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Computational assessment of external force acting on beam elastic foundation

Elastik kiriş temeline etki eden dış kuvvetin hesaplamalı değerlendirmesi

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Abstract

In this paper, we present and employ some numerical techniques for the computational solution of fourth-order differential beam equation. The equation describes a beam system on elastic foundation with an ill-posed situation. We formulate suitable algorithms to aid the computation of the Exponentially Fitted Collocation Method (EFCM), Hybrid Block Method (HBM), Homotopy Perturbation Method (HPM), and Differential Transformation Method (DTM) to reduce and overcome stress involves during evaluation. The formulated algorithms are further used for numerical comparison of the results. The results show that the algorithms are efficient and numerical methods prove to be highly effective for solving beam problems.

Keywords: Ill-Posed fourth-order beam problems, Exponentially fitted collocation method (EFCM), Hybrid block method (HBM), Homotopy perturbation method (HPM), Differential transformation method (DTM).

Öz

Bu makalede, dördüncü mertebeden diferansiyel kiriş denkleminin hesaplamalı çözümü için bazı sayısal teknikler sunuyor ve kullanıyoruz. Denklem, kötü konumlu bir duruma sahip elastik bir temel üzerindeki bir kiriş sistemini tanımlar. Değerlendirme sırasında ortaya çıkan stresi azaltmak ve üstesinden gelmek için Üstel Olarak Yerleştirilmiş Sıralama Yöntemi (EFCM), Hibrit Blok Yöntemi (HBM), Homotopi Pertürbasyon Yöntemi (HPM) ve Diferansiyel Dönüşüm Yöntemi'nin (DTM) hesaplanmasına yardımcı olacak uygun algoritmalar formüle ediyoruz. Formüle edilmiş algoritmalar ayrıca sonuçların sayısal karşılaştırması için kullanılır. Sonuçlar, algoritmaların verimli olduğunu ve sayısal yöntemlerin kiriş problemlerini çözmede oldukça etkili olduğunu kanıtlamıştır.

Anahtar kelimeler: Kötü konumlanmış dördüncü mertebeden kiriş problemleri, Üstel olarak yerleştirilmiş kollokasyon yöntemi (EFCM), Hibrit blok yöntemi (HBM), Homotopi pertürbasyon yöntemi (HPM), Diferansiyel dönüşüm yöntemi.

1 Introduction

Mathematical modelling of physical phenomena usually leads to boundary value problem (BVP) and initial value problems (IVP) which are evolutionary equations from many natural phenomena. Many of these physical events are modelled using linear and non-linear differential equations. The solution to such problems is essential for deep meaning and understanding of the natural phenomenon involved. Therefore, the modelling and study of efficient solution to the problems arising are highly imperative.

Ordinary differential equations continue to have relevance in structural and civil engineering, particularly in beam system. A beam system is defined as structurally bar members whose purposely function is to support transverse loading and carry it to the supports. The external force acting of the beam will continuously deflect, which would distribute reaction forces in the supporting medium. The level of these reaction forces at a given point is proportional to the deflection of the beam $y(t)$.

Several authors including [1]-[12] have worked on various numerical and computational techniques to solve ordinary differential equations such as adomian decomposition method (ADM), differential transformation method (DTM), homotopy perturbation method (HPM), variational iteration method (VIM), Runge-Kutta method (RKM) and homotopy analysis method (HAM) have been developed to numerically solve various type of problems since analytical approach are in most cases not available or inefficient in handling complex problems in applied sciences and computational engineering. In this paper, we investigate the nature of the acting force $E(t)$ acting on elastic foundation beam equation with free ends of the form:

$$\frac{d^4y(t)}{dt^4}+y(t)=E(t) \tag{1}$$

with initial conditions:

$$\begin{cases} y(t_0)=\mu \\ y(t_0)^{/}=\varphi \\ y(t_0)^{//}=\gamma \\ y(t_0)^{///}=\psi \end{cases} \tag{2}$$

Where $E(t)$ is external force acting on beam elastics foundation and μ,φ,γ,ψ are constants

2 Description of numerical techniques

2.1 Exponentially fitted collocation approximation method (EFCAM)

Authors [13] proposed exponentially fitted collocation approximation method (EFCAM) to solve Bessel equation of

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order zero. The use of power series as a basis function was employed and obtain $4th$ derivative of $y(t)$ was carried out which eventually substitute into a slightly perturbed equation (4) added with perturbation term the right-hand side of the equation which is to minimize the error obtain in the problem under consideration [14].

To employ exponentially collocation approximation method for the numerical solution of equation (1), we consider a power series of the form:

$$y(t)=\sum_{N=0}^{m} y_N\, t^N \tag{3}$$

and the Exponentially fitted approximate solution of the form:

$$y(t)=\sum_{N=0}^{m} y_N\, t^N+\tau_4 e^t \tag{4}$$

where t is dependent variables, τ_4 is tau-parameter with the highest derivative of equation (1) and y_N ,$(N\geq 0)$ are the unknown constants to be determined, m is the length of computation and degree of Chebyshev polynomials.

Obtaining the fourth derivative of equation (3), lead to

$$y^{(iv)}(t)=N(N-1)(N-2)(N-3)y_N\, t^{N-4} \tag{5}$$

Substitute equations (5) and (3) into equation (1), we obtain

$$N(N-1)(N-2)(N-3)y_N t^{N-4}+y_N t^N = E(t) \tag{6}$$

Expansion of equation (6) and collect the like terms, we have

$$\begin{cases} y_0+y_1t+y_2t^2+y_3t^3+(24t+t^4)y_4+\cdots+ \\ (N(N-1)(N-2)(N-3)t^{N-4}+t^N)y_N = E(t) \end{cases} \tag{7}$$

Slightly perturbed and collocate (7), we have

$$\begin{cases} y_0+y_1t_i+y_2{t_i}^2+y_3{t_i}^3+(24t_i+{t_i}^4)y_4+\cdots+ \\ (N(N-1)(N-2)(N-3){t_i}^{N-4}+{t_i}^N)y_N \\ = E(t_i)+H(t_i) \end{cases} \tag{8}$$

$$\begin{cases} y_0+y_1t_i+y_2{t_i}^2+y_3{t_i}^3+(24t_i+{t_i}^4)y_4 \\ +\cdots+(N(N-1)(N-2)(N-3){t_i}^{N-4}+{t_i}^N)y_N \\ -\tau_1T_m(t_i)-\tau_2T_{m-1}(t_i)-\tau_3T_{m-2}(t_i) \\ -\tau_4T_{m-3}(t_i)=E(t_i) \end{cases} \tag{9}$$

Where $t_i=a+\frac{(b-a)i}{m+2}$; $i=1,2,3\ldots\ldots..m+1$

Thus, equation (9) leads to (m+1) unknown constants linear algebraic system of equations. In order to make the system of equations a square matrix, four extra equations are added by fitting one (τ_4) tau-parameter to given initial conditions (2).

$$\begin{cases} y(t_0)=\sum_{N=0}^{m} y_N t^N+\tau_4e^{t_0}=\mu \\ y'(t_0)=\sum_{N=0}^{m} Ny_N t^{N-1}+\tau_4e^{t_0}=\varphi \\ y''(t_0)=\sum_{N=0}^{m} N(N-1)y_N t^{N-2}+\tau_4e^{t_0}=\gamma \\ y'''(t_0)=\sum_{N=0}^{m} N(N-1)(N-2)y_N t^{N-3}+\tau_4e^{t_0}=\psi \end{cases} \tag{10}$$

Altogether, we obtained $(m+5)$ algebraic linear equations in $(m+5)$ unknown constants. Thus, we put the $(m+5)$ algebraic equations in a square matrix form:

$$VK=G \tag{11}$$

Where

$$V=\begin{pmatrix} v_{11}\ v_{12}\ v_{13}\ \ldots\ldots.v_{1m} & -T_m(t_1) & -T_{m-1}(t_1) & -T_{m-2}(t_1) & -T_{m-3}(t_1) \\ v_{21}\ v_{22}\ v_{23}\ \ldots\ldots.v_{2m} & -T_m(t_2) & -T_{m-1}(t_2) & -T_{m-2}(t_2) & -T_{m-3}(t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ v_{m1}\ v_{m2}\ v_{m3}\ \ldots\ldots.v_{mm} & -T_m(t_{m+1}) & -T_{m-1}(t_{m+1}) & -T_{m-2}(t_m+1) & -T_{m-3}(t_{m+1}) \\ 1\ \ 0\ \ \ 0\ldots\ldots.0 & 0\ \ 0 & 0\ \ 0 & 0\ \ 0 & e^{t_0} \\ 0\ \ 1\ \ \ 0\ldots\ldots.0 & 0\ \ 0 & 0\ \ 0 & 0\ \ 0 & e^{t_0} \\ 0\ \ 0\ \ \ 2\ldots\ldots.0 & 0\ \ 0 & 0\ \ 0 & 0\ \ 0 & e^{t_0} \\ 0\ \ 0\ \ \ 0.\ldots.6 & 0\ \ 0 & 0\ \ 0 & 0\ \ 0 & e^{t_0} \end{pmatrix}$$

$K=(y_0,y_1,y_2\ldots\ldots\ldots..y_m\ \tau_1\ \tau_2\ \tau_3\ \tau_4\ )^T$
$G=(E(t_1),E(t_2),E(t_3)\ldots\ldots\ldots..E(t_m),\alpha\ ,\tau,\gamma,\delta)^T$

2.1.1 Exponentially fitted collocation algorithm (EFCA)

In this section, the computational procedures using Maple 18 software are employed to formulate three steps algorithm using equations (3) to (11) to reduce the time involved in evaluating the EFCAM. At the same time, efficiency and accuracy of the method are guaranteed.

Restart:
Step 1:
$Y:=t^N$
$\boldsymbol{for\ i\ from\ 0\ to\ m}$
$P:=(diff(Y,t,t,t,t)+Y)*y(0);$
$P[0]:=eval(P,N=0);$
$P:=(diff(Y,t,t,t,t)+Y)*y(1);$
$P[1]:=eval(P,N=1);$
$P:=(diff(Y,t,t,t,t)+Y)*y(2);$
$P[2]:=eval(P,N=2);$
$P:=(diff(Y,t,t,t,t)+Y)*y(3);$
$P[3]:=eval(P,N=3);$
$\vdots \qquad \vdots \qquad \vdots$
$P:=(diff(Y,t,t,t,t)+Y)*y(m)$
$P[m]:=eval(P,N=m);$
$\boldsymbol{end\ do}$
Step 2:
$L:=P[0]+P[1]+P[2]+P[3]\ldots+P[m]-T(m)\tau_1$
$-T(m-1)\tau_2-T(m-2)\tau_3-T(m-3)\tau_4=E(t);$
$C[1]:=eval\left(L,t=\frac{1}{m+2}\right);$
$C[2]:=eval\left(L,t=\frac{2}{m+2}\right);$
$C[3]:=eval\left(L,t=\frac{3}{m+2}\right);$
$\vdots$
$C[m+1]:=eval\left(L,t=\frac{m+1}{m+2}\right);$
Step 3:
$eqn1:=evalf(C[1]);$
$eqn2:=evalf(C[2]);$
$eqn3:=evalf(C[3]);$
$\vdots$
$eqn\boldsymbol{m}+\boldsymbol{1}:=evalf(C[m+1]);$
$eqn\boldsymbol{m}+\boldsymbol{2}:=y(0)+e^{t_0}\tau_4=\mu;$
$eqn\boldsymbol{m}+\boldsymbol{3}:=y(1)+e^{t_0}\tau_4=\varphi;$
$eqn\boldsymbol{m}+\boldsymbol{4}:=2*y(2)+e^{t_0}\tau_4=\gamma\ ;$
$eqn\boldsymbol{m}+\boldsymbol{5}:=6*y(3)+e^{t_0}\tau_4=\psi;$

$solve(\{eqn1,eqn2,eqn3,\ldots\ldots,eqn[\boldsymbol{m}+\boldsymbol{5}]\},\{y(0),y(1),y(2),y(3),\ldots\ldots,y(m),\tau_1,\tau_2,\tau_3,\tau_4\});$

Output: See Table 2
Exponentially fitted collocation approximate solution
$y(t) \approx y(0) + y(1)t + y(2)t^2 + y(3)t^3 + \cdots + y(m)t^m + \tau_4 e^t$

Where m is computational length, $T(m)$ is equivalent shifted Chebyshev polynomial of degree m and $\mu, \varphi, \gamma, \psi$ are constants.

2.2 Hybrid block method (HBM)

Hybrid Block Method (HBM) was proposed by [15] to solve general fourth-order ordinary differential equations in a direct way approach using power series as the basis function. The technique involves the uses of collocation and interpolation techniques to derive a continuous implicit three steps linear multistep method. The HBM for direct solution of general fourth-order initial value problems was specifically with three steps approach with two off-step points at collocation and interpolation. Considering the general fourth-order differential equation of the form;

$$y^{iv}(x) = f(x, y, y', y'', y''') \tag{12}$$

where f is continuous$f: \mathbb{R}^{n+1} \to \mathbb{R}^n$. Likewise, we consider the power series of the form,

$$y(x) = \sum_{j=0}^{t+c-1} a_j x^j \tag{13}$$

where $a_j \in \mathbb{R}, j = 0(1)t + c - 1, y \in C^m$, c and t are collocation and interpolation points, respectively. Taking the fourth derivative of (13), then equate the result of the fourth derivatives to (12);

$$f(x, y, y', y'', y''') = \sum_{j=0}^{t+c-1} j(j-1)(j-2)(j-3) a_j x^{j-4} \tag{14}$$

$a_j \in \mathbb{R}$ are parameters to be determined. Collocating (12) at $= x_{n+i}$; $i = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, 3$, interpolating (14) as well at $x = x_{n+i}$; $i = 0, \frac{1}{2}, 1, \frac{3}{2}$ and evaluating at the endpoint $x = x_{n+i}$; $i =$ 2,3 gives a system of non-linear equation. The resulting system of equations is solved to obtain the desired Hybrid Block Method (HBM) as;

$$\begin{aligned} y_{n+\frac{1}{2}} = y_n + \frac{1}{2} h z_n + \frac{1}{8} h^2 w_n + \frac{1}{48} h^3 q_n + h^4 (&\frac{11849}{6967296} f_n \\ &- \frac{451}{2322432} f_{n+2} + \frac{1469}{174182400} f_{n+3} \\ &- \frac{139}{110592} f_{n+1} + \frac{11}{15552} f_{n+\frac{3}{2}} \\ &+ \frac{5947}{3628800} f_{n+\frac{1}{2}}) \end{aligned} \tag{15}$$

$$\begin{aligned} y_{n+1} = y_n + h z_n + \frac{1}{2} h^2 w_n + \frac{1}{6} h^3 q_n + h^4 (&\frac{23}{1215} f_n \\ &+ \frac{139}{4050} f_{n+\frac{1}{2}} - \frac{149}{7560} f_{n+1} \\ &+ \frac{187}{17010} f_{n+\frac{3}{2}} - \frac{17}{5670} f_{n+2} \\ &+ \frac{11}{85050} f_{n+3}) \end{aligned} \tag{16}$$

$$\begin{aligned} y_{n+\frac{3}{2}} = y_n + \frac{3}{2} h z_n + \frac{9}{8} h^2 w_n + \frac{9}{16} h^3 q_n + h^4 (&\frac{1467}{20480} f_n \\ &- \frac{9477}{143360} f_{n+1} + + \frac{387}{8960} f_{n+\frac{3}{2}} \\ &- \frac{243}{20480} f_{n+2} + \frac{243}{1400} f_{n+\frac{1}{2}} \\ &+ \frac{369}{716800} f_{n+3}) \end{aligned} \tag{17}$$

$$\begin{aligned} y_{n+2} = y_n + 2h z_n + 2h^2 w_n + \frac{4}{3} h^3 q_n + h^4 (&\frac{1528}{8505} f_n \\ &+ \frac{1024}{2025} f_{n+\frac{1}{2}} - \frac{104}{945} f_{n+1} + \frac{1024}{8505} f_{n+\frac{3}{2}} \\ &- \frac{86}{2835} f_{n+2} + \frac{8}{6075} f_{n+3}) \end{aligned} \tag{18}$$

$$\begin{aligned} y_{n+3} = y_n + 3h z_n + \frac{9}{2} h^2 w_n + \frac{9}{2} h^3 q_n + h^4 (&\frac{9}{14} f_n \\ &+ \frac{729}{350} f_{n+\frac{1}{2}} + \frac{9}{14} f_{n+\frac{3}{2}} + \frac{9}{1400} f_{n+3}) \end{aligned} \tag{19}$$

Where $z_{n+j} = y'_{n+j}, w_{n+j} = y''_{n+j}, q_{n+j} = y'''_{n+j}; j = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, 3$

Thus, equations (15) - (19) are the hybrid block method (HBM) for direct the solution of general fourth-order beam equation (1).

2.3 Homotopy perturbation method (HPM)

Author [16] proposed a Homotopy Perturbation Method to solve various problems in applied mathematics and engineering sciences. To illustrate the basic ideas of the HPM, he considers

$$A(y) - f(\tau) = 0 \tag{20}$$

With boundary condition

$$C\left(y, \frac{\partial y}{\partial \tau}\right) = 0 \qquad \tau \in \partial\Omega \tag{21}$$

$$L(y) + N(y) - f(\tau) = 0 \tag{22}$$

Here y and t are dependent and independent variables respectively, A and C is general differential and boundary operators respectively while $\partial\Omega$ is the boundary of the domain Ω and $f(\tau)$ is a known analytic function. Eventually, equation (20) can be rewritten as described in [17] follow.

$$H(\tau, p): \Omega \, \mathrm{X}[0,1] \longmapsto \mathbb{R} \tag{23}$$

Satisfies,

$$H(v, p) = L(v) - L(y_0) + pL(y_0 + p[N(v) - f(\tau)]) = 0 \tag{24}$$

where, $p \in [0,1], \tau \in \Omega$ and p is called homotopy parameter, H is homotopy function and y_0 is an initial approximate solution for equation (20) which satisfies the given boundary conditions. By equation (24), we have

$$H(v, 0) = L(v) - L(y_0) = 0 \tag{25}$$

and

$$H(v, 1) = L(v) + N(v) - f(\tau) = 0 \tag{26}$$

Thus, the solution of equation (24) can be written as a series in q as follows:

$$V = v_0 + qv_1 + q^2v_2 + q^3v_3 + \cdots = \sum_{k=0}^{\infty} q^k v_k \tag{27}$$

2.3.1 Homotopy perturbation algorithm (HPA)

The computational procedures using Maple 18 software are employed to formulate three steps algorithm using equations (20) to (27) in order to reduce the time involved in the evaluation of the HPM while efficiency and accuracy of are guaranteed.

Restart:

Step 1:

$f := E(t);$

$y[0] := \mu + \varphi t + \gamma\left(\frac{t^2}{2!}\right) + \psi\left(\frac{t^3}{3!}\right);$

$N = \mathbb{R};$

Step 2:

$y[1] := -value\left(\begin{matrix} Int((y[0], t, t, t, t) + y[0] - f), \\ [t = 0 \ldots t, t = 0 \ldots t, t = 0 \ldots t, t = 0 \ldots t.] \end{matrix}\right);$

for n from* 2 *to N do

$y[n] := -value\left(Int(y[n-1]), \begin{bmatrix} t = 0 \ldots t, t = 0 \ldots t, \\ t = 0 \ldots t, t = 0 \ldots t. \end{bmatrix}\right);$

end do

Step 3:

$V := sum([j], j = 0 \ldots N);$

for n from 0 *by* 0.1 *to* 1 *do*

$evalf(eval(V, t = n));$

end do

Output: See Table 2

$$y(t) \approx \alpha + \beta t + \gamma\left(\frac{t^2}{2!}\right) + \psi\left(\frac{t^3}{3!}\right) + \cdots + G\left(\frac{t^N}{N!}\right)$$

2.4 Differential transformation method (DTM)

Zhou [18] was the first author to proposed and applied differential transformation method to solve linear and non-linear initial value problems in electric circuit analysis. This basic idea of DTM is to reconstruct a semi-analytical numerical technique using traditional Taylor series for the solution of ordinary differential equations and partial differential equations in the form of a polynomial. The iterative procedures for obtaining analytic Taylor series solutions of differential equations [19],[20].

To solve equation (1) using DTM, we consider an arbitrary functions y(t) which differential transform of $Y(k)$ is defined as.

$$Y(k) = \frac{1}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=0} \tag{28}$$

Where y(t) the original function while Y(k) is the transformed function.

The Inverse transform of Y(k) is given as

$$y(t) = \sum_{k=0}^{\infty} t^k\, Y(k) \tag{29}$$

Substitute equation (28) into equation (29), we have

$$y(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k y(t)}{dt^k}\right]_{t=0} \tag{30}$$

The fundamental mathematical operations performed by the differential transform method are.

2.4.1 Differential transformation algorithm (DTA)

The computational procedures of Maple 18 software are employed to formulate four steps algorithm using Table 1 as follows:

Table 1. One dimensional differential transformation.

Functional Form $y(t)$	Transformed Form $Y(k)$
$\alpha g(t) \pm \beta h(t)$	$\alpha G(k) \pm \beta H(k)$
t^n	$\delta(k-n) = \begin{cases} 1 & if\ k = n \\ 0 & if\ k \neq n \end{cases}$
e^t	$\frac{1}{k!}$
$e^{\beta t}$	$\frac{\beta^k}{k!}$
$g(t)h(t)$	$\sum_{r=0}^{k} G(r)\, H(k-r)$
$\frac{d^n g(t)}{dt^n}$	$(k+1)(k+2)\ldots(k+n)G(k+n)$
$\sin(\alpha t + \beta)$	$\frac{\alpha^k}{k!}\sin\left(\frac{k\pi}{2} + \beta\right)$

Restart:

Step 1:

$Y[0] := \mu;\ Y[1] := \varphi;\ Y[2] := \gamma;\ Y[3] := \psi; N = \mathbb{R};$

$$E(t) := \begin{cases} \frac{(-2)^k}{k!} sin\left(\frac{k\pi}{2} + 1\right) \\ 2\delta[k-1] - \delta[k] \\ -e^1 + \frac{(-2)^k}{k!} \end{cases}$$

Step 2:

$\delta(0) := 1;$

$\delta(1) := 0;$

for n from* 2 *to N do

$\delta(n) := 0;$

end do

Step 3:

for k from* 0 *to N do

$Y[k+4] := -\frac{1}{(k+1)(k+2)(k+3)(k+4)}(Y[k] - E(t));$

end do

Step 4:

$y := sum\left(\frac{Y[j] * t^j}{j!}, j = 0 \ldots N + 4\right);$

for n from 0 *by* 0.1 *to* 1 *do*

$Y[n] := evalf(eval(y, t = n));$

end do

Output: See Table 2 and Table 3.

$y(t) \approx Y(0) + Y(1)t + Y(2)t^2 + Y(3)t^3 + \cdots + Y(N)t^N$

2.5 Numerical results

Table 2. Comparison of analytical solution and numerical solutions.

t	Solutions	$E(t)=\sin(-2t+1)$	$E(t)=-2t+1$
	Analytical	0.973417925	0.973418513
	EFCA	0.973417925	0.973418513
0.1	HBM	0.973417925	0.973418513
	HBA	0.973417925	0.973418513
	DTA	0.973417983	0.973418518
	Analytical	0.977024404	0.977032808
	EFCA	0.977024404	0.977032808
0.2	HBM	0.977024404	0.977032808
	HBA	0.977024405	0.977032808
	DTA	0.977035099	0.977025706
	Analytical	1.010982745	1.011020830
	EFCA	1.010982743	1.011020831
0.3	HBM	1.010982745	1.011020830
	HBA	1.010982744	1.011020830
	DTA	1.011038898	1.010994144
	Analytical	1.075414134	1.075522312
	EFCA	1.075414133	1.075522312
0.4	HBM	1.075414134	1.075522313
	HBA	1.075414133	1.075522313
	DTA	1.075601316	1.075468316
	Analytical	1.170376836	1.170615381
	EFCA	1.170376836	1.170615381
0.5	HBM	1.170376837	1.170615382
	HBA	1.170376837	1.170615382
	DTA	1.170865239	1.170560235

Table 3. Comparison of analytical solution and numerical solutions.

t	Solutions	$E(t)=e^{-2t+1}$	$E(t)=-\frac{1}{2t+1}$
	Analytical	0.973425402	0.973410504
	EFCA	0.973425401	0.973410504
0.1	HBM	0.973425401	0.973410504
	HBA	0.973425400	0.973410504
	DTA	0.973425600	0.973410504
	Analytical	0.977139110	0.976909533
	EFCA	0.977139110	0.976909533
0.2	HBM	0.977139110	0.976909533
	HBA	0.977139110	0.976909533
	DTA	0.977145228	0.976909463
	Analytical	1.011541289	1.010420346
	EFCA	1.011541287	1.010420346
0.3	HBM	1.011541289	1.010420346
	HBA	1.011541289	1.010420346
	DTA	1.011585818	1.010420247
	Analytical	1.077117303	1.073696084
	EFCA	1.077117303	1.073696084
0.4	HBM	1.077117303	1.073696084
	HBA	1.077117303	1.073696084
	DTA	1.077297539	1.073696174
	Analytical	1.174401026	1.166325524
	EFCA	1.174401026	1.166325524
0.5	HBM	1.174401026	1.166325524
	HBA	1.174401026	1.166325524
	DTA	1.174930388	1.166323461

3 Computational experiments

In this section, we perform some numerical experiment on equation (1) coupled with the initial conditions in (2) with $\mu=1, \varphi=\cos(2), \gamma=3, \psi=\frac{1}{5}$. To aid the numerical computations, Maple 18 software package is used. We shall consider trigonometric, algebraic, exponential and inverse functions cases for four cases of $E(t)$ as follows

$$E(t)=\begin{cases}\sin(-2t+1)\\ -2t+1\\ e^{-2t+1}\\ -\dfrac{1}{2t+1}\end{cases} \tag{31}$$

Equation (1) together with the initial conditions in (2) and the defined $E(t)$, we apply the numerical methods discussed in section (2) to solve the initial value problems for the cases. The exact solution and numerical results for each of the methods were tabulated in section 2.5. The numerical results obtained for case 1, 2, 3 and 4 when the external force acting on a beam system was presented in Table 2 and Table 3.

4 Discussion and conclusion

4.1 Discussion

Figures (1, 2, 3, 4, 5) depict the plot of the deflection occur at beam elastic foundation and examination of external force $E(t)$ (trigonometric function, algebraic function, exponential function and inverse function) are carried out to obtain corresponding y(t) (see Table 2 and Table 3) for each case which play a significant role in structural engineering.

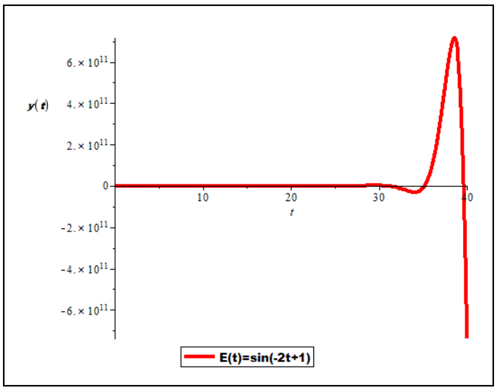

Figure 1. Beam at trigonometrical external force function.

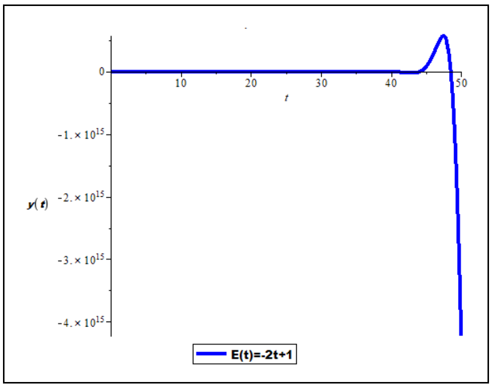

Figure 2. Beam at algebraic external force function.

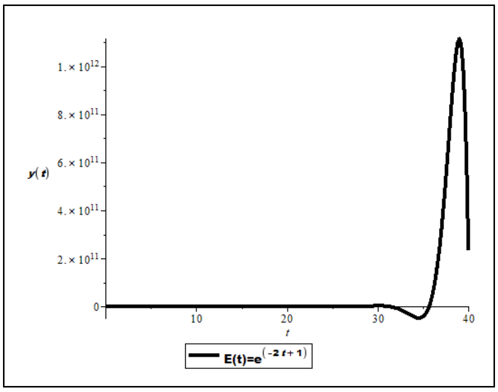

Figure 3. Beam at exponential external force function.

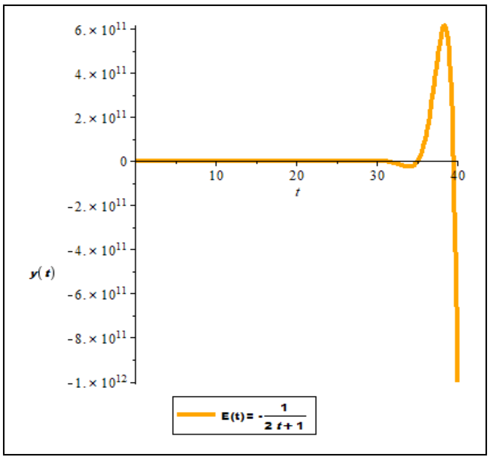

Figure 4. Beam at inverse external force function.

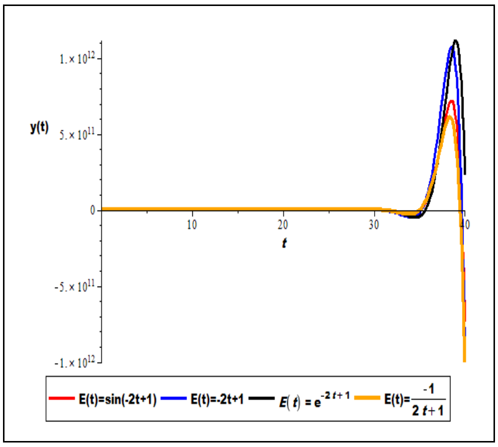

Figure 5. Beam at all Cases of external force considered.

Moreover, the displace cover during beam vibration. To afford collapses of the building, the structural engineers need to consider the effect and amount of external force load and maximum load analysis test to afford structure failure.

4.2 Conclusion

In this paper, the computational study of the External forces that occur during beam system has been carried out. The fourth-order boundary values of governing beam model was numerically solved by employing Exponentially Fitted Collocation Algorithm (EFCA), Hybrid Block Method (HBM), Homotopy Perturbation Algorithm (HPA) and Differential Transformation Algorithm (DTA). Finally, the results are more realistic compare with analytical solutions, and from the computational viewpoint, Exponentially Fitted Collocation Algorithm (EFCA), Hybrid Block Method (HBM) and Homotopy Perturbation Algorithm (HPA) are closer to analytical solutions compare to that Differential Transformation Algorithm (DTA).

5 Author contribution statements

In this study, Abd'gafar Tunde TIAMIYU involves in conceptualization, literature review, software, writing and visualization. Falade Iyanda KAZEEM supervise, validate the results and algorithms, and methodology. Abdullahi Shuaibu ABUBAKAR reviews the writing, spell-check and editing.

6 Ethics committee approval and conflict of interest statement

There is no need to obtain permission from the ethics committee for the article prepared.

There is no conflict of interest with any person or institution in the article prepared.

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