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Adaptive Kalman Filter for Free GPS Localization with Fuzzy Intersection Method

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Abstract: Global Positioning System (GPS) signal outage and noise in the sensor reading impact the accuracy of vehicle position. Thus, noise covariance must be regularly adjusted. A priori knowledge about noise statistics in vehicle positioning applications is difficult to obtain. This study proposes the adaptive Kalman filter (KF) and the fuzzy intersection method for free GPS localization. The KF's parameters were adapted using the fuzzy intersection method and fuzzy model. First, a dataset based on map information was developed to capture road coordinates and predict noise covariance. Second, fuzzy intersection method obtained a good initial state vector. Third, a fuzzy clustering algorithm based on a weighted fuzzy expected value was used to conclude the problem space into the cluster prototypes. Fourth, the fuzzy parameter model was learned from the clustering algorithm in the previous step without expert systems. This study used two road network configurations characterized as single and multiple road entry points. The position accuracy was estimated using the root mean square error. In the first network, the proposed method achieved 1 m accuracy compared to 4, 7, and 9 m accuracies in other related papers, while in the second network; it achieved 2 m accuracy compared to 5, 7, and 10 m accuracies in other related works.

Keywords: Kalman filter, Free GPS localization, Fuzzy logic, V2I communication, Intersection method.

1. Introduction

Kalman filter (KF) is a real-time estimator for dynamic state-space problems. It is designed to solve linear systems. The noise followed a Gaussian distribution. KF does not keep a large amount of historical data for future prediction. Therefore, it is known as a parametric algorithm. The problem space can be adapted by variance analysis [1, 2].

The results of KF may be divergent or stable in nonlinear systems and non-Gaussian and Gaussian distributions with inaccurate noise covariance. It is difficult to have a priori knowledge about noise covariance. Different variants of the standard KF have been proposed to solve nonlinear problems, such as extended Kalman filter (EKF) and unscented Kalman filter (UKF). They also require priori knowledge of variance statistics to avoid divergence or stability [3].

Various adaptive filter methods, such as the Bayesian, maximum likelihood, correlation, and covariance matching methods, are believed to prevent divergent results. They adapt noise covariance matrices in the state to make them static. Covariance matching methods are common adaptive filter methods. They search for the matching degree between the theoretical variance analysis and the current variance analysis. There are two common assumed methods: the degree of mismatching (DOM) and degree of divergence metrics (DOD). The DOM searches for the inconsistency between the theoretical variance and the actual variance in measurements. The DOD estimates the scalar value that describes the traces between the theoretical and actual covariance values [4, 5]. The methods are easily implemented. Their accuracy depends on the window updating size, as well as the degree of measurement accuracy.

With its variations in vehicle positioning applications, KF is used extensively in different navigation systems such as the global positioning system (GPS) and the inertial navigation system (INS) to improve estimated position accuracy purposes. The GPS is basedon satellites positioning system that provides an absolute position that is accurate for a long period, although the GPS's accuracy is reduced owing to the GPS outage signals, non-line-of-sight multipath effect, a smaller number of visible satellites, and signal propagation delay. The INS is a set of customized sensor nodes that detect velocity and angular rates in three dimensions, such as accelerometers and gyroscopes. The INS provides a relative position with high accuracy for a short period. Its accuracy is impaired by malfunction, systematic errors, and random errors. These errors accumulate over time. Different INS and GPS integration modes are assumed. They estimate INS errors using a filtering algorithm. They do not provide enough solutions owing to previous drawbacks [6, 7, 8]. The free GPS cooperative localization has recently emerged as an alternative solution. This enables another source of measurement that can be used to increase the measurement accuracy. It depends on the vehicle-to-infrastructure (V2I) communications. A large number of roadside units (RSUs) are configured on one or two roadsides. RSUs exchange the packets with the nearby vehicles.

Vehicles use V2I to improve the GPS/INS positions or to estimate initial positions. KF, with its variations, is used to enhance the initial positions. The accuracy of KF is reduced owing to the lack of priori knowledge of how to adapt noise covariance. In our previous study [9], EKF was used to improve the initial vehicle positions estimated by the V2I communication and intersection method. Vehicles received beacon messages from at least two RSUs. The intersection points were estimated to be equal initial state vector. The intersection method was less effective in the case of increasing measurement errors. Furthermore, the accuracy of EKF was related to adapting noise covariance in the process and measurement instead of using them constantly to express more uncertainty.

This study adapts the noise covariance and handles the nonlinearity using the fuzzy logic theory [10, 11]. Additionally, fuzzy triangle numbers express the uncertainty around a vehicle position, and therefore, they handle the uncertainty in distance measurements. This study can be summarized as follows:

1. The use of digitized map information allows us to determine road coordinates, predict noise in the vehicle position, and map the vehicle position to road coordinates.
2. The fuzzy intersection method handles the uncertainty around the vehicle position and distance measurements.
3. The V2I communication is used to initialize the position vector.
4. A fuzzy clustering algorithm based on a weighted fuzzy expected value (WFEV) divides a road map into multiple clusters or regions. The cluster prototypes are used to conclude the problem space and then used as the initial noise covariance in the process and measurement.
5. A fuzzy model is built using cluster prototypes without needing expert systems. Sometimes, they are unavailable. The number of fuzzy rules equals the number of clusters, and there is a direct relationship between several input variables and clusters. Furthermore, the fuzzy model is used to update the noise in the measurements.

The experimental results show that the proposed method against other related research papers [9, 25, 26]. In complex road network configurations, the proposed method gives a positional accuracy of about 2 m compared to 5, 7, and 10 m provided in other studies. In a simple road network configuration, the proposed method gives a positional accuracy of about 1 m compared to 4, 7, and 9 m provided in other studies.

The rest of this paper is structured as follows: Section 2 explains the standard KF algorithm. Section 3 explains the related. Section 4 explains that the proposed adaptive KF method via a free GPS localization and fuzzy intersection method. Section 5 provides the experimental results. Section 6 provides a discussion and analysis of the experimental results. Section 7 outlines the proposed algorithm and explains the directions for future work.

2. Standard Kalman filter

Standard KF is designed for linear systems. The following equation can be used to describe a linear system [12, 13]:

$$x_{k+1,k} = AX_{k,k} + BU_k + w_k \tag{1}$$

where $x_{k+1,k}$ is a posterior state at time k + 1, $X_{k,k}$ is a priori state at time k, and U_k is a control variable such as velocity and acceleration. A and B are the transition matrices. W_k is a process white noise covariance matrix with variance Q and zero mean. It is made up of prediction and correction steps. In the prediction step, the predicted error covariance matrix $p_{k+1,k}$ is calculated as follows:

$$p_{k+1,k} = Ap_{k,k}A^T + Q_k \quad (2)$$

Where Q is a noise covariance matrix in the process. The innovation $\nu_{k+1,k}$ is defined as the difference between the actual measurement z_{k+1} and its predicted state vector $x_{k+1,k}$. It represents the errors in the prediction step [14]:

$$\nu_{k+1,k} = z_{k+1} - Hx_{k+1,k} \quad (3)$$

Where H is a transition matrix. The correct measurement updates the predicted posterior state based weighted difference between an actual measurement Z_{k+1} and a predicted state $x_{k+1,k}$ as follows:

$$x_{k+1,k+1} = x_{k+1,k} + K(Z_{k+1} - Hx_{k+1,k}) \quad (4)$$

Where $\hat{x}_{k+1,k+1}$ is a posterior state at time k + 1. The Kalman gain (K) is a weight factor representing the minimal square errors between the predicted state and the noise covariance in the measurements R:

$$K = P_{k+1,k}\, H^T\left(HP_{k+1,k}\ H^T + R\right)^{-1} \quad (5)$$

The residual is defined as the difference between the actual measurement Z_{k+1} and the posteriori state $x_{k+1,k+1}$ [14]. This represents the errors in the correction step:

$$\xi_{k+1,k+1} = z_{k+1} - Hx_{k+1,k+1} \quad (6)$$

The predicted error covariance matrix $P_{k+1,k+1}$ is updated to posteriori predicted error covariance $P_{k+1,k+1}$ as follows:

$$P_{k+1,k+1} = (I - K.H)P_{k+1,k} \quad (7)$$

3. Related studies

This section discusses related research papers. The studies are split into two categories. The first category is adaptive noise covariance in KF algorithms [15-23, 27]. The studies belonging to the first category worked on the integration mode of the GPS and INS. The second category is cooperative localization methods [24-26]. They are based on V2V or V2I communication. They reduced the noise in the measurement.

The authors in [15, 16] proposed an adaptive KF based on covariance matching methods. The Mamdani model was used to estimate a scalar value that equaled the noise covariance updating rate. This model uses the single-input, single-output system. The input variable was a DOM or DOD, while the output variable was a noise covariance variable. The parameters of membership functions were determined by an expert system. The model' results might lead to bounded. The expert systems were not available at all times.

In [17], the authors proposed adaptive neuro-fuzzy extended Kalman filtering for robot localization. The authors used a DOM method and the delta of DOM, to measure the inconsistent degree and the changing rate in DOM, respectively. The steepest gradient descent algorithm was used to determine the parameters of ANFIS. The complexity degree of ANFIS was attributed to its structure, i.e., five layers. Its accuracy was more related to tuning parameters and the gradient method. This caused slow running of the learning algorithm. This is not agreed upon in safety applications.

In [18], the authors proposed a dual-optimized adaptive Kalman filtering (DO-AKF) algorithm based on the backpropagation (BP) neural network and variance compensation principle. The BP neural network was built based on the compensation error. A principle signifying errors between the measured and the best-predicted vector. The input layers consist of three variables: innovation, residual, and Kalman gain. The output layer was noise. After training the BP neural network, the compensation error was used to update the posterior state vector, as shown in Eq. (4). It did not account for the measurement uncertainty. Accuracy is related to the predefined BP neural network parameters.

In [19], the authors used the Mahalanobis distance and BP neural network to increase the accuracy of the INS/GPS final position. The Mahalanobis distance was estimated between INS and GPS measurements. Newton's method was used to minimize the errors in distance measurement. This was used to initialize the noise covariance in the KF algorithm. BP neural network was trained to adapt noise covariance in GPS and INS measurements in case of a GPS outage signal. The position accuracy was related to the precise degree of GPS/INS measurements and with the tuning parameters of the BP neural network.

In [20], the authors proposed an adaptive EKF based on fuzzy innovation covariance for improving the INS position. One EKF algorithm was used to estimate INS noise in case of the lack of GPS measurements. The authors used two Mamdani models to adapt the noise covariance. The input variables of the first model were position dilution of precision and the number of apparent satellites. The output variable was the position error. The second model used the innovation and output of the first model as input variables. The output variable was the noise covariance. In an urban environment, the non-line-of-sight, a smaller number of visible satellites (i.e., less than four satellites), and GPS outage signal have reduced the accuracy of GPS measurement.

In [21], the authors used a fuzzy neural network to train the input variables (i.e., GPS, INS, and odometer) and output variable (i.e., velocity correction) to enhance INS measurement. The training process was applied in case of the availability of GPS data. Therefore, measurement accuracy was reduced in urban environments and with a smaller number of visible satellites. In [22], the authors proposed a mixed prediction approach to enhance the INS's vehicle position. It consisted of a radial basis function neural network, time series analysis, and UKF algorithms. The predicted errors depended on the radial basis neural network and time series in line-of-sight conditions. The output of this prediction was used to adjust the noise covariance in UKF in GPS to block the signal. The experimental results showed that using a mixed prediction approach rather than using each prediction alone achieved high accuracy. It took more computation time and longer training time to adapt the parameters.

In [23], the author used Sage–Husa adaptive method to adapt noise covariance in the process and measurements. Sage–Husa adaptive method updated the noise covariance based on previous innovation data and constant parameters to prevent the divergence and guarantee positive definite noise covariance matrices. A singular matrix problem could be generated in the case of bound measurements.

In [24], the authors proposed GPS-free localization using a directional antenna. There were two RSUs installed on one roadside. Each RSU had a directional antenna with a fixed direction. The vehicle estimated the angle of arrival upon receiving the beacon messages. There were no intersection lines on the road. The final vehicle position was determined using the straight-line equation. The position accuracy depends on the strength degree of the received signals and the multipath effect.

In [25], the authors used GPSs/INSs to obtain the initial positions. The EKF was then used to improve the vehicle position. In the EKF algorithm, the nonlinear distance function, i.e., nearby vehicles, and the kinematic mathematical model were linearized to solve nonlinear problems and correct the INS measurement using V2V communication. The noise covariance matrix in the process and measurement was constant. The position errors might accumulate over time. The results showed divergence because of increasing uncertainty in measurement. As the number of neighbouring vehicles moving in the same direction increased, the position accuracy improved. This is difficult owing to lane-change scenarios and urban environments.

In [26], a cooperative vehicle localization improvement using the distance information (COVALID) algorithm was proposed. This algorithm is an improved version of the VANET location improve (VLOCI) algorithm. The weighted centroid method was used to estimate the initial coordinates; the unknown vehicle position determined a scale factor for each neighbour based on the measured distance. The distances measured by the GPS/INS systems differed, and this was specified. The rules for similar triangles were applied to find neighbours that relax on a straight line or have a linear link with the unknown vehicle. The final position determined by the GPS was updated by the difference in distance between the INS and the GPS. The COVALID algorithm reduced GPS position error by 63%; this is still not a good enough solution, especially on highways. The similarity metric is also influenced by the multipath effect in the distance estimation between neighbouring vehicles. They give constant weight to each neighbour based on different distance ranges. Constant weights do not express neighbours well. Therefore, there may be a chance to find vehicles outside the network boundaries.

In [27], the authors used standalone GPS/INS to trace the autonomous vehicle. This vehicle was equipped with two GPS receivers and an INS (i.e., an attempt to get more measurements). The authors used an agglomerative clustering algorithm and then the singular value decomposition method to adapt the noise covariance matrix in the process and measurement. The multipath effect, GPS signal outage, and random noise in measurement decreased the precision of the vehicle position. The GPS and INS errors accumulate over time.

4. Adaptive Kalman filter and fuzzy intersection method

The adaptive KF and fuzzy intersection method are discussed in detail. It is based on V2I communication. The RSUs are installed on two roadsides, and they perform the following tasks:

1. Each RSU has global information about the road map coordinates in its ROI.

2. RSUs are responsible for dividing the network into N clusters or regions. Each cluster is characterized by cluster identifier, prototype, and cluster variance.
3. Each RSU sends periodic beacon messages with its coordinates and additional information in point 2.

Each vehicle receives the beacon messages. It calculates the distance to each RSU at the time of arrival. Each vehicle registers the information in the topology table, i.e., cluster identifier, cluster prototypes, cluster variances, and RSU position. This information will be further used to initialize the noise covariance in the process and measurement. Each vehicle searches for which the cluster has a membership by finding the minimum distance between itself and each cluster prototype.

There are four steps in the adaptive KF process. The first step extracts road data to map vehicle coordinates to road coordinates. The second step uses the fuzzy intersection method to get a good initial state vector. The third step uses a fuzzy clustering algorithm to divide the road data into different regions or clusters. The last step uses the Mamdani model to update noise covariance in the measurement.

4.1 First step: map information

This step defines more information about the road coordinates. The benefit of this step is mapping the vehicle position to road coordinates to get more precise vehicle locations. Recently, Google Maps and Open Street Map have been extensively used to extract the road coordinates. This is known as global information, which can be programmed into RSUs. The random dataset is drawn according to the road coordinates, and it consists of x–y road coordinates.

$$x_i = rand.X_{coord} \quad i = 1 \dots n \tag{8}$$

$$y_i = rand.Y_{coord} \quad i = 1 \dots n \tag{9}$$

X_{coord}, Y_{coord} are the road coordinates. n denotes number of nodes.

4.2 Second step: fuzzy intersection method

The standard intersection formula [28] depends on the precise degree of distance measurement and suitable radical line (i.e., R12), as shown in Fig. 1 [9]. There is a direct relationship between them. The fuzzy intersection method is the fuzzification of the standard intersection formula [28]. The fuzzy triangle numbers and fuzzy singleton functions are

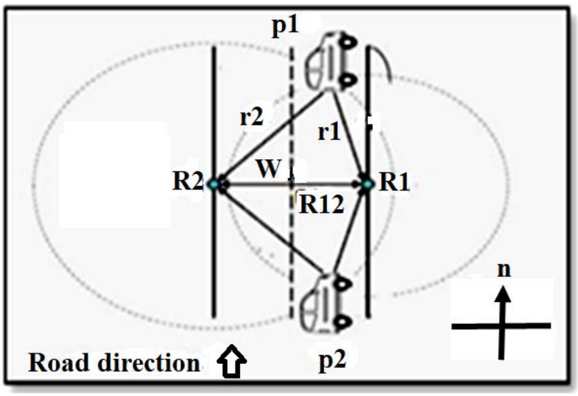


Figure. 1 Intersection point

used to indicate the uncertainty in the distance and circle center's coordinates, respectively. This step is used to draw a dataset by each RSU and then by each vehicle. The linguistic terms "about x" and "about y" represent the fuzzy coordinates of the intersection point X_p respectively, as shown in Fig. 2. The mf(x) and mf(y) are membership values for x and y coordinates. x_L, x_m, x_U , y_L, y_m, y_U are the lower, middle, and upper parameters of two fuzzy triangle numbers [10, 11]. These parameters are determined by the following equation:

$$X_L = X_m - \alpha_x . X_m = X_m .(1 - \alpha_x) \tag{10}$$

$$Y_L = Y_m - \alpha_y . Y_m = Y_m .(1 - \alpha_y) \tag{11}$$

$$X_U = X_m + \alpha_x . X_m = X_m .(1 + \alpha_x) \tag{12}$$

$$Y_U = Y_m + \alpha_y . Y_m = Y_m .(1 + \alpha_y) \tag{13}$$

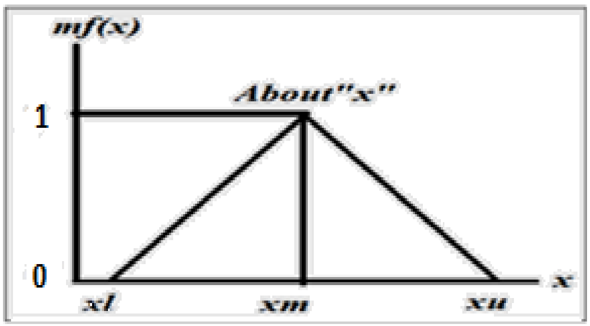
$$X_m = X_p, Y_m = Y_p \tag{14}$$

where α_x, α_y equal the value between 0 and 1$\in [0, 1]$. The middle parameters are equal to the actual point coordinate. Fuzzy singleton functions are used to represent two fixed RSU coordinates. All membership values for all parameters (i.e., lower, upper, and middle) equal 1. The distance between the RSUs and each point is calculated using the vertex distance method as shown in the following equations:

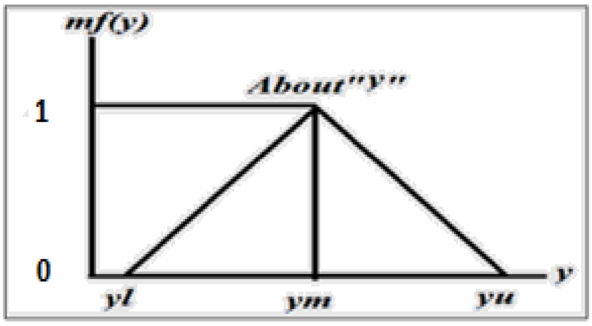
$$d_{ij}^x = \sqrt{\frac{1}{3}[(x_l^j - RX_{Li})^2 + (x_u^j - RX_{ui})^2 + (x_m^j - RX_{mi})^2]} \tag{15}$$

$$d_{ij}^y = \sqrt{\frac{1}{3}[(y_l^j - RY_{Li})^2 + (y_u^j - RY_{ui})^2 + (y_m^j - RY_{mi})^2]}$$

$$i=1,\ 2,\ \ j=1 \dots n \tag{16}$$



(a)



(b)
Figure. 2 Triangle fuzzy number :(a) “about x” and (b) “about y”

where d_{ij}^{x} and d_{ij}^{y} are the square root of the center of the area in the x–y dimensions. $RX_{Li}, RX_{ui}, RX_{mi}, RY_{Li}, RY_{ui}, RY_{mi}$ are the lower, upper, and middle parameters of fuzzy singleton functions for two RSU dimensions. The final distance r_{ij} equals the sum of the d_{ij}^{x} and d_{ij}^{y} as follows:

$$r_{ij} = d_{ij}^{x} + d_{ij}^{y} \tag{17}$$

In the standard intersection method, the lamba λ_i and epsilon ϵ_i represent the change in the x-coordinate and y-coordinate, respectively. The lamba λ_i equals the cosine rule to express horizontal changes between the intersection point and the RSU. The epsilon ϵ_i equals the changing y in vertical space to express the remainder of the estimated distance between the intersection point and the RSU. Therefore, fuzzy triangle numbers can express the uncertainty in the x and y coordinates. In the standard intersection formula, they are estimated using the following equations:

$$\lambda_i = \frac{r_{i1}^2 - r_{i2}^2 + R_{12}^2}{2R_{12}^2} \tag{18}$$

$$\epsilon_i = \sqrt{\left|\frac{r_{i1}^2}{R_{12}^2} - {\lambda_i}^2\right|} \tag{19}$$

The three parameters (i.e., lower, middle, and upper) of each fuzzy triangle number for λ_i and ϵ_i are determined using the following equations:

$$\lambda_L = \lambda_i - \alpha_\lambda \lambda_i = \lambda_i(1 - \alpha_\lambda) \tag{20}$$

$$\lambda_U = \lambda_i + \alpha_\lambda \lambda_i = \lambda_i(1 + \alpha_\lambda) \tag{21}$$

$$\lambda_m = \lambda_i \tag{22}$$

$$\epsilon_L = \epsilon_i - \alpha_\epsilon \epsilon_i = \epsilon_i(1 - \alpha_\epsilon) \tag{23}$$

$$\epsilon_U = \epsilon_i + \alpha_\epsilon \epsilon_i = \epsilon_i(1 + \alpha_\epsilon) \tag{24}$$

$$\epsilon_m = \epsilon_i \tag{25}$$

where $\alpha_\epsilon = \alpha_\lambda$. We substitute $\lambda_L, \lambda_U, \lambda_m, \epsilon_L, \epsilon_U, \epsilon_m$ in the following equations to get three intersection points (x_l, x_m, x_u), (y_l, y_m, y_u).

$$x_{pj} = x_{R1} + \lambda_i x_d \mp \epsilon_i y_d \tag{26}$$

$$y_{pj} = y_{R1} + \lambda_i y_d \pm \epsilon_i x_d \tag{27}$$

The center of area method is calculated to get the crisp intersection point (j) as follows:

$$x_j = \frac{x_l + x_m + x_u}{3} \tag{28}$$

$$y_j = \frac{y_l + y_m + y_u}{3} \tag{29}$$

The final point position is (x_j, y_j).

4.3 Third step: fuzzy clustering with expected value

The fuzzy clustering algorithm based on WFEV is an expansion of the FCM algorithm [29, 30]. It minimizes the objective function as follows:

$$obj = \sum_{i=1}^{c} \sum_{j=1}^{n} u_{ij}^{m} \parallel x_j - c_i \parallel^2 + \sum_{i=1}^{c} \sigma_i \sum_{j=1}^{n} (u_{ij}^{m} . \log u_{ij}^{m} - u_{ij}^{m}) \tag{30}$$

where σ_i is a variance of a cluster i. u_{ij}^{m} is a membership function for point j in the cluster (i), and c_i is a cluster center or prototypes. It is a Gaussian fuzzy membership value. The point near a cluster center is given more weight by the Gaussian membership function than is the farther point. Therefore, the noisy points have a low membership value and has little influence on cluster centers. Each data point has D dimensions as follows:

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1. The x–y coordinates are drawn randomly based on the road coordinates in the first step.
2. The $x^{\sim}-y^{\sim}$coordinates are estimated using the fuzzy intersection point.
3. $N_x = |x - x^{\sim}|$, $N_y = |y - y^{\sim}|$ are the absolute difference value or error between the actual point and estimated coordinates.
4. The root mean square error (RMSE) is a square root of the sum N_x and N_y.

$$RMSE = \sqrt{N_x^2 + N_y^2} \tag{31}$$

These dimensions represent the actual coordinates and the uncertainty around the data points. The following steps are the procedures for a fuzzy clustering algorithm with a WFEV and a modified merging algorithm.

First step: Get the cluster centers and variances after running the K mean cluster algorithm on the dataset.

Second step: The "eliminating" function removes the cluster with fewer cluster members or no cluster member.

Third step: The weighted fuzzy variance (WFV_i) and fuzzy weight (FW_{ij}^t) are calculated from the following equations, respectively:

$$WFEV_i = C_i^{t+1} = \sum_{j=1}^{M} FW_{ij}^t \, x_j \tag{32}$$

$$WFV_i = (\sigma_i^2)^{t+1} = \sum_{j=1}^{M} FW_{ij}^t \, (x_j - C_i^t)^2 \tag{33}$$

$$FW_{ij}^t = u_{ij}^t = \frac{exp\left[-\frac{(x_j - C_i^t)^2}{2\sigma_i^2}\right]}{\sum_{j=1}^{M} exp\left[-\frac{(x_j - C_i^t)^2}{2\sigma_i^2}\right]} \tag{34}$$

where M is the number of points in a cluster C_i^t, x_j is a data point, and σ_i^2 is a variance for cluster (i).

Fourth step: each data point is allocated to the closest cluster center based on the shortest Euclidean distance.

Fifth step: The crisp variances are estimated in-state weight fuzzy variance (WFV) after little rounds to reach convergence quickly.

Sixth step: Test convergence test is done. If it is achieved, go to step 7; otherwise, go to step 3.

Seventh step: The merging algorithm

7-1: generate the squared similarity matrix S.

7-2: for each row, we get max (S).

7-2-1 If max S (.) >= Threshold

7-2-1- 1 merges two clusters.

A similarity measure is calculated between each pair of cluster centers. It is based on two parameters: cluster centers and variance [31]. Therefore, the optimal number of clusters can be gained. The distance is not a good reference to fuse clusters because the clusters may be disconnected, but these clusters are overlapped.

4.4 Fourth step: adaptive Kalman filter based on the Mamdani fuzzy model

Each vehicle performed this step after receiving the beacon messages from the RSUs. The Mamdani model [10, 11] is built from the clustering algorithm output without the need for expert systems. The Mamdani output is biased for the specified fuzzy membership function parameters. The parameter identification may not express the real problem. Furthermore, sometimes experts are not available. The Mamdani model uses a multi-input, single-output system. The input variables are estimated as $x^{\sim}-y^{\sim}$using the fuzzy intersection method and noise in two coordinates N_x, N_y. The output variable is the root mean square error (RMSE). The following steps explain the construction of the fuzzy model:

Fuzzification: is done using a fuzzy clustering algorithm based on the WFEVs. The Gaussian membership function represents the linguistic terms for the input and output variables. Examples of the output membership functions result from clustering algorithms, as shown in Fig. 3.

Rule Base: consists of a set of "if–then" rules. The number of rules equals the number of clusters, as shown in Table 1.

Defuzzification: is done at the center of the area to get a crisp output.

The purpose of this step updates the noise covariance in R. The following example explains the procedures of adaptive KF based on the Mamdani model. The priori state vector is equal to the vehicle coordinates after applying the fuzzy intersection method in the prediction step:

$$X_{k,k} = \begin{bmatrix} x_j \\ y_j \end{bmatrix} \tag{35}$$

the posterior state based on the priori state becomes

Table 1. Rule base

R_1	If $Est_x$is $A_1$and $Est_y$is B_1 and N_x is C_1 and N_y is D_1	Then RMSE is RES_1
R_2	If $Est_x$is A_2 and $Est_y$is B_2 and N_x is C_2 and N_y is D_2	Then RMSE is RES_2
R_3	If $Est_x$is A_3 and $Est_y$is B_3 and N_x is C_3 and N_y is D_3	Then RMSE is RES_3
R_4	If $Est_x$is A_4 and $Est_y$is B_4 and N_x is C_4 and N_y is D_4	Then RMSE is RES_4

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$$x_{k+1,k} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_j \\ y_j \end{bmatrix} \quad (36)$$

the predicted error covariance matrix becomes

$$p_{k+1,k} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_{i1} & 0 \\ 0 & c_{i2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} c_{i5} & 0 \\ 0 & c_{i6} \end{bmatrix} \quad (37)$$

where c_{i1}, c_{i2} are the first two dimensions of cluster center (i), which has the minimum distance to the vehicle (j). c_{i5}, c_{i6} are the fifth and sixth dimensions of cluster center (i), respectively. The noise covariance in measurements (R) equals the third and fourth dimensions of a cluster (i). Kalman gain (g) is estimated using Eq. (4). The posterior state at time k + 1 is then employed:

$$X_{k+1,\mathrm{k}+1} = \begin{bmatrix} x_j \\ y_j \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22}1 \end{bmatrix} \left(\begin{bmatrix} X_{ins} \\ Y_{ins} \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_j \\ y_j \end{bmatrix} \right) \quad (38)$$

the Mamdani fuzzy model is used to update the noise covariance matrix in measurement R to further reduce the position errors. The following values consist of the input vector:

$$R = \mathcal{F}_{\mathcal{M}} \left[x_j \quad y_j \quad Res_{11} \;\; Res_{21} \right] \quad (39)$$

where $\mathcal{F}_{\mathcal{M}}$ is a fuzzy model and Res is estimated residual vector using Eq. (6). z_{k+1} is an INS measurement. Update measurement steps are repeated with the output of the fuzzy model (R) to update the posteriori state at time k + 1.

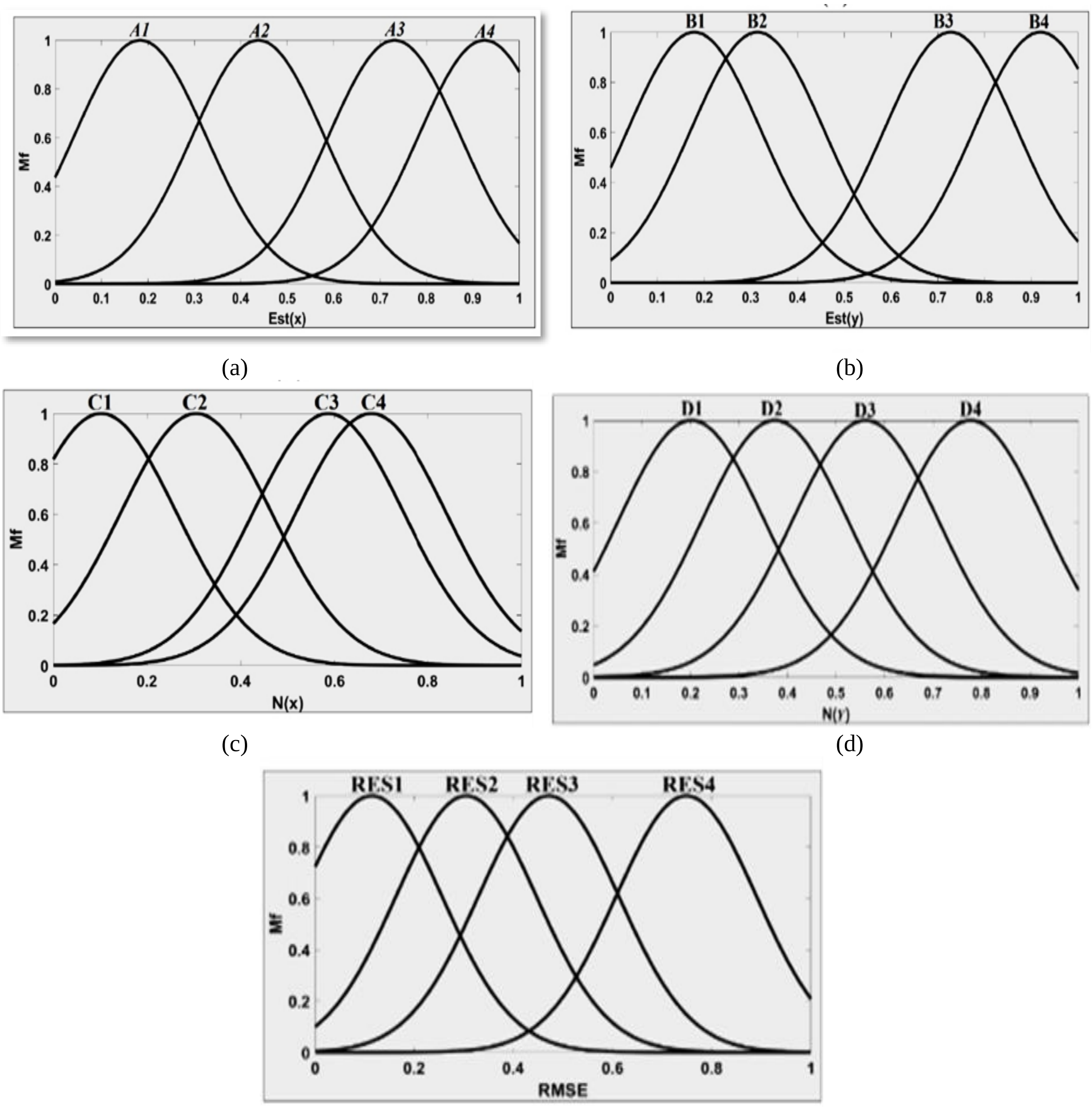
5. Simulation scenario

The purpose of this section is to study the effect of KF-adapted parameters in the proposed method on vehicle position accuracy and compares to our previous work [9]. At the same time, studying the integration between KF and fuzzy logic effect in adapting noise covariance matrixes instead of using them constant. Additionally, proposed method also compares to other related research papers [25, 26] to test the efficiency in handling nonlinear problems on position accuracy. In [9], the authors used classical formula of the intersection method to provide a good initial state vector. EKF linearize the distance function and reduce noise in measurement. In the case of a failed RSU, the virtual RSU was estimated on the basis of mobility measurement to continue estimate vehicle position. In [25], the authors used the GPS/INS integration mode to get the position vector. EKF was used to linearize the distance function between nearby vehicles and reduce noise in measurement. Furthermore, EKF also linearize the INS's mathematical equations, i.e., the kinematic model. The reliance on V2V communication overcomes the limitations of GPS/INS in urban environments. In [26], the authors used a weighted centroid method to improve the initial position state vector. Nearby vehicles exchange the GPS/INS measurements. Weight factors were used to represent the uncertainty in distance measurement. The rules for similar triangles were used to search for a neighbour, which has a linear relation with the target vehicle and less noise in measurements. EKF minimizes the distance function.

In [25, 26] cooperative localization methods used constant values in the noise covariance matrix except proposed method. We assume GPS position errors from 0.5 to 3m in [25, 26]. The performance of the previous methods is evaluated by applying them to two different scenarios. The first scenario was a one-way road in Enlargen where vehicles entered from one entry point. The second scenario was more complex and concerned different roads in KarradaIn. Vehicles entered from different entry points with different directions and opposite road coordinates. Each road consisted of a single lane in a single direction. The RMSE measures the difference between a real position and an estimated position, as in Eq. (29).

The simulation scenarios are performed using network simulator OMNET++ [32], and fuzzy logic is implemented using MATLAB. The VEINS framework is a specialized framework used for interpreting XML files that represent road traffic networks [33]. These road traffic networks are generated using a SUMO framework [34], which is a bi-directional coupled program to a network simulator.

Road coordinates can be extracted from SUMO. The simulation parameters are summarized in Table 2. The IEEE 802.11p supports communication in vehicular networks. The lognormal shadowing model used is the propagation model. The path exponent equals 1.5. The noise follows a Gaussian distribution with zero mean and variance σ. A new vehicle is added every 1 s. The mobility model is the Krauss model, which is one of the car-following models. It describes the manner of the back vehicle based on the manner of the front vehicle. The simulation time equals 100 s in the proposed GPS-free localization, and in the fuzzy intersection method, the size of the drawn randomly dataset equals 900 points in two scenarios. A mean of the results was obtained at the end of each experiment. Each scenario ran five times, and the mean was again taken.



(e)

Figure. 3 Gaussian membership functions: (a) estimated x dimension, (b) estimated y dimension, (c) noise in x, (d) noise in y, and (e) root mean square errors

Table 2. Simulation parameters

Parameters	Values
IEEE standard	802.11p standard
Number of vehicles	1 vehicle/s
Propagation model	Lognormal shadowing
Mobility model	Krauss
Pause time	1 s
Acceleration	2.6 m/s^2

5.1 Scenario 1

In Enlargen city, the RSUs are located at each roadside. They were placed at (600, 10) and (400, 800) in [25,26], respectively. They were also placed at (0, 1500) and (12, 1500) in [9] and in the proposed cooperative localization, respectively. Fig 4 shows the RMSE values for distance errors ranging from 0.5 to 3 m. The RMSE of the proposed cooperative localization is lower than those of the localization algorithms in [9, 25, 26]. The reasons return to use fuzzy logic and adapting the noise covariance according to the mapping to road coordinates return. The RMSE in [9] is lower than that in research papers [25, 26]. This results to the method for choosing the initial state vector and the limitation of using GPS/INS in an urban environment.

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5.2 Scenario 2

In KarradaIn city, the RSUs are located at (700, 3000) and (1000, 3000) in the proposed localization as in our previous work [9]. Fig 5 shows the RMSE for several different distance errors ranging from 1.5 to 3 m in eight clusters. The proposed method achieves an RMSE value lower than those of other localization algorithms [9, 25, 26]. This reflects the power of the proposed method to adapt noise covariance based on on-road data. In [9, 25, 26], the accuracy results of the EKF algorithm explain the need to adapt noise covariance instead of using them constantly. Fig 6 shows the RMSE for the proposed algorithm with high distance errors from 10 to 30 m and the same number of clusters. The increased errors do not affect the RMSE more because the adaptive noise covariance is performed from numerical data and according to road coordinates.

Furthermore, the fuzzy intersection method's power handles uncertainty in distance measurements, and it gives a chance to increase road width. Therefore, well initial intersection points.

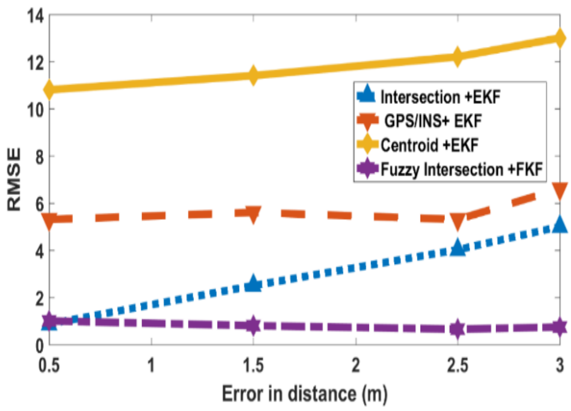

Figure. 4 Scenario 1: RMSE at different distance errors and 8 clusters

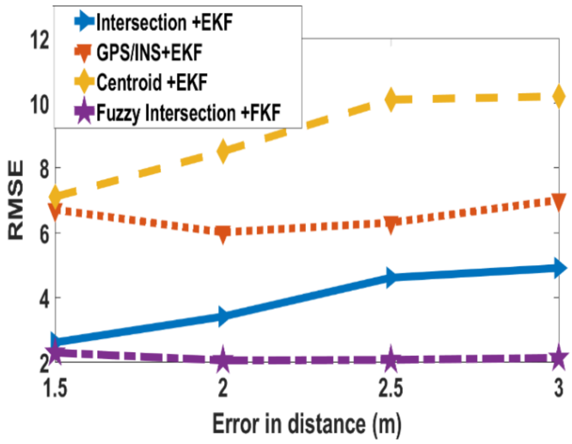

Figure. 5 Scenario 2: RMSE at different distance errors in eight clusters

Fig 7 shows the RMSE for the proposed algorithm at different distance errors and different cluster numbers. The nature of the drawn dataset needs more clusters to become more separated; hence, increasing the number of clusters positively impacts RMSE.

6. Discussion

The KF, with its variations, requires adapting its parameters online. It is challenging without a priori knowledge of statistical data analysis in advance. The nonlinear nature of localization algorithms and uncertainty in measurements add difficulty to the degree of obtaining a more precise vehicle position.

In the proposed localization method, map information is used to map the vehicle position to road coordinates and predict noise in the estimated position for each region. The nonlinear nature of localization and adapting noise covariance is handled by the fuzzy logic theory.

The fuzzy intersection method is proposed to deal with the uncertainty in distance measurements. The Gaussian membership function gives lower weight to noise data, and therefore, they become less affected by position accuracy. In [9], the noise in the

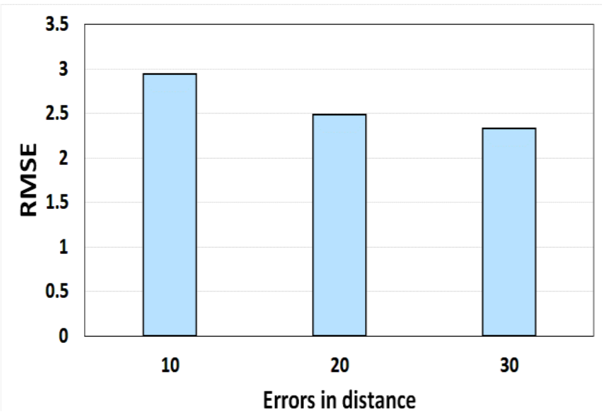

Figure. 6 Scenario 2: RMSE at different distance errors in eight clusters

Figure. 7 Scenario 2: RMSE at different distance errors in different clusters

process and measurement is made by finding the variance as constant values along the simulation running. The initial state vector is obtained from the standard formula of the intersection method. EKF handles the uncertainty in measurements. EKF is still sensitive to well initial state vector and adapts to noise covariance in measurement. In [25], the authors used data fusion from GPS and INS to get the initial position vector. It is further improved by V2V communication to reduce the uncertainty in measurement. EKF was used to linearize the distance function and kinematic model. The position accuracy is more related with the GPS/INS limitations. In [26], the weighted centroid method is not a good method to get the initial parameter of EKF. The use of similar triangle rules for finding neighbours with linear relation is not suitable for adapting noise in the process and measurement.

7. Conclusion and future works

An adaptive KF for free GPS localization and fuzzy intersection method were introduced in this paper. The proposed method consists of four steps: the first step maps vehicles to road coordinates and predicts noise. The second step uses the fuzzy intersection method to get a good position vector and overcomes the errors in distances. The third step used fuzzy clustering based on WFEVs and a modified merging algorithm to establish the parameters of the Mamdani model in the last step. The Mamdani model is used to find the scalar value of u. In Enlargen city, the performance is approached 1 m approximately and above little 2 m in KarradaIn city. This is the best accuracy due to the increased errors in the distance and among the related research papers.

Other parameters, such as angle, will be studied in the future to improve position accuracy. A hybrid commutation pattern is used to reduce the cost of deploying RSU on the road. Another variance statistical analysis method is used to adapt the noise covariance.

Conflicts of Interest

The authors declare no conflict of interest.

Author Contributions

Conceptualization, Walaa Afifi; methodology, Walaa Afifi and Hesham A. Hefny; software, Walaa Afifi; validation, Walaa Afifi, and Hesham A. Hefny; formal analysis, Walaa Afifi and Hesham A. Hefny; investigation, Walaa Afifi; resources, Walaa Afifi; data curation, Walaa Afifi; writing—original draft preparation, Walaa Afifi; writing—review and editing, Walaa Afifi; visualization, Nagy R. Darwish; supervision, Nagy R. Darwish; project administration, Nagy R. Darwish; funding acquisition, Walaa Afifi.

References

[1] S. Brown and S. Rutan, "Adaptive Kalman Filtering", *International Journal of Research of The National Bureau of Standards*, Vol. 90, No. 6, pp. 403-408, 1985.

[2] C. Hu, W. Chen, Y. Chen, and D. Liu, "Adaptive Kalman Filtering for Vehicle Navigation", *International Journal of Global Positioning Systems*, Vol. 2, No. 1, pp. 42-47, 2003.

[3] B. Zheng, P. Fu, B. Li, and X. Yuan, "A Robust Adaptive Unscented Kalman Filter for Nonlinear Estimation with Uncertain Noise Covariance", *International Journal of Sensors*, Vol. 13, No 3, pp. 1-15,2018.

[4] M. Osman, A. Hussein, A. A. Kaff, F. Garc´ıa and J. Armingol, "Online Adaptive Covariance Estimation Approach for Multiple Odometry Sensors Fusion", In: *Proc. of Intelligent Vehicles Symposium*, pp. 355-360, 2018.

[5] A. Werries and J. Dolan, "Adaptive Kalman Filtering Methods for Low-Cost GPS/INS Localization for Autonomous Vehicles", *Carnegie Mellon University. Journal Contribution*, pp. 1-6, 2018.

[6] U. Iqbal, A. Okou, and A. Noureldin, "An Integrated Reduced Inertial Sensor System-RISS/GPS for Land Vehicles", In: *Proc. of Position, Location and Navigation Symposium, Monterey*, pp. 1014-1021, 2008.

[7] M. Cossaboom, J. Georgy, T. Karamat, and A. Noureldin, "Augmented Kalman Filter and Map Matching for 3D RISS/GPS Integration for Land Vehicles", *International Journal of Navigation and Observation*, Vol. 2012, pp. 1-16, 2011.

[8] A. M. Aravindan, J. Antonio, and J. Vila, "GNSS/INS Integration using Software Defined Radio Receiver", *University Poliecnica de Catalunya*, 2012.

[9] W. Afifi, H. Hefny, and N. Darwish, "A cooperative localization method based on V2I communication and distance information in vehicular networks", *International Journal of Computer Networks and Communications*, Vol. 13, No. 6, pp. 53-71,2021.

[10] E. Cox, "Fuzzy fundamentals", *International Journal of IEEE Spectrum*, Vol. 29, No. 10, pp. 58-61 ,1992.

[11] L. Zadeh, "Fuzzy Logic, Neural Networks, and Soft Computing", *International Journal of Fuzzy Systems*, Vol. 37, No. 3, pp. 77-84, 1994.

[12] W. Afifi, H. Hefny, N. Darwish, and I. Fahmy, "Relative Position Estimation in Vehicle Ad-Hoc Network", *IoT and Cloud Computing Advancements in Vehicular Ad-Hoc Networks*, pp. 48-83, 2020.

[13] G. Welch and G. Bishop, "An introduction to the Kalman Filter", *UNC Chapel Hill*, TR 95-041, Vol. 8, pp. 1-16,2006.

[14] S. Akhlaghi, N. Zhou, and Z. Huang, "Adaptive adjustment of noise covariance in Kalman filter for dynamic state estimation", In: *Proc. of Power & Energy Society General Meeting*, pp. 1-5, 2017.

[15] J. Sasiadek and Q.Wang, "Fuzzy Adaptive Kalman Filtering for INS/GPS Data Fusion and Accurate Positioning", In: *Proc. of Automatic Control in Aerospace*, Vol. 34, No. 15, pp. 410-415, 2001.

[16] P. Ambrosio and N. Mort, "Adaptive Kalman Filtering through fuzzy logic", In: *Proc. of Fuzzy Systems, Recent Advances and Practical Applications of Fuzzy, Neuro-Fuzzy, and Genetic Algorithm-Based Fuzzy Systems*, pp. 67-73, 2000.

[17] R. Havangi, M. Nekoui, and M. Teshnehlab, "Adaptive Neuro-Fuzzy Extended Kalman Filtering for Robot Localization", *International Journal of Computer Science Issues*, Vol. 7, No. 2, pp. 15-23, 2010.

[18] S. Zhou, C. Shen, L. Zhang, N. Liu, T. He, B. L. Yu, and J. Song, "Dual-optimized adaptive Kalman filtering algorithm based on BP neural network and variance compensation for laser absorption", *International Journal of Optics Express*, Vol. 27, No. 22, pp. 31875-31888, 2019.

[19] M. Aslinezhad, A. Malekijavan, and P. Abbasi, "ANN-assisted robust GPS/INS information fusion to bridge GPS outage", *International Journal of Wireless Communications and Networking*, Vol. 129, No. 2020, pp. 1-28, 2020.

[20] R. Woo, E. Yang, and D. Seo, "A Fuzzy-Innovation-Based Adaptive Kalman Filter for Enhanced Vehicle Positioning in Dense Urban Environments", *International Journal of Sensors*, Vol. 19, No. 8, pp. 1-24, 2019.

[21] Z. Li, J. Wang, B. Li, J. Gao, and X. Tan, "GPS/INS/Odometer Integrated System Using Fuzzy Neural Network for Land Vehicle Navigation Applications", *International Journal of Navigation*, Vol. 67, No. 6, pp. 967-983, 2014

[22] D. Zhao, H. Qian, and D. Xu, "Research on a mixed prediction method to vehicle integrated navigation systems", *International Journal of Advanced Robotic Systems*, Vol. 16, No. 6, pp. 1-11, 2019.

[23] K. Badshah and Q. Yongyuan, "Tightly Coupled Integration of a Low-Cost MEMS-INS/GPS System using Adaptive Kalman Filtering", *International Journal of Control and Automation*, Vol. 9, No. 2, pp. 179-190, 2016.

[24] Parveen, S. Kumar, and R. Singh, "GPS-Free Localization in Vehicular Networks Using Directional Antennas", In: *Proc. of Data Engineering for Smart Systems. Lecture Notes in Networks and Systems*, Vol. 238, pp. 173-182, 2022.

[25] L. Du, L. Chen, X. Hou, and Y. Chen, "Cooperative Vehicle Localization Base on Extended Kalman Filter in Intelligent Transportation System", In: *Proc. of Wireless and Optical Communication*, pp. 1-5, 2019.

[26] F. Lobo, D. Grael, H. Oliveira, L. Villas, A. Almehmadi, and K. E. Khatib, "Cooperative Localization Improvement Using Distance Information in Vehicular Ad Hoc Networks", *International Journal of Sensors*, Vol. 19, No. 23, pp. 1-27, 2019.

[27] A. Khitwongwattana and T. Maneewarn, "Extended Kalman Filter with Adaptive Measurement Noise Characteristics for Position Estimation of an Autonomous Vehicle", In: *Proc. of Mechtronic and Embedded Systems and Applications*, pp. 505-509, 2008.

[28] J. Vine, "Geometry for Computer Graphics", *Springer-Verlag*, 2005.

[29] W. Afifi and H. Hefny, "Modified Merging Algorithm of Fuzzy Clustering with Expected Value", In: *Proc. of Computer Engineering and Systems*, pp. 161- 166, 2015.

[30] W. Afifi and H. Hefny, "Adaptive Takagi–Sugeno Fuzzy Model using Weighted Fuzzy Expected Value in Wireless Sensor Network", In: *Proc. of Hybrid Intelligent Systems*, pp. 225-231, 2014.

[31] H. Hefny, "Comments on Distinguishability quantification of fuzzy sets", *International Journal of Information Sciences*, Vol. 177, No. 21, pp. 4832-4839, 2007.

[32] OMNET ++ web site: https://omnetpp.org/

[33] Vehicular network simulation framework (VEINS): veins.car2x.org/documentation/.

[34] Simulation of urban mobility (SUMO): https://sumo.dlr.de/docs/.