using equation:

$$\hat{X}^{(0)}(k+1) = \hat{X}^{(1)}(k+1) - \hat{X}^{(1)}(k) \tag{8}$$

Thus,

$$\hat{X}^{(0)}(k+1) = \left(1 - e^{h}\right)\left(C - \tfrac{q}{h}\right)e^{-hk} \tag{9}$$

where C is the initial condition value.

3. PROPOSED ENHANCED GREY SYSTEM MODEL USING PSO OPTIMIZED INITIAL CONDITION

In this study, the particle swarm optimization (PSO) algorithm was used to augment the traditional grey system model, GM(1,1) by choosing an optimum initial condition value to increase its prediction accuracy. The original GM(1,1) takes the first value of the input data set as the initial condition for prediction, however, the proposed model in this work searches for the optimum value of the initial condition of the response function of the whitezination equation of the classical GM(1,1) based on the given data set using the particle swarm optimization algorithm. The main area this paper contributes to existing knowledge is the unique approach in finding an optimum initial condition for the input historical dataset. The objective function for the selection of the optimum initial condition value was formulated by writing the time response function of the whitenization equation in terms of the initial condition (C). This was done by first multiplying $B^T B$ and $B^T Y$ to get a 2 by 1 matrix shown in equation 10.

$$[h,q]^T = \left(B^T B\right)^{-1} B^T Y = \begin{bmatrix} \dfrac{\sum_{k=2}^{n} Z^{(1)}(k) \sum_{k=2}^{n} X^{(0)}(k) - (n-1)\sum_{k=2}^{n} Z^{(1)}(k) X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^2 - \left(\sum_{k=2}^{n} Z^{(1)}(k)\right)^2} \\ \dfrac{\sum_{k=2}^{n} Z^{(1)}(k)^2 \sum_{k=2}^{n} X^{(0)}(k) - \sum_{k=2}^{n} Z^{(1)}(k) X^{(0)}(k) \sum_{k=2}^{n} Z^{(1)}(k)}{(n-1)\sum_{k=2}^{n}\left(Z^{(1)}(k)\right)^2 - \left[\sum_{k=2}^{n} Z^{(1)}(k)\right]^2} \end{bmatrix} \tag{10}$$

Hence, substituting for *h* and *q* into the response function of the whitenization equation (9):

$$\hat{X}^{(0)}(k+1) = \left(1 - e^{\frac{\sum_{k=2}^{n} Z^{(1)}(k) \sum_{k=2}^{n} X^{(0)}(k) - (n-1)\sum_{k=2}^{n} Z^{(1)}(k) X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^2 - \left(\sum_{k=2}^{n} Z^{(1)}(k)\right)^2}}\right) \times \left(C - \frac{\dfrac{\sum_{k=2}^{n} Z^{(1)}(k)^2 \sum_{k=2}^{n} X^{(0)}(k) - \sum_{k=2}^{n} Z^{(1)}(k) X^{(0)}(k) \sum_{k=2}^{n} Z^{(1)}(k)}{(n-1)\sum_{k=2}^{n}\left(Z^{(1)}(k)\right)^2 - \left[\sum_{k=2}^{n} Z^{(1)}(k)\right]^2}}{\dfrac{\sum_{k=2}^{n} Z^{(1)}(k) \sum_{k=2}^{n} X^{(0)}(k) - (n-1)\sum_{k=2}^{n} Z^{(1)}(k) X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^2 - \left(\sum_{k=2}^{n} Z^{(1)}(k)\right)^2}}\right) \times e^{-\left(\frac{\sum_{k=2}^{n} Z^{(1)}(k) \sum_{k=2}^{n} X^{(0)}(k) - (n-1)\sum_{k=2}^{n} Z^{(1)}(k) X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^2 - \left(\sum_{k=2}^{n} Z^{(1)}(k)\right)^2}\right)k} \tag{11}$$

The first value of the input data used as the initial condition value $X^0(1)$ is replaced with a variable C. The objective function, *f(c)* for the PSO optimum search is given by the sum of squared errors shown in (11).

$$f=\sum_{k=1}^{n}\left(\left(X^{(0)}(k)\right)-\left(1-e^{\frac{\sum_{k=2}^{n}Z^{(1)}(k)\sum_{k=2}^{n}X^{(0)}(k)-(n-1)\sum_{k=2}^{n}Z^{(1)}(k)X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^{2}-\left(\sum_{k=2}^{n}Z^{(1)}(k)\right)^{2}}}\right)\times\left(C-\frac{\frac{\sum_{k=2}^{n}Z^{(1)}(k)^{2}\sum_{k=2}^{n}X^{(0)}(k)-\sum_{k=2}^{n}Z^{(1)}(k)X^{(0)}(k)\sum_{k=2}^{n}Z^{(1)}(k)}{(n-1)\sum_{k=2}^{n}\left(Z^{(1)}(k)\right)^{2}-\left[\sum_{k=2}^{n}Z^{(1)}(k)\right]^{2}}}{\frac{\sum_{k=2}^{n}Z^{(1)}(k)\sum_{k=2}^{n}X^{(0)}(k)-(n-1)\sum_{k=2}^{n}Z^{(1)}(k)X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^{2}-\left(\sum_{k=2}^{n}Z^{(1)}(k)\right)^{2}}}\right)\times e^{-\left(\frac{\sum_{k=2}^{n}Z^{(1)}(k)\sum_{k=2}^{n}X^{(0)}(k)-(n-1)\sum_{k=2}^{n}Z^{(1)}(k)X^{(0)}(k)}{(n-1)\sum_{k=2}^{n}\left(-Z^{(1)}(k)\right)^{2}-\left(\sum_{k=2}^{n}Z^{(1)}(k)\right)^{2}}\right)k}\right)^{2},\quad k=1,2,3,\ldots,n \tag{12}$$

3.1. Testing Criterion

To validate the proposed PSOOIC GM (1,1), sets of randomly generated monotonic increasing and monotonic decreasing data and two actual datasets were used. The monotonic data was used because it is easier to identify outliers or anomalies that may affect the accuracy of any developed prediction model. Sample monotonic sets for testing improved grey system model accuracy are shown in [14]. In this paper, monotonic increasing and decreasing data were generated using equations 13 and 14, respectively.

$$x(t) = 10e^{t} \qquad t = 1, 2, 3\ldots\ldots\ldots\ldots\ldots\ n \tag{13}$$

$$x(t) = 10e^{-t} \qquad t = 1, 2, 3\ldots\ldots\ldots\ldots\ldots\ n \tag{14}$$

Also, two sets of actual data were used because real-life data reflects the real-world conditions under which the model will be used, provides a more accurate assessment of a model's performance, helps to identify weaknesses, proves the model's credibility, and checks its applicability. For the real datasets, the yearly value of lost load (VoLL) (a measure of the economic cost of power outages) and the value of lost load per GDP (VoLL/GDP) from 2016 to 2019 for the Ghana power system using dependency factor, value-added, and electricity consumption data for the residential, commercial, and industrial sectors. The dependency factor was determined using a limited customer survey. The value-added and electricity consumption data were collected from the Ghana statistical service, the technical utility-regulatory company, and the Energy Commission of Ghana [18,19]. The GDP data was collected from the Bank of

Ghana [20]. The composite VoLL used as the input dataset was the sum of the individual sector VoLL and the VoLL per GDP was the ratio of the yearly VoLL to the yearly GDP.

The performance of the improved model to predict the four data sets was assessed using the maximum average percentage error (MAPE) and the average minimum, maximum absolute error chart in excel. The MAPE is shown in equation 15 below

$$MAPE = \frac{1}{2}\sum_{i=1}^{k}\frac{\left|x_p^0(k) - x^0(k)\right|}{x^0(k)} \times 100\% \tag{15}$$

4. RESULTS AND ANALYSIS

From Table 1 the proposed PSOOIC predicted an optimized initial condition value of 18.9241 and this gave the best prediction as compared to the traditional grey system model and an improved initial condition approach model in [12]. The original grey system model would have chosen a value of 16.4872 as the initial condition of the response function of the whitenization equation of the model which represents the first value of the monotonic decreasing data. The fig 1 shows the average, maximum and minimum chart (Average-Max-Min chart) which displays the average, maximum, and minimum absolute error values where the average is plotted with greater importance. From *fig. 1* the predictions from the proposed PSO optimized initial condition (PSOOIC), had the best average absolute error value of 1.0375 as compared to the Original Grey and Improved IC in [12] with values of 5.5011 and 1.0933 respectively. This leverage that the initial condition value is an important parameter in the traditional grey system model. Though the improved IC in [12] had a good minimum and maximum absolute error, the overall performance of the proposed PSOOIC was better on the monotonic decreasing data. Also, the overall mean absolute percentage error (MAPE) in fig 2 revealed that the PSOOIC had the lowest percentage error value compared to the improved IC in [12] and the grey system model. While the proposed model gave an MAPE of 0.1798%, the improved model in [12] and the original grey system model gave MAPE of 0.2108% and 0.8779% respectively.

Table 1. Prediction of monotonic increasing data

Actual Data	Original Grey	Improved IC in [12]	Optimum C	PSOOIC GM(1,1)
16.4872	16.2276	17.3466	18.9241	17.1714
27.1828	26.4843	28.3106		28.0245
44.8169	43.2237	46.2043		45.7375
73.8906	70.5433	75.4078		74.6468
121.8249	115.1302	123.0693		121.8258
200.8554	187.8983	200.8554		198.8259

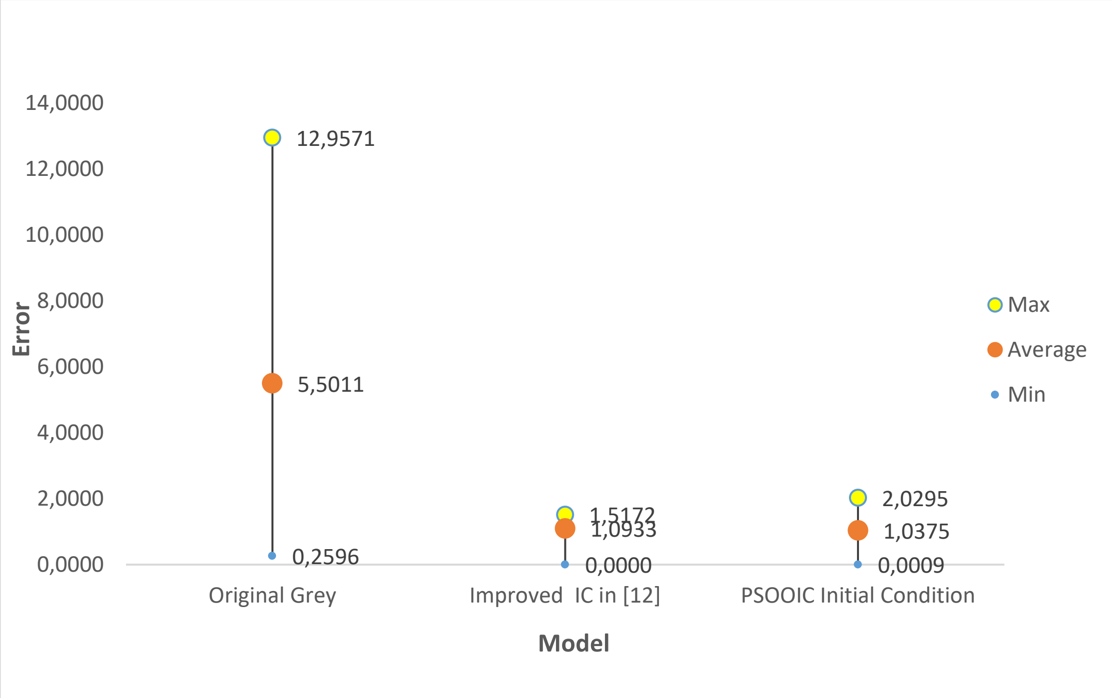

Fig. 1. Average-Max-Min chart of monotonic increasing data

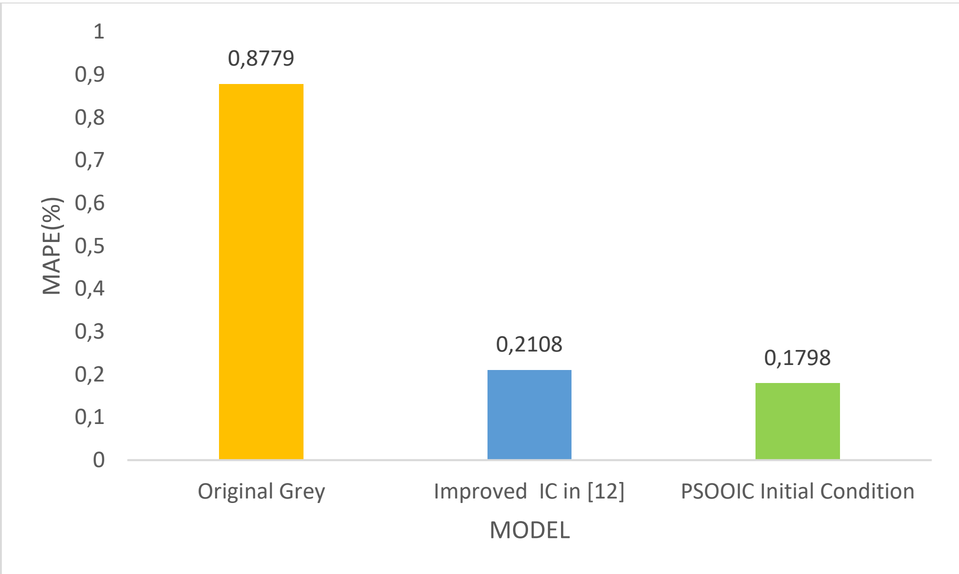

Fig. 2. Mean absolute percentage error of monotonic increasing data

Table 2 shows the prediction of the proposed PSOOIC, improved IC model [12] and the original grey model. The optimum initial condition value of the input data was found to be 5.916. The PSOOIC in *fig. 3* had a lower Maximum and Average Absolute Error of 0.0616 and 0.0263 respectively as compared to the higher values of the Maximum and Average Absolute Error of the original Grey and Improved IC model in [12]. However, the Minimum Absolute Error of the Improved IC in [12] had no forecast error. This notwithstanding the overall performance of the models assessed using the MAPE shown in *figure 4* depicts a better prediction accuracy.

Table 2. Prediction of monotonic decreasing data

Actual Data	Original Grey	Initial Condition	Optimum C	Optimized Initial Condition
6.0653	5.9094	5.7651	5.9160	6.0037
3.6788	3.6209	3.5324		3.6787
2.2313	2.2186	2.1644		2.2540
1.3534	1.3594	1.3262		1.3811
0.8208	0.8329	0.8126		0.8462
0.4979	0.5104	0.4979		0.5185

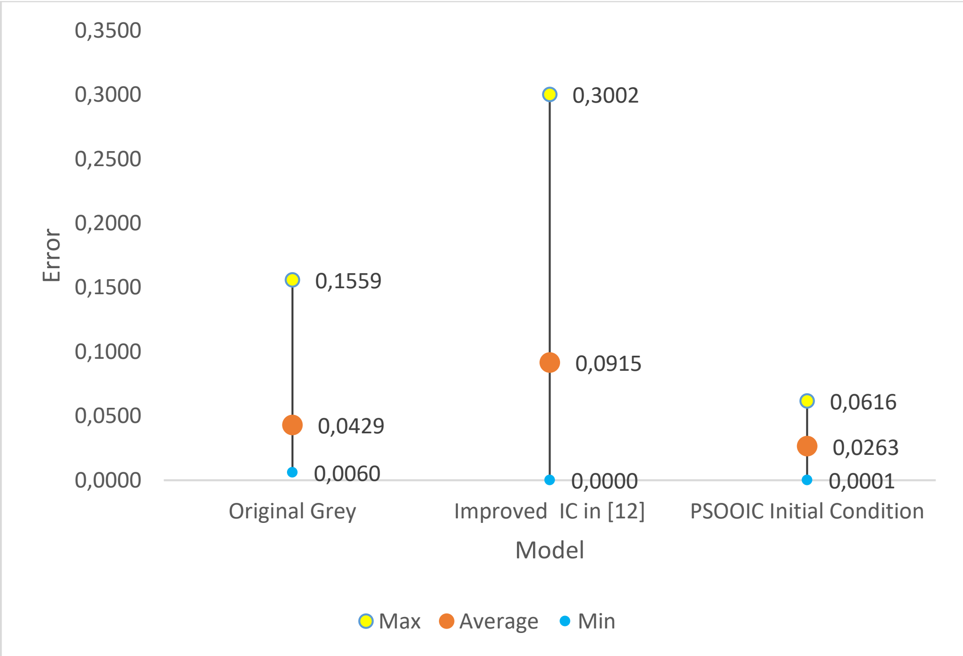

Fig. 3. Average-Max-Min chart of monotonic decreasing data

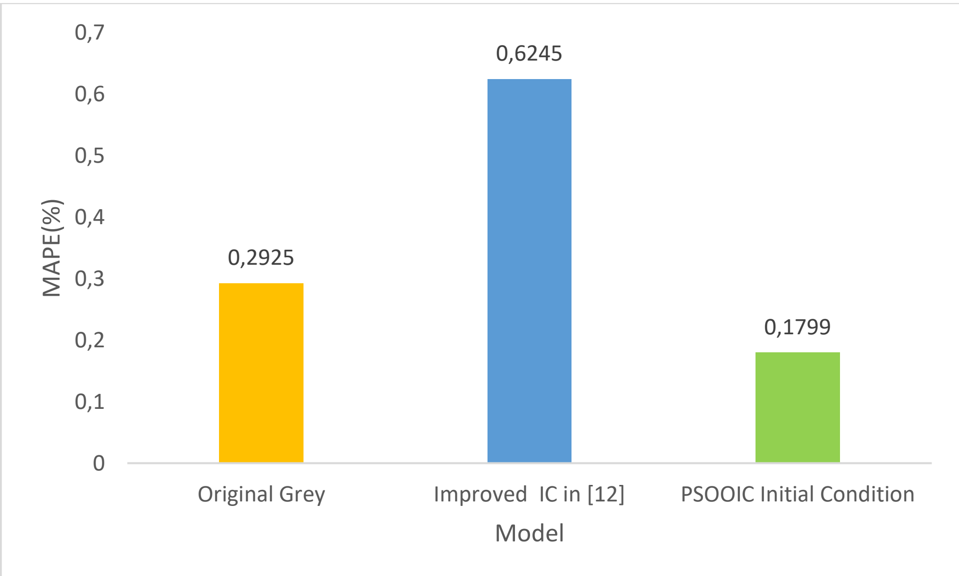

Fig. 4. Mean absolute percentage error of monotonic decreasing data

From table 3 the PSOOIC shows a good prediction when the initial condition value was chosen as 5.0203. This value was realized to be an optimum value that resulted in the wide gap in prediction accuracy between the PSOOIC and the model that tend to improve the initial condition in [12]. The proposed model also outperformed the original grey model; however, the original grey also outperformed the improved model in [12]. It is shown in *figure 5* that the maximum absolute error and the average absolute error of the PSOOIC were 0.4371 and 0.2369 respectively and they were lower than that of the original grey model and improved IC in [12]. The maximum absolute error and the average absolute error of the original grey were 0.6340 and 0.2869 respectively.

From *fig. 6*, it is observed that the degree of deviation of the predicted values of the PSOOIC is smaller than that of the Original Grey and Improved IC in [12] models. The MAPE of PSOOIC was the lowest value of 2.1359%, slightly lower than the Original Grey model with 2.587%, indicating a better forecast. In this VoLL per GDP dataset, the original grey model performed better than the improved IC model in [12] since its MAPE is 4.0876%. This shows that the proposed PSOOIC is a better-improved model for prediction.

Table 3. Value of lost load per GDP

VoLL per GDP	Original Grey	Initial Condition	Optimum C	Optimized Initial Condition
3.9250	4.5590	5.0431	5.0203	4.1041
3.3370	3.2213	3.5634		2.8999
2.0490	2.2761	2.5178		2.0490
1.7790	1.6082	1.7790		1.4478

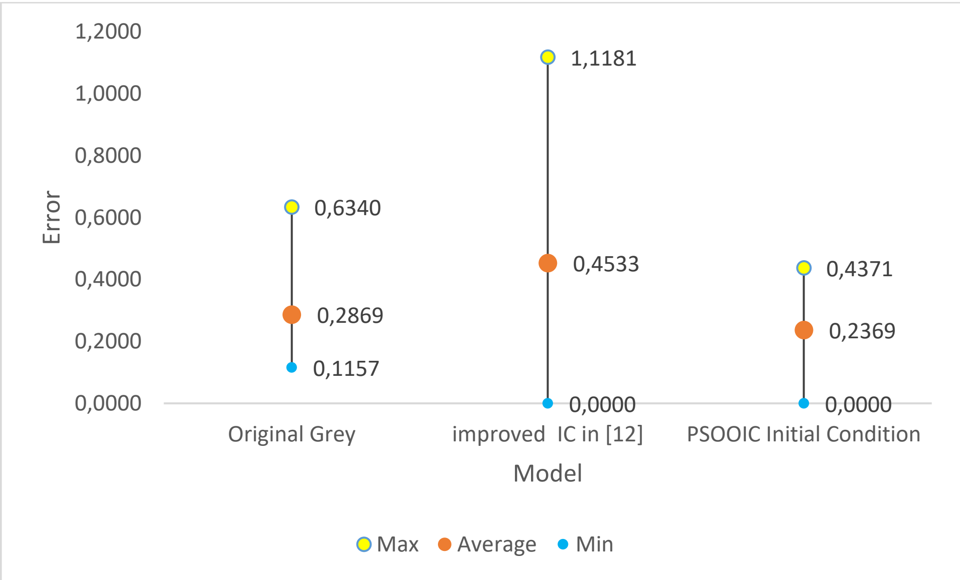

Fig. 5. Average-Max-Min chart of value of lost load per GDP data

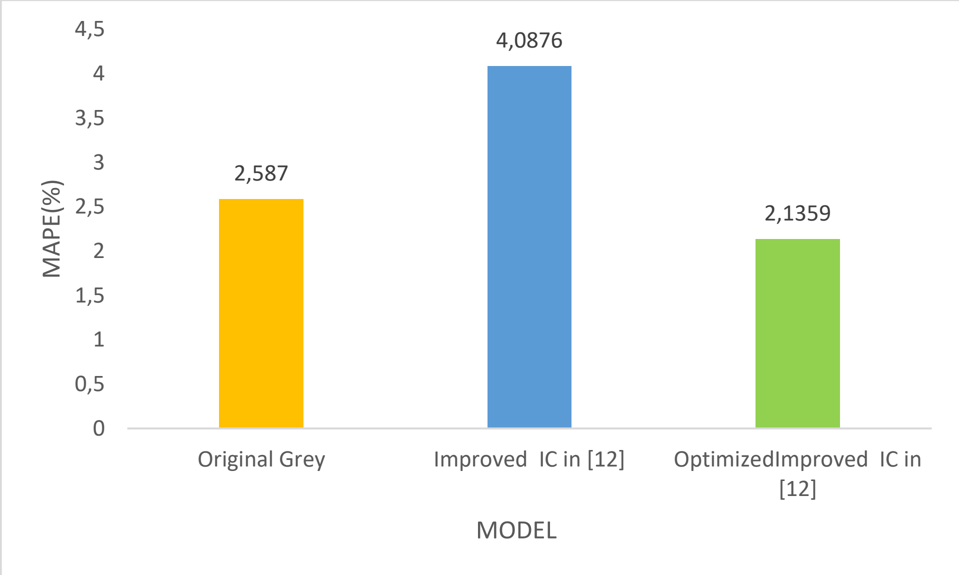

Fig. 6. Mean absolute percentage error of value of lost load per GDP data

It is shown in table 4 that the prediction values provided by the improved model in [12] deviated greatly from the Actual data. This shows that the proposed model and the original grey model performed better than the improved model in [12]. However, the predictions of the proposed PSOIC GM (1,1) that chose an optimum initial condition value outperformed the classical grey model. With a maximum absolute error value of 1.67×10^9 and an average absolute error of 8.68×10^8, as shown in fig 7, the performance of the improved initial condition in [12] compared with the original grey system model and the proposed model can be seen to be very poor. Considering the model that sought to improve the initial condition value in [12], a better minimum absolute error was observed in fig 7 compared to the proposed PSOOIC and the traditional grey system model. This depicts that the strength of the improved model in [12] always gives a better minimum absolute error. However, the maximum absolute error and the average absolute error of the PSOOIC give the lowest values of 1.40×10^9 and $7.50 \times 10^{9,}$ respectively. Moreover, the overall MAPE of the PSOOIC had the smallest value, as shown in fig 8, depicting a better prediction accuracy. From fig 8, the PSOOIC had the smallest value of MAPE (11.2813%) in comparison to Original Grey (12.8168%) and Improved in IC [12] (13.0617%), making the PSOOIC a better model due to its low MAPE that strengthens its predictive precision ability.

Table 4. Prediction of value of lost load

VoLL (Dollars)	Original Grey	Initial Condition	Optimum C	Optimized Initial Condition
2.170×10^9	0.5326×10^9	0.4954×10^9	3120012789	0.7657×10^9
1.981×10^9	0.7058×10^9	0.6565×10^9		1.0148×10^9
1.3445×10^9	0.9354×10^9	0.8700×10^9		1.3449×10^9
1.153×10^9	1.2396×10^9	1.1530×10^9		1.7823×10^9

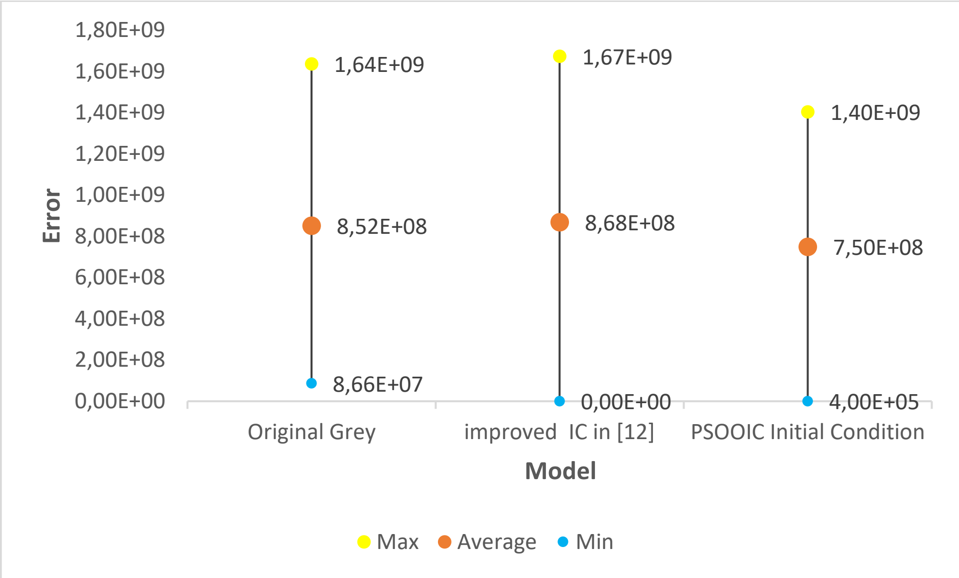

Fig. 7. Average-Max-Min chart of value of lost load

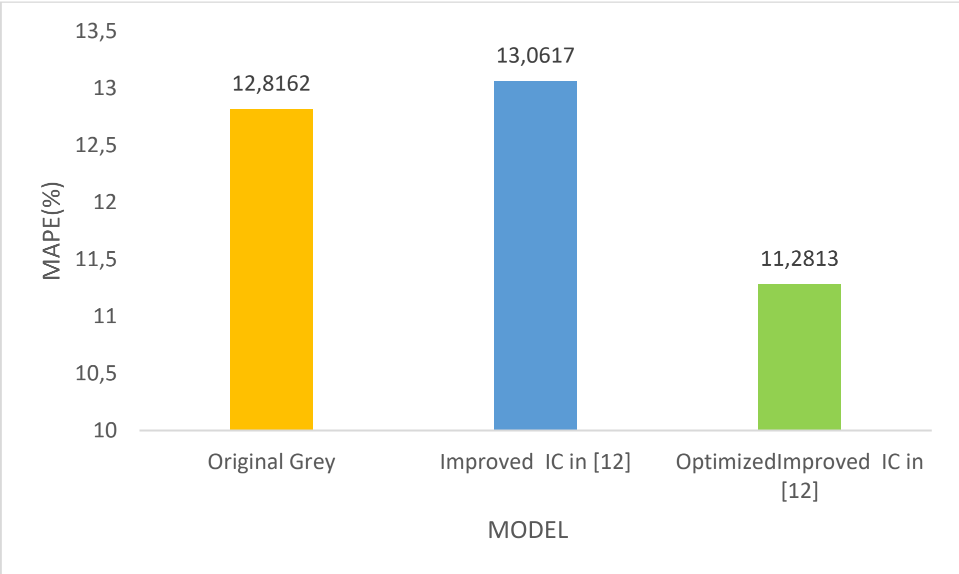

Fig. 8. Mean absolute percentage error of value of loss load data

5. CONCLUSION

In this paper enhanced model of the grey system model has been proposed. This model chooses an optimum initial condition value from the input data using the particle swarm optimization algorithm. The optimum initial condition chosen becomes the initial condition value for the response function of the whitenization equation of the original grey system model which uses the first input of the input data. The accuracy of the proposed model was tested using a generated monotonic increasing and decreasing data, value of lost load and value of lost load per GDP. The proposed PSOOIC had the best performance as compared to the traditional grey system model and a model that also improved the initial condition. The results showed that the enhanced model using the PSO to choose an optimum initial condition is a better modification of the grey system model.

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