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Issue Article

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SOLUTION OF THE DIRICHLET PROBLEM FOR THE POISSON EQUATION IN A COMPLEX DOMAIN IN THE MAPLE SYSTEM ENVIRONMENT

Abstract: *This work is devoted to modeling the solution of problems of bending and vibration of elastic and viscoelastic plates of arbitrary configuration for various friction models. The article discusses the Dirichlet boundary value problem for the Poisson equation in a complex domain. The proposed solution algorithm is shown in the Maple environment. As a result of the work, the corresponding solutions were obtained using the package of applied programs (procedures-libraries) developed to solve these issues. On the basis of the proposed algorithm, the problems of mechanics of a deformable solid body of arbitrary and complex configuration are solved.*

Key words: *Dirichlet problem, stiffness of viscoelastic plates, bending, Poisson's equations, Poisson's coefficient, R – function.*

Language: *English*

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Introduction

The construction and study of mathematical models of physical phenomena is associated with the study of the solution of the problem of mathematical physics. It is appropriate to trace the main stages of the origin and development of mathematical physics. Mathematical physics has developed since the time of Newton in parallel with the development of physics and mathematics. First, differential and integral calculus was discovered, the methods of mathematical physics began to form in the study of vibrations of strings and rods, as well as in solving problems related to acoustics and hydrodynamics. At the same time, the foundations of analytical mechanics were laid. Then the ideas of mathematical physics received a new development in connection with the problems of heat conduction, diffusion, elasticity and viscoelasticity, optics, and electrodynamics. During this period, the theory of potential and the theory of stability of motion are created, the problems of quantum physics and the theory of relativity are included, as well as new problems of gas dynamics, particle transport and plasma physics. Many problems of classical mathematical physics are reduced to boundary value problems for differential (integral-differential) equations—the equations of mathematical physics.

The main mathematical tools for studying these problems are the theory of differential equations (including related areas - integral equations and the calculus of variations), the theory of functions, functional analysis, probability theory, approximate methods, and computational mathematics.

In papers [1,2], the problems of studying dynamic stability and numerical solution of nonlinear problems of the dynamics of viscoelastic systems are considered. Mathematical modeling of dynamic problems of vibrations and stability of viscoelastic systems is also very relevant due to the fact that, on the one hand, the possibilities of using materials with pronounced viscoelastic properties in the aviation industry and other branches of mechanical engineering are expanding, and on the other hand, when using hereditary models [3,4,5] to describe the internal damping of a material, the equations of oscillations of elastic systems are written in the same form as for viscoelastic systems, however, in a viscoelastic formulation, the problem has not been studied in a complex form.

When studying complex technological processes, apparatuses and physical phenomena, a researcher cannot take into account all the factors: some are the most important, and some can be neglected. To take into account these factors, an object model is created, which is then transformed into the “model-algorithm-program” triad, which is supplemented when solving many problems with the stages of primary processing of the experimental results, checking the adequacy and correction of the model, testing the program and analyzing the results of a computational experiment. The value of a computational experiment can hardly be overestimated, especially if a full-scale experiment is dangerous, expensive or simply impossible. Only a reasonable combination of analytical and numerical methods is a necessary condition for success in solving practical problems.

Mathematical formulation of the problem and methods of solution.

The mathematical model of the problems of bending viscoelastic plates is described by the equation

$$\frac{\partial^2 M_x^*}{\partial x^2} + 2\frac{\partial^2 M_{xy}^*}{\partial x \partial y} + \frac{\partial^2 M_y^*}{\partial y^2} + q(x,y,t) = 0 \quad (1)$$

If the hypothesis of the constancy of the Poisson's ratio is used when formulating the basic physical relations, the bending and torque moments are determined by the following dependencies:

$$M_x^* = -D(1-R^*)\left\{\frac{\partial^2 W}{\partial x^2} + \mu\frac{\partial^2 W}{\partial y^2}\right\};$$

$$M_y^* = -D(1-R^*)\left\{\frac{\partial^2 W}{\partial y^2} + \mu\frac{\partial^2 W}{\partial x^2}\right\};$$

$$M_{xy}^* = -D(1-\mu)(1-R^*)\frac{\partial^2 W}{\partial x \partial y} \quad (2)$$

where,

D - the stiffness of the viscoelastic plates; R^* – an integral operator with relaxation kernels $R(t)$, i.e.

$$R^*W = \int_0^t R(t-\tau)W(x,y,\tau)\,d\tau;$$

$W(x,y,t)$ - plate deflection;

μ - Poisson's coefficient;

$q(x,y,t)$ - the intensity of the external load.

If the hypothesis of the elasticity of volumetric deformations is used, then the following dependences are valid for the bending and torque moments:

$$M_x^* = -\left[2G(1-R_c^*)\frac{\partial^2 W}{\partial x^2} + L^*\left\{\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right\}\right]\cdot\frac{h^3}{12};$$

$$M_y^* = -\left[2G(1-R_c^*)\frac{\partial^2 W}{\partial y^2} + L^*\left\{\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial y^2}\right\}\right]\cdot\frac{h^3}{12}; \quad (3)$$

$$M_{xy}^* = -D(1-\mu)(1-R_c^*)\frac{\partial^2 W}{\partial x \partial y}\cdot\frac{h^3}{12}$$

where,

$G = E/2(1+\mu)$ - shear modulus; E – the modulus of elasticity; R_c^* - integral operator with shear relaxation kernels $R_c(t)$; L^* - an integral operator, i.e.

$$L^* = \left\{\frac{2}{3} + K[2G(1-R_c^*)]^{-1}\right\}^{-1}\left[K - \frac{2}{3}G(1-R_c^*)\right];$$

$K = E/3(1-2\mu)$ - bulk modulus of elasticity; h is the plate thickness.

To obtain the equation of motion, it is sufficient to substitute the expression $q(x,y,t) - \rho h\frac{\partial^2 W}{\partial t^2}$ instead of $q(x,y,t)$ in equations (1) and obtain the following equation for an oscillating fine viscoelastic plate

$$\frac{\partial^2 M_x^*}{\partial x^2} + 2\frac{\partial^2 M_{xy}^*}{\partial x \partial y} + \frac{\partial^2 M_y^*}{\partial y^2} + q(x,y,t) = \rho h\frac{\partial^2 W}{\partial t^2} \quad (4)$$

where ρh is the mass of the slab per unit surface.

Equations (1) and (4) are solved under the appropriate boundary and initial conditions

$$L_i W\,\big|_{\Gamma_i} = \varphi_i(x,y); \quad \Gamma = \cup_{i=1}^{n}\Gamma_i;$$

$$W_{t=0} = W_0; \quad \frac{\partial W}{\partial t}\Big|_{t=0} = W_0^t \quad (5)$$

where L_i are differential operators depending on the boundary conditions; Γ is the boundary of the region; W_0 0 and W_0^t are initial values.

Systems of coordinate functions (SCF) W and the boundary of the studied area Γ are constructed using the R-function method of V.L. Rvachev [6].

Note that when solving the problems of bending and vibrations of elastic and viscoelastic plates of complex shape, orthonormal SCFs are used with respect to the biharmonic and unit operators, respectively. Here, the use of orthonormal SCF greatly facilitates the solution of systems of integral and integral-differential equations (IDEs). To solve autonomous systems of integral and IDE, a numerical method based on the use of quadrature formulas is used.

Solving the Dirichlet problem for the Poisson equation in a complex domain in the Maple environment.

In many cases, the solution of the problem of modeling problems of the theory of viscoelasticity of a complex shape is associated with the study of the solution of the Dirichlet problem for the Poisson equation in a complex domain. The work is devoted to modeling the solution of problems of bending and oscillations of elastic and viscoelastic plates of arbitrary configuration for various viscosity models in the Maple system medium [7,8,9].

The Dirichlet problem for the Poisson equation is one of the classical problems of mathematical physics. To solve partial differential equations, as a rule, grid methods are used, often with the help of a computer, a grid is built in the domain of definition, and a difference equation is compiled in which the desired unknowns are the values of the function at the grid nodes. The solution of the difference equation can also be sought in different ways. In practice, iterative methods are widely used. The computational scheme in this case describes how the next state of the grid depends on the previous one. As a result of computing on a computer, an approximate solution of partial differential equations is obtained [10,11,12,13,14].

In this paper, an application package (procedures-libraries) is developed for solving the Dirichlet problem for the Poisson equation in a complex domain in the Maple system.

To solve the problem of modeling these tasks, we connect the developed RFM package:

```
> restart:
>
libname:="F:/Maple/MathPhysics/RFM/lib",libname;
```

libname := "F:/Maple/MathPhysics/RFM/lib", "C:\Progra

```
> with(RFM);
```

Choosing a system of R-operations:

```
> SetRSystem(0);
```

Set up base areas:

```
> F1:= nStripX(0,1);
```

$$F1 := (x, y) \to \frac{1^2 - (y - 0)^2}{2\ 1}$$

```
> F2:= nStripY(0,1);
```

$$F2 := (x, y) \to \frac{1^2 - (x - 0)^2}{2\ 1}$$

```
> F3:= nStripX(1,0.5);
```

$$F3 := (x, y) \to \frac{0.5^2 - (y - 1)^2}{2\ 0.5}$$

```
> F4:= nStripY(1,0.5);
```

$$F4 := (x, y) \to \frac{0.5^2 - (x - 1)^2}{2\ 0.5}$$

```
> F5:= nCircle(-0.5,-0.5,0.2);
```

$$F5 := (x, y) \to \frac{0.2^2 - (x - (-0.5))^2 - (y - (-0.5))^2}{2\ 0.2}$$

```
> F6:= nHalfPlane(0,-0.5,-0.5,-0.5):
> F7:= nHalfPlane(-0.5,-0.5,-1,-1):
```

We build a function that describes the boundary of the area under study:

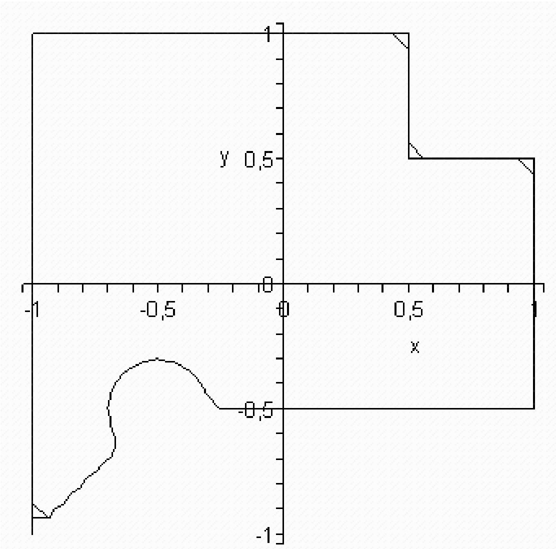
```
> Omega:= (((F1 &In F2) &In &Not(F3 &In F4)) &In &Not(F5)) &In &Not(F6 &In F7):
```

We set a rectangle bounding the study area:

```
> BoundRect:= [-1,-1,1,1];
```

We graphically depict the constructed function:

```
> PlotDomain(Omega,BoundRect);
```



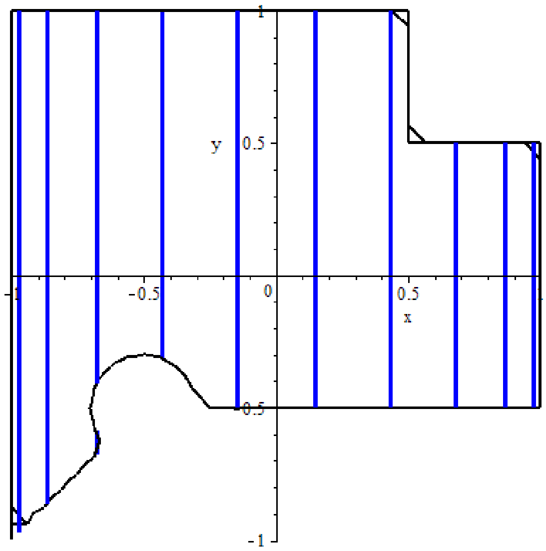
We start the procedure for preparing for integration:

> PrepareForDbInt10(Omega, BoundRect, Mx1, My1, 20);

$$[-0.9739065285, 0.9999999980]$$
$$[-0.8650633669, 1.]$$
$$[-0.6794095684, -0.5883866890, -0.4116133110, 1.]$$
$$[-0.3114162616, 0.9999999986]$$
$$[-0.4999992370, 1.]$$
$$[-0.5000000001, 0.9999999987]$$
$$[-0.4999992370, 1.]$$
$$[-0.4999992370, 0.5000007630]$$
$$[-0.4999992370, 0.5000000005]$$
$$[-0.5000000000, 0.5000000002]$$

We graphically represent the segments of integration:

> PlotIntPaths(Omega, BoundRect, Mx1, My1);



Create an array of power polynomials:

> Basis:= GenPowerPolynoms(3);

$$Basis := [1, x, y, x\,y, y^2, x^2, x^2\,y, x\,y^2, y^3, x^3]$$

We generate coordinate functions:

> w:= GenCoordFunctions(Omega(x,y), Basis):

We set the common term of the left side of the Ritz system:

> GenTerm:= (i,j) -> (Grad(w(x,y)[i]) &. Grad(w(x,y)[j])):

We carry out integrations and fill in the matrix of the Ritz system:

> A:= CreateLeftMatrixHF(GenTerm, Mx1, My1, BoundRect, true);

$$A := \begin{bmatrix}
0.6942063854 & -0.04513281540 & 0.1385147851 & -0.0000982744665 & 0.1715200111 & 0.2301433417 & 0.01983115336 & -0.02807837605 & 0.08334872851 & -0.04517829880 \\
-0.04513281540 & 0.2865889179 & -0.0000982745394 & 0.03253575799 & -0.02807837609 & -0.05243371668 & -0.008344164835 & 0.06069662706 & -0.004291975076 & 0.1670247356 \\
0.1385147851 & -0.0000982745394 & 0.2279655869 & -0.03170608505 & 0.1087579377 & 0.01983115335 & 0.06288746886 & -0.009471109679 & 0.1127190883 & -0.003165030067 \\
-0.0000982744665 & 0.03253575799 & -0.03170608505 & 0.07127043960 & -0.009471109660 & -0.008344164723 & -0.02929792981 & 0.02218216688 & -0.02312255893 & 0.01050035715 \\
0.1715200111 & -0.02807837609 & 0.1087579377 & -0.009471109660 & 0.1211020590 & 0.05231365627 & 0.01772423457 & -0.02463779327 & 0.08166675620 & -0.02461267152 \\
0.2301433417 & -0.05243371668 & 0.01983115335 & -0.008344164723 & 0.05231365627 & 0.1775985470 & 0.01249155120 & -0.02764314020 & 0.01374184635 & -0.04387217476 \\
0.01983115336 & -0.008344164835 & 0.06288746886 & -0.02929792981 & 0.01772423457 & 0.01249155120 & 0.04116565672 & -0.008337508358 & 0.03030876456 & -0.006795209218 \\
-0.02807837605 & 0.06069662706 & -0.009471109679 & 0.02218216688 & -0.02463779327 & -0.02764314020 & -0.008337508358 & 0.03499769565 & -0.01059571466 & 0.03521635089 \\
0.08334872851 & -0.004291975076 & 0.1127190883 & -0.02312255893 & 0.08166675620 & 0.01374184635 & 0.03030876456 & -0.01059571466 & 0.08453949808 & -0.004544472821 \\
-0.04517829880 & 0.1670247356 & -0.003165030067 & 0.01050035715 & -0.02461267152 & -0.04387217476 & -0.006795209218 & 0.03521635089 & -0.004544472821 & 0.1340964095
\end{bmatrix}$$

We set the modulus of elasticity, plate thickness and Poisson's coefficient, since they enter the right side of the equation through the cylindrical stiffness:

> GenTermB:= i -> w(x,y)[i];

$$GenTermB := i \rightarrow w(x, y)_i$$

> B:= CreateRightVector(GenTermB, Mx1, My1,BoundRect);

$$B := \begin{bmatrix} 0.3543012636 \\ -0.02354012016 \\ 0.07677632291 \\ -0.01536378468 \\ 0.06252726133 \\ 0.08385394667 \\ 0.01346953997 \\ -0.01170908109 \\ 0.03358586553 \\ -0.01368908724 \end{bmatrix}$$

We solve the equation and get:

> Solut:= SolveSystem(A, B):

$$Solut := \begin{bmatrix} 0.729222331876494990 \\ -0.0471940335419104296 \\ 0.249472492783038430 \\ -0.222883413342468778 \\ -0.632631291933822171 \\ -0.353691752318328168 \\ -0.249035388264939288 \\ -0.302538571801416911 \\ 0.00560785939283478269 \\ 0.0609115538569656376 \end{bmatrix}$$

Building a solution:

> U:= CreateSolution(Solut, w):

We calculate the value of the deflection at the maximum point:

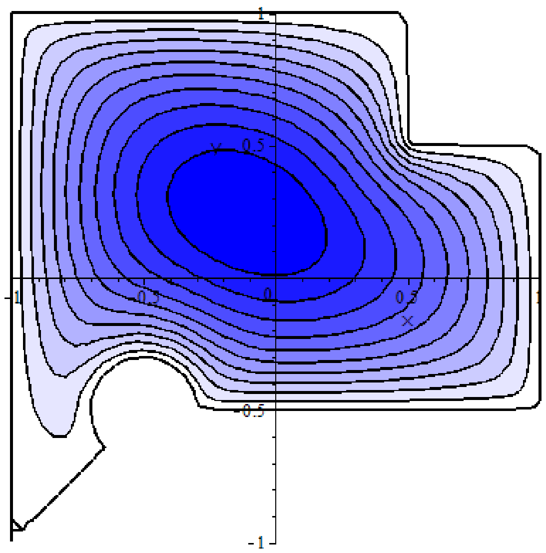
> evalf(U(-0.1,0.25));

0.170509276387391

We graphically depict the resulting solution in the form of level lines:

> with(plots):

> PlotSolution(Omega, U, BoundRect, 10);



The graphical results show that the choice of one or another hypothesis when formulating physical relationships leads to a rather significant change in the stress-strain state of the plates.

Conclusions

In the course of this work, a numerical-analytical method for solving boundary value problems of mathematical physics was studied using the example of the inhomogeneous Dirichlet problem for the Poisson equation in a complex domain in the Maple system environment. A numerical implementation of the computational method was carried out and a computational experiment was carried out.

On the basis of the proposed algorithm of tasks in the environment of the Maple system, an automated system of a software package has been developed, with the help of which problems in the mechanics of a deformed solid body with an arbitrary configuration are quickly solved. With the help of the developed automated system, a number of practical important boundary value problems of the theory of elasticity and the hereditary theory of viscoelasticity with a complex shape of the region boundary were studied.

A comparative analysis shows that a partial change in the shape of the region boundary leads to a rather significant change in the stress-strain state of the plates. The convergence of the computational algorithm is investigated and the reliability of the results obtained using an automated system by comparing them with the exact solution or solutions obtained by other authors is shown.

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