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SCMA CODEBOOK DESIGN BASED ON GRADIENT ASCENT ALGORITHM

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ABSTRACT

Sparse Code Multiple Access (SCMA) is a non-orthogonal multiple access scheme, which improves spectrum efficiency. SCMA codebooks determine the bit error rate (BER) performance. There are various methods of constructing codebooks, but most of them are based on the formation of a mother constellation (MC). In this paper, we propose two methods to construct mother constellations with good characteristics, such as minimum Euclidian distance (MED) and peak-factor (PAPR). Also, we propose the method for optimizing user operators based on the Gradient Ascent Algorithm (GAA) to maximize the MED between superimposed codewords and superimposed constellation points. We consider the MCs built on various principles: rotation of the base constellation and interleaving; mapping of the golden angle modulation (GAM) points; mapping of the constellation points of existing modulation methods with interleaving The BER performance of the proposed methods outperforms existing codebook design schemes in uncoded SCMA systems. The proposed optimization method can be applied to arbitrary MC. The proposed methods for designing and optimizing codebooks can be used to build SCMA codebooks of different sizes with good characteristics.

1. INTRODUCTION

Every year technological progress rises the requirements for wireless networks: there is a need for higher transmission speed, bandwidth, the number of devices, and low latency. To increase the spectral efficiency in wireless communication systems of future generations, it is proposed to use non-orthogonal multiple access methods (NOMA) (Dai et al., 2015), among which is the multiple access based on sparse codes (SCMA) (Nikopour et al., 2013). In the SCMA, user bits are directly converted into sparse multidimensional codewords. By introducing controlled interference, it is possible to use SCMA for networks with the number of subscribers exceeding the number of available resources. To eliminate the influence of inter-to-user interference, it is proposed to use the message passing algorithm (MPA) with less complexity compared with the classical maximum a posteriori (MAP) method (Bayesteh et al., 2013).

SCMA signals are formed on basis of multidimensional codebooks that influence the performance of systems with

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SCMA. Currently, it is an immediate problem to design codebooks with high parameters and performance. At the moment, designing codebooks with good parameters and performance is an actual issue. In (Nikopour et al., 2013; Taherzadeh et al., 2014), a general formulation of the problem of finding optimal codebooks is given and a multilevel optimization approach is proposed to obtain a suboptimal solution. Thus, most works on designing codebooks are based on two steps: obtaining the mother constellation (MC) with the required parameters and transformations over the MC to obtain the codebooks of users (layers). The MC is designed and optimized in such a way as to maximize the normalized minimum Euclidean distance. MC transformations are necessary for obtaining user codebooks and are optimized, so that interfering signals of different layers in the same resource are clearly distinguishable during decoding. These transformations can include complex conjugation, phase rotation, and dimensional permutation. Research (Alam & Zhang, 2017) proposes an optimization criterion of subscriber codebooks based on any MC to improve noise immunity. However, this work does not describe the optimization approaches themselves.

The formation of codebooks based on the phase rotation of the MC is well suited for the downlink channel. However, due to the fact that the codewords of the subscribers pass through different channels in the uplink channel, the phase rotation of the MC loses its meaning (Taherzadeh et al., 2014). In addition, some works are aimed at reducing the PAPR, which is critical for systems based on OFDM.

In this article, we build on the methods of generating codebooks (Cai et al., 2016) and (Mheich et al., 2018), but the proposed optimization methods can be applied to other books based on the phase rotation of the MC. Work (Cai et al., 2016) proposes a simple method for generating codebooks, called MD-SCMA, which is not based on an optimization parameter. Work (Mheich et al., 2018) suggests the formation of codebooks based on GAM points with better MC characteristics compared to MD-SCMA. In this article, we propose the MC optimization method, as well as two methods for optimizing layer operators based on phase rotation: maximizing the Euclidean distance between superimposed codewords and superimposed constellation points. The proposed optimization methods have a slight computational complexity compared to other similar works (Zhang et al., 2021).

The following notation is accepted: let $\mathbb{B},\mathbb{Z},\mathbb{C}$ denote binary, integer, and complex numbers, respectively. To denote scalars, vectors, and matrices, the notations x, $\mathbf{x}$, and $\mathbf{X}$ are used; the j-th column of $\mathbf{X}$ is denoted $\mathbf{X}^j$, the i-th row of $\mathbf{X}$ is denoted $\mathbf{X}_i$; $\|\mathbf{x}\|$ is the norm of the vector $\mathbf{x}$, and $\mathbf{I}_L$ is the identity matrix of size $L \times L$. To denote the element-wise multiplication of matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \odot \mathbf{B}$ is used; diag($\mathbf{v}$) is a diagonal matrix with $\mathbf{v}$ elements in the main diagonal. $(\cdot)^H$ denotes a hermitian conjugation, $(\cdot)^*$ denotes a complex conjugation, $(\cdot)^T$ denotes a transposition, and blockdiag **A**, **B**,...) is a block-diagonal matrix with a diagonal of matrices **A**, **B**, etc.

2. SYSTEM MODEL

Let us consider an SCMA system with J users, K orthogonal subcarriers, and the modulation index M, while each user uses N subcarriers, and $1 < N < K$. The information bits $\mathbf{b} \in \mathbb{B}^{\log_2 M}$ are transformed into an N-dimensional complex constellation point $\mathbf{c}$:

$$g_j : \mathbb{B}^{\log_2 M} \to \mathbf{C}_j,\ \mathbf{C}_j \subset \mathbb{C}^{N \times M}$$

then $\mathbf{c} = g_j(\mathbf{b})$. Then, using a layer mapping matrix $\mathbf{V}_j \in \mathbb{B}^{K \times N}$, the sparse codeword can be obtained as

$$\mathbf{x}_j = \mathbf{V}_j \cdot \mathbf{c},\ \mathbf{x}_j \in \mathbf{X}_j$$

where $\mathbf{X}_j \subset \mathbb{C}^{K \times M}$ is the codebook of the j-th layer.

In the uplink channel, the received signal can be written as

$$\mathbf{y} = \sum_{j=1}^{J} \mathrm{diag}(\mathbf{h}_j) \cdot \mathbf{x}_j + \mathbf{n} \qquad (1)$$

where $\mathbf{h}_j = (h_1, \ldots, h_K)$ is the vector of channel coefficients, and $h_k \sim CN(0,1)$, $\mathbf{n} \sim CN(0, N_0 \cdot \mathbf{I}_k)$ is the additive white Gaussian noise. For the downlink channel in (1), $\mathbf{h}_j = \mathbf{h}$.

For the multi-user detection, the MPAis used in the receiver.

3. MC DESIGN

In this section, we look into the MC design. In the general case, when designing the MC, two parameters are considered: the normalized minimum Euclidean distance (MED) and PAPR. The average power of the subscriber's codeword is determined by the formula:

$$P_{avg} = \frac{1}{M} \sum_{m=1}^{M} \left\| \mathbf{x}_j(m) \right\|^2 \qquad (2)$$

The normalized MED is defined as:

$$d_{min} = \frac{d_{min}}{\sqrt{P_{avg}}} \qquad (3)$$

where

$d_{min} = \min(\|\mathbf{x}_j(m) - \mathbf{x}_j(n)\|),\ \forall\ m,n \in \{1,\ldots,M\},\ m \neq n.$

The PAPR is defined as

$$PAPR[dB] = 10\lg\left(\frac{\max\limits_{1 \le m \le M} \left\| \mathbf{x}_j(m) \right\|^2}{P_{avg}} \right) \qquad (4)$$

As a rule, the design of books is aimed at reducing the PAPR and maximizing d_{min}. Also, in order to improve the noise immunity in the Rayleigh channels at high values of the signal-to-noise ratio (SNR), works (Gao et al.; Chen et al) propose to increase the minimum product distance (MPD) of the subscribers' codebooks, defined as

$$MPD_j = \min_{\substack{l \neq k \\ 1 \le l \le M \\ 1 \le k \le M}} \left\{ \prod_{n=1}^{N} \left| c_{j,n}^{l} - c_{j,n}^{k} \right| \right\},\ \mathbf{c}_j \in \mathbf{C}_j$$

Since the subscribers' codebooks are based on the phase rotation of the MC, MPD_j is equal to the MPD of MC. We will use the normalized value.

$$MPD = MPD / P_{avg}$$

3.1 MD-SCMA codebooks

Step 1: Building a basic constellation based on pulse amplitude modulation (M-PAM). Let $\mathbf{bc} = (bc_1, ..., bc_M)$ be a vector-string whose elements $bc_m = (2m-1-M)(i+1)/k$ for $1 \le m \le M$. We have introduced a normalization coefficient k to change P_{avg}. As will be shown later, this parameter has no effect on d_{min}. and the PAPR.

Step 2: Obtaining an N-dimensional mother M-point constellation. The mother constellation is obtained by phase rotation of **bc** for each dimension at a certain angle

$$\mathbf{M} = \begin{pmatrix} \mathbf{bc} \cdot e^{i\theta_1} \\ \vdots \\ \mathbf{bc} \cdot e^{i\theta_n} \\ \vdots \\ \mathbf{bc} \cdot e^{i\theta_N} \end{pmatrix}$$

where $\theta_n = (n-1)\pi / MN$ for $1 \le n \le N$.

Step 3: Interleaving even dimensions to reduce the PAPR. For even l ($2 \le l \le N$), , a permutation is introduced according to the following rule

$$\mathbf{M}_l = \left(-M_{l,\frac{M}{2}+1}, \ldots, -M_{l,\frac{3M}{4}}, M_{l,\frac{3M}{4}+1}, \ldots, M_{l,M}, \right.$$
$$\left. -M_{l,M}, \ldots, -M_{l,\frac{3M}{4}+1}, M_{l,\frac{3M}{4}}, \ldots, M_{l,\frac{M}{2}+1} \right)$$

For the MD-SCMA, expressions (2)-(4) are written as

$$P_{avg} = \frac{2N(M^2-1)}{3k^2}$$

$$d_{min} = \begin{cases} \sqrt{24(N_o + 4N_e)/(2N(M^2-1))} & M = 4 \\ \sqrt{24N} & M > 4 \end{cases}$$

$$PAPR[dB] = 10\lg\left(\frac{3(N_o(M-1)^2 + N_e)}{N(M^2-1)} \right)$$

where N_o and N_e is the number of odd and even dimensions, respectively. The normalization coefficient k is chosen in such a way that $P_{avg} = 1$.

3.2 (θ, ρ)-GAM codebooks

(θ, ρ)-GAM codebooks are based on the golden angle modulation, information on which can be found on the list of references in work [7].

Step 1: Generating GAM points

$$x_n = \sqrt{\frac{2P(n+\rho)}{N_p+1}} \cdot \frac{e^{i2\pi(\varphi+\theta)}}{k},\ n = 1, \ldots, N_p \quad (5)$$

where $P = 1$ is the power constraint, $N_p = NM/2$ is the number of GAM points; $\varphi = (1-\sqrt{5})/2$ is the golden angle; θ and ρ are the optimization parameters that give additional degrees of freedom when designing the codebook; and k is the normalization coefficient.

Step 2: Mapping GAM points to an N-dimensional mother M/2-point constellation. The algorithm of the mapping is presented in [7]. The result of this mapping is $\mathbf{M}^m$ for $1 \le m \le M/2$.

Step 3: Getting the remaining points of the mother constellation. We get $\mathbf{M}^{m+M/2} = -\mathbf{M}^m$ for $1 \le m \le M/2$.

For (θ, ρ)-GAM, expressions (2) and (4) are written as

$$P_{avg} = \frac{2P}{(N_p+1)k^2} \cdot \left(\alpha + \frac{N(N_o - N_e)(M+2)}{4} \right)$$

$$PAPR[dB] = 10\lg\left(\frac{2MN(N_o - N_e) + 4\alpha}{N(N_o - N_e)(M+2) + 4\alpha} \right) \quad (6)$$

where $\alpha = N_o(N_o + \rho - N) + N_e(NM/2 + N_e + \rho + 1)$ is for $\rho > -1$. The normalization coefficient k is chosen in such a way that $P_{avg} = 1$.

3.3 Proposed (θ, ρ)-GAM codebooks codebooks with zero PAPR

Analyzing expression (6), we can conclude that for even N, the PAPR will be zero, and for odd N, the PAPR will be non-zero. This is due to the algorithm for mapping GAM points to an N-dimensional mother M/2-point constellation. In Algorithm 1, which is given in Appendix A, we propose a new way of mapping GAM points to obtain zero PAPR. After obtaining $\mathbf{M}^m$ for $1 \le m \le M/2$, the remaining points of the mother constellation are defined as $\mathbf{M}^{m+M/2} = -\mathbf{M}^m$.

Expressions (2) and (4) are written as

$$P_{avg}=\begin{cases}\dfrac{2P}{(N_p+1)k^2}\cdot\left(\alpha+\dfrac{M(N_o-N_e)(M+2)}{4}\right) & N \text{ is even}\\ \dfrac{2P}{(N_p+1)k^2}\cdot\left(\alpha+\dfrac{M(N_o-N_e-1)(M+2)}{2}\right) & N \text{ is odd}\end{cases}$$

$$PAPR[dB]=\begin{cases}10\lg\left(\dfrac{M^2(N_o-N_e)+2\alpha}{0.5M(N_o-N_e)(M+2)+2\alpha}\right) & N \text{ is even}\\ 10\lg\left(\dfrac{2MN(N_o-N_e-1)+2\alpha}{N(N_o-N_e-1)(M+2)+2\alpha}\right) & N \text{ is odd}\end{cases}$$

where

$$\alpha=(N-1)(N_o(N_o-1)+N_e^2)+(\rho+1)(N_o+N_e)+M\left(\frac{N_eM}{2}-N_o\right)$$

if N is even and

$$\alpha=N(N_eM-2N_o+3)+\rho(N_o+N_e)+N_e(N_e+1)+N_o(N_o-2)+1$$

if N is odd.

Algorithm 1. Mapping for downlink

if N is even
  for $k = 1 : N$
    if k is odd
      for $m = 1 : M/2$
        $\mathrm{Map}(k,m)=(N-1)(k-1)+1+M(m-1)$
      endfor
    else (k is even)
      for $m = 1 : M/4$
        $\mathrm{Map}(k,m)=(N-1)(k-1)+1+M(M/2-m)$
      endfor
      for m = $M/4+1 : M/2$
        $\mathrm{Map}(k,m)=-[(N-1)(k-1)+1+M(M/2-m)]$
      endfor
    endif
  endfor
  $N_p=(N-1)^2+1+M(M/2-1)$
else (N is odd)
  for $k = 1 : N$
    if k is odd
      if $k = 1$ or $k = N$
        for $m = 1 : M/2$
          $\mathrm{Map}(k,m)=k+N(m-1)$
        endfor
      else
        for $m = 1 : M/2$
          $\mathrm{Map}(k,m)=k+2N(m-1)$
        endfor
      endif
    else (k is even)
      for $m = 1 : M/4$
        $\mathrm{Map}(k,m)=k+2N(M/2-m)$
      endfor
      for m = $M/4+1 : M/2$
        $\mathrm{Map}(k,m)=-[k+2N(M/2-m)]$
      endfor
    endif
  endfor
  $N_p=N(M-1)-1$
endif
for $k = 1 : N$
  for $m = 1 : M$
    $n=\mathrm{Map}(k,m)$
    if $n > 0$
      $\mathrm{M}_k^m=x_n$ by formula (5)
    else ($n < 0$)
      $\mathrm{M}_k^m=-x_{|n|}$ by formula (5)
    endif
  endfor
endif

3.4 Proposed codebooks with zero PAPR

In this subsection, we propose Constant Modulus (CM) codebooks with high d_{min} and zero PAPR. We can write an N-dimensional point **c** of MC as

$$\mathbf{c}=\mathbf{A}\odot\begin{pmatrix}e^{i\varphi_{1,1}} & \cdots & e^{i\varphi_{1,M}}\\ \vdots & \ddots & \vdots\\ e^{i\varphi_{N,1}} & \cdots & e^{i\varphi_{N,M}}\end{pmatrix}\cdot\mathbf{e}=\mathbf{A}\odot e^{i\mathbf{\Phi}}\cdot\mathbf{e} \quad (7)$$

where **A** is a matrix of complex amplitudes of size $N\times M$, $\mathbf{\Phi}$ is a matrix of phases of the same size, and **e** is a vector of the M length determining the index of the codeword. **e** has M-1 zeros and one unit at the position defining the codeword index. There are M combinations of vector **e** in total.

The difference between two MC points can be written as

$$\Delta\mathbf{c}^{n,p}=\mathbf{A}\odot e^{i\mathbf{\Phi}}\cdot\Delta\mathbf{e}^{n,p}$$

where $\Delta\mathbf{e}^{n,p}=\mathbf{e}^n-\mathbf{e}^p, \forall\ p,n\in\{1,...,M\},\ p\neq n.$ Then, the Euclidean distance between two MC points is defined as

$$d^{n,p}=\sqrt{(\Delta\mathbf{c}^{n,p})^H\cdot\Delta\mathbf{c}^{n,p}} \quad (8)$$

To increase d_{min}, it is necessary to maximize the function

$$F=\min\{d^{n,p}\}$$

To maximize F, it is proposed to use the Gradient Ascent Algorithm (GAA), which is given in Algorithm 2.

Algorithm 2. Gradient Ascent Algorithm

Input: step size $\alpha>0$, initial phase matrix $\mathbf{\Theta}_0$, number of iterations T, $\{\Delta\mathbf{e}^{n,p}\}$, coefficient $0<\beta<1$
Output: $\mathbf{\Theta}_{opt}=\mathbf{\Theta}$
Initialization: k = 0, find F and corresponding $\Delta\mathbf{e}^{n,p}$, calculate **G** with $\Delta\mathbf{e}^{n,p}$
while $k < T$ **do**
  $\overline{\mathbf{\Theta}}=\mathbf{\Theta}+\alpha\mathbf{G}$
  Find $\overline{F}$ and corresponding $\Delta\mathbf{e}^{n,p}$, calculate $\overline{\mathbf{G}}$
  if $\overline{F}<F$
    $\alpha=\beta\alpha$

7: **endif**
8: $\mathbf{\Theta} = \overline{\mathbf{\Theta}}$
9: $F = \overline{F}$
10: $\mathbf{G} = \overline{\mathbf{G}}$
11: $k = k+1$
12: **endwhile**

The gradient of function (8) is shown at the top of the next page. The general algorithm for designing MC is presented below.

Step 1. Choosing matrix of complex amplitudes **A** in (7). Matrix **A** is chosen in such a way as to provide unit P_{avg} and zero PAPR.

Step 2. Optimization of Apply the GAA with (8) and (9), then

$$\mathbf{G}^m = \frac{i\Delta\mathbf{e}_m^{n,p}\begin{bmatrix}(\Delta\mathbf{c}^{n,p})^H \cdot \mathrm{diag}\left(e^{i\mathbf{\Phi}^m}\right)\cdot \mathrm{diag}\left(\mathbf{A}^m\right) - \ldots \\ \ldots - \left(\mathrm{diag}\left(\left(\mathbf{A}^m\right)^*\right)\cdot \mathrm{diag}\left(e^{-i\mathbf{\Phi}^m}\right)\cdot \Delta\mathbf{c}^{n,p}\right)^T\end{bmatrix}}{2\sqrt{(\Delta\mathbf{c}^{n,p})^H \cdot \Delta\mathbf{c}^{n,p}}}$$

Step 3. Generation of MC. The mother constellation is obtained as $\mathbf{M} = \mathbf{A} \odot e^{i\mathbf{\Phi}_{opt}}$.

4. CONSTRUCTION OF SCMA CODEBOOKS

4.1 SCMA codebook based on the MC phase rotation

The codebooks of each user are generated on basis of a sparse matrix **F** showing which subcarriers are used by each user. Let d_k be the number of users transmitting their data on the same subcarrier. The layer books are obtained by rotating the MC by a certain angle (Nikopour et al., 2013; Nikopour et al., 2013; Cai et al., 2016, Gao et al., 2018):

$$\varphi_u = (u-1)\frac{2\pi}{Md_k} + e_u\frac{2\pi}{Md_k}, \forall\, u = 1, \ldots, d_k \quad (9)$$

where e_u is an arbitrary member of . In works (Nikopour et al., 2013; Nikopour et al., 2013; Cai et al., 2016, Gao et al., 2018) $e_u = 0$. These angles are assigned to the non-zero elements of **F** in a certain order. The matrix **F** with a Latin structure provides better performance. The Latin structure used is shown below

$$\mathbf{F} = \begin{pmatrix} e^{i\varphi_1} & e^{i\varphi_2} & e^{i\varphi_3} & 0 & 0 & 0 \\ e^{i\varphi_2} & 0 & 0 & e^{i\varphi_3} & e^{i\varphi_1} & 0 \\ 0 & e^{i\varphi_3} & 0 & e^{i\varphi_1} & 0 & e^{i\varphi_2} \\ 0 & 0 & e^{i\varphi_1} & 0 & e^{i\varphi_2} & e^{i\varphi_3} \end{pmatrix} \quad (10)$$

The method for obtaining the Latin structure is presented in (Zhou et al., 2017).

After that, we define a user operator as

$$\mathbf{\Delta}_j = \mathrm{diag}(\mathbf{f}_j), \forall\, j = 1, \ldots, J$$

where $\mathbf{f}_j$ is the j-th column of (11) without zero elements. The user mapping matrix $\mathbf{V}_j$ is found on basis of **F**, satisfies $\mathbf{F}^j = \mathrm{diag}(\mathbf{V}_j \mathbf{V}_j^T)$, and represents $\mathbf{I}_N$ with $K-N$ zero rows. The subscriber's codebook is obtained as

$$\mathbf{X}_j = \mathbf{V}_j \cdot \mathbf{\Delta}_j \cdot \mathbf{M} \quad (11)$$

Analyzing expression (12), we can conclude that for fixed $\mathbf{V}_j$ and **M**, it is necessary to find optimal $\mathbf{\Delta}_j$ by some criterion to improve noise immunity.

4.2 Optimal rotation angles for maximizing MED between superimposed codewords

For the downlink channel, the signal at the output of the transmitter is the superimposed codeword (SCW) **r**, which is written as

$$\mathbf{r} = \sum_{j=1}^{J} \mathbf{x}_j \quad (12)$$

Codebooks providing a larger MED between SCWs with all possible implementations of vector **b** ensure better noise immunity [11]. For codebooks generated by (12), we can rewrite (13) as

$$\mathbf{r} = \overline{\mathbf{V}} \cdot \mathrm{diag}(e^{i\boldsymbol{\varphi}}) \cdot \overline{\mathbf{M}} \cdot \mathbf{e}$$

where $\overline{\mathbf{V}} = (\mathbf{V}_1, \ldots, \mathbf{V}_J)$, $\boldsymbol{\varphi}$ is a phase vector of the NJ length; $\overline{\mathbf{M}} = \mathrm{blckdiag}(\mathbf{M}_1, \ldots, \mathbf{M}_J)$ and $\mathbf{M}_j = \mathbf{M}$; $\mathbf{e} = (\mathbf{e}_1^T, \ldots, \mathbf{e}_J^T)^T$. $\mathbf{e}_j$ has M-1 zeros and one unit at the position defining the codeword index of the j-th subscriber. There are M^J combinations of vector **e** in total. The difference between two superimposed codewords can be written as

$$\Delta\mathbf{r}^{n,p} = \overline{\mathbf{V}} \cdot \mathrm{diag}(e^{i\boldsymbol{\varphi}}) \cdot \overline{\mathbf{M}} \cdot \Delta\mathbf{e}^{n,p}$$

where $\Delta\mathbf{e}^{n,p} = \mathbf{e}^n - \mathbf{e}^p, \forall\, p, n \in \{1, \ldots, M\},\ p \neq n.$ Then, the Euclidean distance between two superimposed codewords is defined as

$$d^{n,p} = \sqrt{(\Delta\mathbf{r}^{n,p})^H \cdot \Delta\mathbf{r}^{n,p}} \quad (13)$$

The gradient of function (14) is:

$$\mathbf{G} = \frac{i\begin{bmatrix}\mathrm{diag}\left((\Delta\mathbf{r}^{n,p})^H \cdot \overline{\mathbf{V}}\right)\cdot \mathrm{diag}\left(e^{i\boldsymbol{\varphi}}\right)\cdot \overline{\mathbf{M}} \cdot \Delta\mathbf{e}^{n,p} - \ldots \\ \ldots - \mathrm{diag}\left(e^{-i\boldsymbol{\varphi}}\right)\cdot \mathrm{diag}\left(\overline{\mathbf{M}} \cdot \Delta\mathbf{e}^{n,p}\right)\cdot \overline{\mathbf{V}}^H \cdot \Delta\mathbf{r}^{n,p}\end{bmatrix}}{2\sqrt{(\Delta\mathbf{r}^{n,p})^H \cdot \Delta\mathbf{r}^{n,p}}} \quad (15)$$

To maximize the MED, it is necessary to use the GAA with (14) and (15).

After applying it, $\boldsymbol{\varphi}_{opt}=\boldsymbol{\Theta}_{opt}=(\boldsymbol{\psi}_1,\dots,\boldsymbol{\psi}_J)$ and user operators $\boldsymbol{\Delta}_j$ are computed as

$$\boldsymbol{\Delta}_j=\mathrm{diag}(e^{i\boldsymbol{\psi}_j})$$

The $\Delta\mathbf{e}^{n,p}$ vector has $M^J\big/\left(2!\cdot(M^J-2)!\right)$ combinations. The proposed optimization method has a high computational complexity, but it can be reduced by using unique vectors $\Delta\mathbf{e}^{n,p}$. For example, for $M=4, K=4$ and $J=6$, there are 8386560 combinations of $\Delta\mathbf{e}^{n,p}$, among which 2413404 are unique. Thus, the computational complexity of this method can be reduced.

4.3 Optimal rotation angles for maximizing MED between constellation points

In [12], for the downlink channel, it was proposed to increase the MED between superimposed constellation points (SCPs) to improve the distinguishability of constellation points in the receiver. The set of points of the constellation of the *k*-th subcarrier is defined as

$$\Xi_k=\sum_{j\in\xi_k}x_{k,m_j}$$

where ξ_k is a set of non-zero positions in the *k*-th row of **F**. Any *p*-th point of Ξ_k can be written as

$$\Xi_k^p=\overline{\mathbf{V}}_k\cdot\mathrm{diag}(e^{i\boldsymbol{\varphi}})\cdot\mathbf{M}\cdot\mathbf{e}^p$$

where $\boldsymbol{\varphi}$ is a phase vector of the Nd_k length, $\mathbf{M}=\mathrm{blckdiag}(\mathbf{M}_1,\dots,\mathbf{M}_{d_k})$ and $\mathbf{M}_j=\mathbf{M}$; $\mathbf{e}=(\mathbf{e}_1^T,\dots,\mathbf{e}_{d_k}^T)^T$. $\mathbf{e}_j$ has *M*-1 zeros and one unit at the position defining the codeword index of the *j*-th subscriber. There are M^{d_k} combinations of vector **e** in total.

The difference between two superimposed constellation points can be written as

$$\Xi_k^{n,p}=\overline{\mathbf{V}}_k\cdot\mathrm{diag}(e^{i\boldsymbol{\varphi}})\cdot\mathbf{M}\cdot\Delta\mathbf{e}^{n,p}$$

Then, the Euclidean distance between two superimposed constellation points is defined as

$$d^{n,p}=\sqrt{(\Xi_k^{n,p})^*\cdot\Xi_k^{n,p}} \qquad (14)$$

The gradient of function (16) is shown at the bottom of the page. To maximize the MED, it is necessary to use the GAA with (16) and (17) for each subcarrier:

$$\mathbf{G}_k=\frac{i\begin{bmatrix}\mathrm{diag}\left((\Xi_k^{n,p})^*\cdot\overline{\mathbf{V}}_k\right)\cdot\mathrm{diag}\left(e^{i\boldsymbol{\varphi}}\right)\cdot\mathbf{M}\cdot\Delta\mathbf{e}^{n,p}-\dots\\ \dots-\mathrm{diag}\left(e^{-i\boldsymbol{\varphi}}\right)\cdot\mathrm{diag}\left(\mathbf{M}\cdot\Delta\mathbf{e}^{n,p}\right)^H\cdot\overline{\mathbf{V}}_k^{\,H}\cdot\Xi_k^{n,p}\end{bmatrix}}{2\sqrt{(\Xi_k^{n,p})^H\cdot\Delta\Xi_k^{n,p}}} \qquad (17)$$

After applying the GAA, $\boldsymbol{\varphi}_k^{opt}=\overline{\mathbf{V}}_k\cdot\mathrm{diag}(e^{i\boldsymbol{\theta}_{opt}})=(\theta_k^1,\dots,\theta_k^{d_k})$, it is necessary to convert $\boldsymbol{\varphi}_k^{opt}$ to $\boldsymbol{\psi}_1$ using the Latin factor graph (11). We will denote the obtained optimal angles as $\boldsymbol{\Psi}=(\boldsymbol{\psi}_1,\dots,\boldsymbol{\psi}_J)$.

The $\Delta\mathbf{e}^{n,p}$ vector has $M^{d_k}\big/\left(2!\cdot(M^{d_k}-2)!\right)$ combinations. For example, for $M=4, K=4$ and $d_k=3$, there are 2016 combinations of $\Delta\mathbf{e}^{n,p}$, among which 1098 are unique. The optimization method described in the previous subsection requires processing 2413404 combinations, and the method proposed in this subsection requires processing $K\cdot1098=4392$ combinations. Thus, the optimization method proposed in this subsection can significantly reduce the computational complexity of the procedures for generating codebooks.

5. RESULTS

This section presents the results of modeling MD-SCMA, (θ,ρ)-GAM, and CM codebooks in downlink Single Input Multiple Output with two receiving antennas (1×2 SIMO) Rayleigh channel. The SCMA system parameters are $J=6, K=4$ and $d_k=3$. SIMO modeling is carried out to study the books in MIMO channels. The bit error probability (BER) for all codebooks is estimated using five MPA iterations; the receiver has an ideal channel and noise estimation.

For the CM codebooks, we have tried using QAM, PSK, and APSK constellations with interleaving to get the MC. For the case when M = 4, four columns of a matrix represent the codewords labeled by 00, 01, 10, and 11, respectively. Similarly, for the case when M = 8, eight columns represent the codewords labelled by 000, 001, 010, 011, 100, 101, 110, and 111, respectively.

$$\mathbf{M}_4=\begin{pmatrix}-0.3880-0.5911i & -0.4954-0.5045i\\ 0.3943+0.5870i & -0.6408+0.2989i\\ -0.4220+0.5674i & 0.6376+0.3057i\\ 0.4280-0.5629i & 0.5008+0.4992i\end{pmatrix}^T$$

$$\mathbf{M}_4=\begin{pmatrix}0.3965-0.5855i & -0.6420+0.2963i\\ -0.5855-0.3965i & 0.2963+0.6420i\\ -0.3965+0.5855i & 0.6420-0.2963i\\ 0.5855+0.3965i & -0.2963-0.6420i\\ -0.1440-0.6953i & 0.2346-0.6671i\\ 0.6923+0.1440i & -0.6671-0.2346i\\ 0.1440+0.6923i & -0.2346+0.6671i\\ 0.6923-0.1440i & 0.6671+0.2346i\end{pmatrix}^T$$

Table 1 represents the optimal angles obtained for each of the codebooks.

Table 2 presents the characteristics of the received codebooks. We denote the MED of SCWs as d_{min} and the MED of SCPs on the *k*-th subcarrier as d_k^{min}. We also introduce an additional parameter $d=\sum_{k=1}^{K}\frac{d_k^{min}}{\sqrt{P_k^{avg}}}\Big/K$, where P_k^{avg} is the average power of the constellation of the *k*-th subcarrier.

Table 1. Optimum rotation angles

Codebook	*M*	$\boldsymbol{\Psi}$
MD-SCMA-SCP	4	$\begin{pmatrix} 2.9466 & 5.5828 & 0.6177 & 1.1763 & 2.4944 & 7.4463 \\ 1.5963 & 2.0885 & 4.0282 & -1.8658 & 3.2154 & 2.7101 \end{pmatrix}$
MD-SCMA-SCW	4	$\begin{pmatrix} 0.6085 & 0.1835 & 5.0425 & 0.3729 & 1.9960 & 2.7038 \\ 5.8382 & 4.4312 & 5.5049 & 2.9565 & 4.6604 & 6.0416 \end{pmatrix}$
(0.0635,0)-GAM-SCP [7]	4	$\begin{pmatrix} -4.2597 & 2.8591 & 3.3626 & 0.3669 & -0.0488 & 2.6353 \\ -1.7659 & 6.7080 & 1.8957 & 0.9270 & -5.7139 & 3.3681 \end{pmatrix}$
(0.0635,0)-GAM-SCW [7]	4	$\begin{pmatrix} 6.1433 & 5.0358 & 2.5132 & 4.9515 & 4.2598 & 5.1107 \\ 2.9361 & 1.0947 & 5.5072 & 6.0500 & 0.2452 & 6.0942 \end{pmatrix}$
(0.3054, 1000)-GAM-SCP	4	$\begin{pmatrix} 0.7852 & 3.6215 & 1.0906 & 0.1763 & -1.0882 & 0.3137 \\ 4.9775 & 5.7252 & 2.3708 & -4.3106 & 0.1891 & 0.4938 \end{pmatrix}$
(0.3054, 1000)-GAM-SCW	4	$\begin{pmatrix} 5.8589 & 5.1865 & 2.6469 & 4.8447 & 4.2393 & 5.4010 \\ 3.0634 & 0.7678 & 5.6086 & 6.0866 & 0.4477 & 5.7904 \end{pmatrix}$
CM-SCP	4	$\begin{pmatrix} 1.9546 & 0.9049 & 0.2888 & 1.5578 & -2.6334 & -5.1867 \\ 0.4587 & -5.8522 & -0.6204 & 4.1886 & 4.6131 & 6.2789 \end{pmatrix}$
CM-SCW	4	$\begin{pmatrix} 5.0564 & 3.5509 & 4.8898 & 4.4235 & -0.4114 & 6.4984 \\ 1.9671 & 4.6130 & 5.9353 & -0.5656 & 0.6568 & 2.6510 \end{pmatrix}$
MD-SCMA-SCP	8	$\begin{pmatrix} 3.4961 & 2.0686 & 5.5904 & -1.0926 & 9.3796 & 4.3230 \\ 4.6140 & 1.3654 & 4.4506 & 2.4126 & 2.3564 & 0.9290 \end{pmatrix}$
(0.08,0)-GAM-SCP [7]	8	$\begin{pmatrix} 4.3030 & -0.6546 & 1.0194 & 1.0044 & 2.1337 & 4.4985 \\ 3.7172 & 4.3906 & 1.6650 & 2.3673 & 3.8581 & 3.7146 \end{pmatrix}$
(0.2810,14)-GAM-SCP	8	$\begin{pmatrix} 4.4082 & 3.8022 & 4.8420 & 6.0634 & 3.2164 & 2.9751 \\ 3.5592 & 1.8434 & 0.3227 & 0.1466 & 2.7990 & 0.9650 \end{pmatrix}$
CM-SCP	8	$\begin{pmatrix} 1.2496 & 2.3485 & 0.6613 & 0.0305 & 2.0596 & 6.9043 \\ 2.5077 & -2.5986 & -1.7333 & 0.6598 & 8.9372 & -1.1531 \end{pmatrix}$

Table 2. Characteristics of codebooks

Codebook	*M*	d_{min}	*MPD*	d_1^{min}	d_2^{min}	d_3^{min}	d_4^{min}	d	d_{min}
MD-SCMA	4	1.414	0.8	0.1695	0.0827	0.0827	0.1695	0.1030	0.7830
MD-SCMA-SCP	4	1.414	0.8	0.3162	0.3162	0.3162	0.3162	0.2582	0.6325
MD-SCMA-SCW	4	1.414	0.8	0.1060	0.1825	0.2341	0.2982	0.1675	1.0415
(0.0635,0)-GAM [7]	4	1.289	0.658	0.089	0.0564	0.056	0.060	0.055	0.5331
(0.0635,0)-GAM-SCP [7]	4	1.287	0.658	0.198	0.2119	0.218	0.194	0.1693	0.649
(0.0635,0)-GAM-SCW [7]	4	1.287	0.658	0.081	0.0856	0.026	0.069	0.0544	0.936
(0.3054,1000)-GAM	4	1.414	1	0.004	0.1280	0.128	0.004	0.0539	0.754
(0.3054, 1000)-GAM-SCP	4	1.414	1	0.304	0.3031	0.304	0.303	0.2477	0.526
(0.3054, 1000)-GAM-SCW	4	1.414	1	0.070	0.0014	0.289	0.181	0.1106	0.991
CM	4	1.633	1.155	0.025	0.0557	0.056	0.025	0.0327	0.537
CM-SCP	4	1.633	1.155	0.255	0.2546	0.255	0.255	0.2078	0.518
CM-SCW	4	1.633	1.155	0.127	0.1353	0.093	0.108	0.0945	0.760
MED [11]	4	1.297/1.4	0.005/0.632	0.082	0.0848	0.006	0.003	0.0271	1.297
MD-SCMA	8	0.436	0.095	0.021	0.0202	0.020	0.021	0.0168	--
MD-SCMA-SCP	8	0.436	0.095	0.117	0.1165	0.117	0.117	0.0952	--
(0.08,0)-GAM [7]	8	0.524	0.134	0.012	0.0083	0.013	0.005	0.0079	--
(0.08,0)-GAM-SCP [7]	8	0.524	0.134	0.0363	0.0305	0.036	0.036	0.0284	--
(0.2810,14)-GAM	8	1.16	0.641	0.011	0.0115	0.012	0.001	0.0091	--
(0.2810,14)-GAM-SCP	8	1.16	0.641	0.033	0.0461	0.043	0.035	0.0320	--
CM	8	1.414	0.697	0.008	0.0048	0.005	0.008	0.0052	--
CM-SCP	8	1.4142	0.6965	0.059	0.061	0.062	0.063	0.0499	--

Fig. 1-Fig. 4 show a comparison of the BER performance of codebooks.

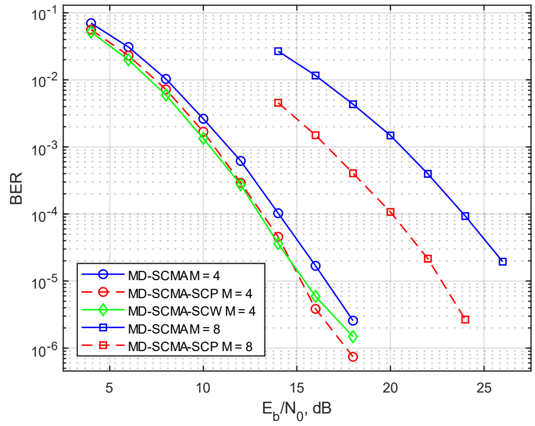

Figure 1. BER performance comparison for MD-SCMA codebooks

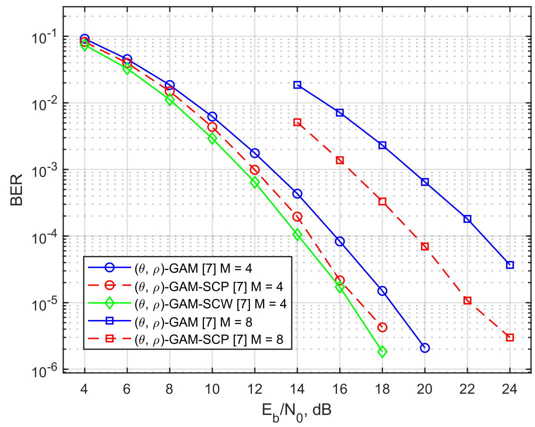

Figure 2. BER performance comparison for (θ, ρ)-GAM codebooks

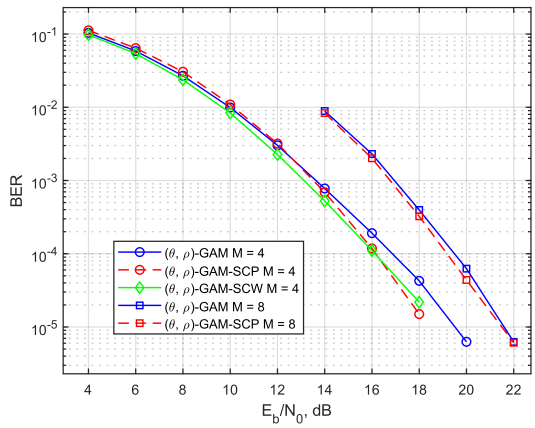

Figure 3. BER performance comparison for proposed (θ, ρ)-GAM codebooks

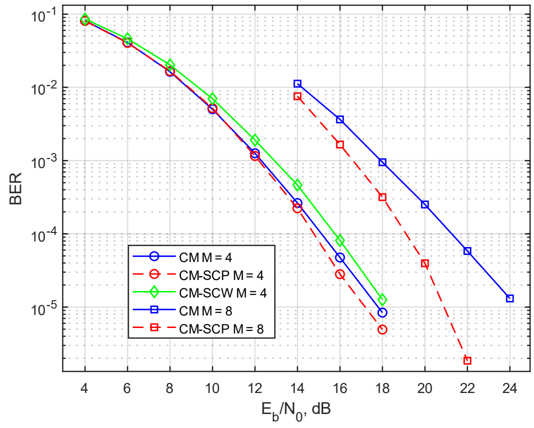

Figure 4. BER performance comparison for proposed CM codebooks

As can be seen from Fig. 1, for $M = 4$, the MD-SCMA-SCP and MD-SCMA-SCW have the same performance and exceed the MD-SCMA by about 1.15 dB at $\mathrm{BER} = 10^{-5}$. For $M = 8$, the MD-SCMA-SCP exceeds the MD-SCMA by 3.8 dB at $\mathrm{BER} = 10^{-4}$.

As can be seen from Fig. 2, for $M = 4$, the (θ, ρ)-GAM-SCP exceeds the (θ, ρ)-GAM by 1.17 dB and the (θ, ρ)-GAM-SCW exceeds the (θ, ρ)-GAM by 1.72 dB at $\mathrm{BER} = 10^{-4}$. For $M = 8$, the (θ, ρ)-GAM-SCP exceeds the (θ, ρ)-GAM by 3.2 dB at $\mathrm{BER} = 10^{-4}$.

As can be seen from Fig. 3, for $M = 4$, the proposed (θ, ρ)-GAM-SCP and the (θ, ρ)-GAM-SCW have the same performance and exceed the proposed (θ, ρ)-GAM by 0.7 dB at $\mathrm{BER} = 10^{-4}$. For $M = 8$, (θ, ρ)-GAM-SCP exceeds the proposed (θ, ρ)-GAM by 0.32 dB at $\mathrm{BER} = 10^{-4}$.

As can be seen from Fig. 4, for $M = 4$, the proposed CM-SCP exceeds the CM by about 0.8 dB at $\mathrm{BER} = 10^{-5}$. For $M = 8$, the CM-SCP exceeds the CM by 2.15 dB at $\mathrm{BER} = 10^{-4}$.

Analyzing Fig. 1-Fig. 4, we have concluded that the increase of d_k^{min} allows increasing the distinguishability of SPCs and improving the performance of MC-based codebooks at high SNRs. The high d_{min} of SCWs makes it possible to improve noise immunity only at low SNRs. Thus, the optimization method proposed in subsection 4.3 has a relatively low computational complexity and better noise immunity at high SNRs.

Fig. 5 shows a comparison of the BER performance of the books under consideration.

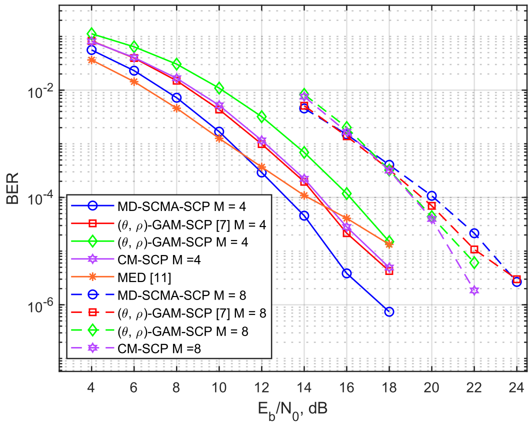

Figure 5. Comparison of BER performance of various books

As can be seen from Fig. 5, the MD-SCMA-SCP has the best noise immunity for $M = 8$, and the CM-SCP – for $M = 4$. Comparing the characteristics from Table 1, we concluded that the books with a high MPD (for $M = 8$) provide the best noise immunity at high SNRs. There is no such dependence for $M = 4$.

6. SIMULATION RESULTS

This paper has considered and refined some methods of the MC design and proposed a method for the MC design with high d_{min} and zero PAPR based on the GAA. The proposed method makes it possible to design the MC not only with high d_{min}, but also with high MPD. In addition, there have been proposed the optimization methods for maximizing the MED between superimposed codewords and superimposed constellation points. Both methods have a low computational complexity relative to similar ones.

The simulation showed that the SCP allows improving the distinguishability of points in the superimposed constellation and increasing noise immunity at high SNR relative to non-optimized MC-based codebooks. In turn, the SCW makes it possible to increase noise immunity at low SNR.

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