



Early Mathematics Learners' Numerical Errors: Consequence of Poor Learners' Comprehension and Teachers' Instructions

Anass Bayaga

*Nelson Mandela University, Gqeberha, SOUTH AFRICA
Faculty of Education*

Ndamase Nzuzo

*University of Fort Hare, East London, SOUTH AFRICA
Faculty of Education*

Michael J. Bossé

*Appalachian State University, Boone North Carolina, UNITED STATES
Department of Mathematical Sciences*

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Abstract

While the coronavirus disease 2019 (COVID-19) is still considered as a pandemic in recent human history, evidence from World Health Organization (2021) so far has recorded a total of 116,521,281 confirmed cases of COVID-19 with 2,589,548 as a total of deaths from over 215 countries or territories worldwide. Recognizing that COVID-19 is not only pandemic since March 11, 2020, but spreading worldwide at unprecedented rate, number of sectors including schools and universities as a measure to minimize person-to-person transmission closed their services. Such an uncertain closure warranted restructuring of services provided by schools and universities. The challenges therefore have necessitated the current research to investigate and alleviate challenges brought about by the COVID-19. In essence, the present research's aim was to report on early mathematics learners (foundation phase) numerical errors, which is as a consequence of poor learners' comprehension and teachers' instructions. Based on the aim, the study was positioned within a cognitive theory in order to examine processing of numerical competence among early mathematics learners. A case study via 80 grade 3 learners with ages 8 and 9 was sampled. A textual analysis was used in unpacking and de-contextualizing processing of numerical competence by early mathematics learners. The evidence revealed learners' mathematical mistakes were caused from limited reading skills and ill-presented problems via teachers. Due to the need to teach children at home (home school) due to the COVID-19, it is hoped that the findings thus assist audience, including non-academic and parents, who grapple with poor instructions coupled with poor learners' comprehension.

Keywords: numerical competence, foundation phase, computational competence, mathematical errors.

1. Introduction

While the coronavirus disease 2019 (COVID-19) is still considered as a pandemic in recent human history, evidence from World Health Organisation (2021) so far has recorded a total of 116,521,281 confirmed cases of COVID-19 with 2,589,548 as a total of deaths from over 215 countries or territories worldwide. Recognizing that COVID-19 is not only pandemic since March 11, 2020, but spreading worldwide at unprecedented rate, number of sectors including schools and universities as a measure to minimize person-to-person transmission closed their services. Such an uncertain closure warranted restructuring of services provided by schools and universities. Thus, the restructuring was fundamentally based upon either the nationwide or partial lockdowns of educational institutions, which included but not limited to schools, colleges, and universities. Such nationwide or partial lockdowns of educational institutions eventually mushroomed into challenges. While not an exhaustive account, many of such challenges were in the form of learning being interrupted, increased drop-out rates coupled with lowered academic achievement (UNESCO, 2020; Verschaffel, Depaepe & Mevarech, 2019). The challenges further encouraged remote learning. Effectively, the remote learning provided prospects for learners, teachers and even parents to be engaged with the content while concurrently studying or working from place of residence.

However, from both South Africa and international perspectives, while foundation phase learners' numerical errors have received increased attention, several other areas remain rife for further research, which are already exasperated by challenges of Covid 19 (Booth, McGinn, Barbieri & Young, 2017; Congdon, Kwon & Levine, 2018; Sethole, Gaba, Adler & Vithal, 2006; Yang, Sherman & Murdick, 2011; Özer & Göksun, 2020; Verschaffel et al., 2019). For instance, Yang et al. (2011) raised concern regarding error pattern analysis of elementary school aged students with limited English proficiency. A growing concern as alluded by Sethole et al. (2006) is the need to fine-tune language of description for mathematics items which incorporate the everyday activities. Unlike Sethole et al. (2006), Archibald (2009) suggests improving for instance spatial abilities to improve mathematics achievement. There have also been calls by Booth et al. (2017) to consider further investigation in misconceptions and learning of basic algebra. Yet, other areas remain widespread for further investigations. Example, the need to investigate how learning to measure through action and gesture could improve children's prior knowledge (Congdon et al., 2018) as well as investigating how visual-spatial and verbal abilities differentially affect processing of gestural vs. spoken expressions (Özer & Göksun, 2020).

Regardless of the ongoing need to examine the aforementioned, one common trend of thought as argued by Özer and Göksun (2020) and Verschaffel et al. (2019) is that such areas of research are as a consequence of poor comprehension by learners and ill-presented instructions from teachers, which tend to contribute to the already existing problems. Consequently, examining how to model the role of the learners' (1) understanding in the context of numerical processing as well as (2) misunderstandings sourced in weak information processing skills could assist in improving foundation phase learners' numerical error, which arguably is as a consequence of poor comprehension and instructions. Thus, the current research was built on extending the body of knowledge of previous studies such as Özer and Göksun (2020) and Verschaffel et al. (2019) in response to addressing the growing questions and fields of investigations.

2. Background to the study

Due in part to prevalence of foundation phase learners' mathematical errors born from weaknesses in reading and ill-presented problems, number of theoretical frameworks have been developed in order to account for their communicative and cognitive functions (Gordon & Ramani, 2021). In other words, mathematical errors born from weaknesses in reading and ill-presented

problems is predicated upon the understanding of cognition and information processing. In spite of the assessment of Gordon and Ramani (2021), various cognitive studies on mathematical errors instigated by weaknesses in reading and ill-presented problems have intensified and also taken various forms of investigations (Congdon, Novack, Goldin-Meadow & James, 2019; Ramani, Jaeggi, Daubert & Buschkuehl, 2017; Rhoads, Miller & Jaeger, 2018; Wakefield, Novack, Congdon, Franconeri & Goldin-Meadow, 2018; Wakefield, Özer & Göksun, 2020; Wiebe, Espy & Charak, 2008; Xenidou-Dervou, De Smedt, van der Schoot & van Lieshout, 2013; Zheng, Swanson & Marcoulides, 2011).

For instance, in the last decade, Zheng et al. (2011) in investigating working memory components suggested a relationship with children's mathematical word problem solving. During the same period, Xenidou-Dervou et al. (2013) suggested that math achievement could be linked with individual differences in kindergarten. Using confirmatory factor analysis, Wiebe et al. (2008) opined that preschool children could better be understood via their information processing. In recent times, it has also been revealed that gesture helps learners learn by guiding their visual attention (Wakefield et al., 2018). Other studies similar to aforementioned included the neural effects of gesture-based instruction in 8-year-old children (Wakefield et al. 2019), domain-specific and domain-general training to improve kindergarten children's mathematics (Ramani et al., 2017), gesturing improving preschoolers' executive function (Rhoads et al., 2018).

Irrespective of the extent literature, while we know one way to conceptualize how children solve mathematics problems and learn mathematics related content within the information processing approach as alluded by Özer and Göksun (2020), one thing stands out. That is, as opposed to single theory, rather an umbrella term has been advocated (Gordon & Ramani, 2021). Gordon and Ramani (2021) call the umbrella approach as embodied cognition (to be explained later). That is to say an integrating or embodied cognition responsible for information processing, which also means a combined model of the role of gesture in children's mathematical environments. While Booth, Barbieri, Eyer and Paré-Blagojev (2014) suggest that persistent and pernicious errors in problem-solving for instance could hamper the progress of learners, in contrast, and in support of the embodied cognition model, Adams, McLaren, Durkin, Mayer, Rittle-Johnson, Isotani and van Velsen (2014) have shown that using erroneous examples could improve mathematics learning. In support of Adams' et al. (2014) assertion, Jarvin (2009) suggests helping one's class to develop their knowledge and understanding of numbers with thought-provoking activities, which could eventually improve mathematics achievement of learners. Such advocacy as alluded by Jarvin (2009), Pardesi (2008) and Murray (2011) suggest a possibility of identifying errors in learners' mathematical thinking and methods of remediation.

Collectively, the evidence for further research points to Yang's et al. (2011) view that error pattern analysis of elementary school-aged learners with limited English proficiency requires further investigation. Against the backdrop of Yang et al. (2011), Gordon and Ramani (2021) suggest that information processing need not be focused only on visual mathematics specific input in order that we better understand how such inputs are relevant but equally for learners' mathematics environment. For instance, in using spatial abilities to improve mathematics achievement, it is important for information processing to consider the body itself (Archibald, 2009). However, Gordon and Ramani (2021) argue that while the information processing may describe the cognitive processes, it does not account for any co-occurring physical behaviors. This is because, Cotton (2010) argues that it could assist in understanding the teaching of primary mathematics. Accordingly, information processing model does not fully explain for instance gesture-specific benefits which sometimes tend to occur within a mathematics-related context or environment, hence need to use the embodied cognition as opposed to information processing approach (to be explained later). As a result, the questions remain unresolved as to how to satisfactorily understand the role of learner's (1) understanding in the context of numerical processing, as well as (2) misunderstandings sourced in weak information processing skills, and

(3) the different types of math stimuli (words and gestures) within the mathematics classroom from teachers’ instructions.

2.1 Implication thus far for current study

Implication for resolving such questions are based on number of claims. Gordon and Ramani (2021) claim that: First, because cognition is situated – meaning, cognitive processing tends to occur in conjunction with the task relevant inputs and outputs within specific mathematics environments, invariably, cognition should not be separated from specific mathematics environments in order that misunderstandings are clarified. The next claim indicates that cognition is time based. What this means is that cognitive processing does require real-time responses to the stimuli in order that errors and misunderstandings are corrected. Lastly, the specific mathematics environments or topics is an integral part of the cognitive system. This is because, given that input (the way we teach) stimuli, cognitive processes, as well as behavioural factors are recurrent or cyclical in nature, such factors may not be considered in isolation. For instance, using fingers in counting as an illustration, is an indication that such a gesture can be applied as a representation of relevant numeric information. That is being able to link number words to objects to keep track of quantity (Gordon & Ramani, 2021).

What could be drawn this far is that the presentation of both the parts of body and environment within which learning takes place are integral to cognition. Consequently, the argument that the specific mathematics environment or topics is an integral part of the cognitive system. Based on such integration, the current paper explores further a model combining central tenets from both information processing and embodied cognition.

2.2 Research questions

As a result of the introduction, background and lastly the implication, the question remains unresolved as to how to model the role of the learners’ (1) understanding in the context of numerical processing, as well as (2) misunderstandings sourced in weak information processing skills.

2.3 Research aims

To examine how to model the role of the learners’ (1) understanding in the context of numerical processing, as well as (2) misunderstandings sourced in weak information processing skills.

3. Theoretical framework – Embodied cognition

There has been growing concerns associated with foundation phase learners’ numerical error, which arguably has been attributed simultaneously to poor comprehension on the part of the learner and poor instructions from the teacher. As reflected in the background and because of such growing concerns, number of theories have been proposed to address similar questions. For instance, within Gordon and Ramani’s (2021: 3) work, embodied cognition is explained as “during a lesson on addition, math input could include a teacher’s speech and gestures in reference to an equation on the chalkboard, while the output could be children’s verbal and gestural response and explanations.” By implication, a comparable study based on embodied cognition by Gordon and Ramani (2021) revealed that by integrating both the embodied cognition and information processing theory, such combined model could explain the role of gesture in

learner's mathematical environments. It has also been showed that early mathematics learners' ability to count set of objects is predicated upon their math-input being the given instructions and countable objects as well as their output which could be the learners' pointing and counting out loud (Gordon & Ramani, 2021).

However, learner's mathematics output could be moderated given that Gordon and Ramani (2021) suggest learner's gesture and speech often contain different but complementary information. For example, recently, Wakefield et al. (2021) was able to show that it is possible to separate mathematics output by modality, based on the fact that both self-produced gestures as well as speech do not relate to learning and retention for learners in the same way that observed gestures do. In another study, Broaders, Cook, Mitchell and Goldin-Meadow (2007) were able to show that learners' gesture tend to bring out implicit knowledge and leads to learning.

Based on the analyses this far, it could be argued as did Chu, Meyer, Foulkes and Kita (2014) that individual differences in frequency and saliency of speech-accompanying gestures play a critical role in cognitive abilities. This as alluded by Chu et al. (1999) indicate the use of gesture and speech, could capture transitions in learning and that any mismatch between say gesture and speech could lead to transitional (intermediate) knowledge acquisition. In essence, learning through action is the foundation for any prior knowledge (Congdon et al., 2018).

A notable conclusion this far too is that simultaneous presentation of speech and gesture (as adaptive learning) in mathematics instruction supports generalization, retention and thus learning (Congdon, Novack, Brooks, Hemani-Lopez, O'Keefe & Goldin-Meadow, 2017). In effect, adaptive learning could lead to sustained enhancement of poor working memory in learners. Thus, there could be number of fields for applying the embodied cognition for further investigations by specifying the components (learner, input, and output) within a mathematics environment. For instance, Ping and Goldin-Meadow (2008) through embodied cognition have showed that using ungrounded iconic gestures (such as hands in the air) could teach learners conservation of quantity. Like Ping and Goldin-Meadow (2008), Rhoads et al. (2018) revealed that gesturing (put your hands up) improves early mathematics learners' executive function.

Notwithstanding the aforementioned studies, the embodied cognition theory does not explain nor do we currently understand for instance how to model the role of the learner's (1) understanding in the context of numerical processing, as well as (2) misunderstandings sourced in weak information processing skills.

4. Methodology

4.1 *Sampling and sample size*

Based on a case study and directed by the aim of the study, a total of 80 learners were selected to participate in this study. The sample size was determined so to be sufficient to hopefully provide a meaningful result, according to McMillan and Schumacher (2006). The researchers included 16 accessible schools. A simple random sample of five learners per school was selected, justifying the sample size of 80 learners. The participating learners' ages ranged from eight and nine. Participating learners were assured of their anonymity in the covering letters, and their confidentiality was maintained at all times. Permission was granted by the relevant school principals to distribute the questionnaires to the learners. Notably, the respective classroom teachers indicated that, of the 80 participating students, 20 (25%) had recognised difficulty reading English (the language of the mathematical instruction and questions) and 15 (19%) had significant weaknesses in reading.

4.2 Instrumentation

Teachers from the participating schools collaborated, developed, and used those same final exams for all the students in their respective schools. These finals were selected as the bases for research since: (A) the teachers could verify that, over the school year, instruction had covered the mathematical material; (B) the material in the final exams reflected mathematical concepts and topics which the teachers considered most important; (C) questions on the final exams were formulated in style consistent with what respective students had seen in their classrooms; and (D) in addition to covering many other topics in mathematics (e.g., algebra, space and shape, measurement, and data handling), they all had questions regarding the topics of this study (number sense, operation, relationships, and patterns). Some of the concepts that the questionnaire tested were - counting backwards and forwards; counting in the twenties, and showing calculations, multiplication and division as well as relationships and patterns. Using the final examination paper was an appropriate method for this study because all the learning outcomes had been taught by this time. Therefore, the learners were expected to be able to produce the outcome of what they had been taught, and they should have been able to extrapolate facts and concepts if required. It is also vital to note that the computational competence used in this study focused mainly on errors or mistakes and aimed to understand why they occurred in the foundation phase.

4.3 Data analysis

From the final exams, questions and responses regarding the mathematical topics of this study were selected. Discourse analysis was employed to investigate the questions and answers on the final exam questions regarding numbers sense and operations (Wertsch, 1990; Wertsch, Hagstrom & Kikas, 1995). Discourses analysis allowed the researchers to analyse the work of individual pupil and look for themes in the answers of many or all pupils. The themes (patterns) were then coded and used for analysis for the data to search for commonalities and differences among learners (Bogdan & Biklen, 2003; Creswell, 2003; Miles & Huberman, 1994; Strauss & Corbin, 1990). Findings from this investigation are provided below.

5. Results

The aim of the current research was to examine numerical processing and acquisition in the foundation phase. The instrument used test questions, for example, to count backwards and forwards, to count in the twenties, and to show calculations, multiplication, and division. For this research, and recall from the methodology section, the examinable questions included but were not limited to data handling (reading the calendar), word problems and measurements, evaluating, and interpreting figures and tables.

Question 1. “A car has four wheels and one in the boot. How many wheels of 12 cars?” The learners were expected to count the number of wheels of one car and thereafter count the wheels of 12 cars. Of the 80 learners included in the study, only 40 learners were able to count the wheels for 12 cars correctly; 18 learners only counted the wheels of the 12 cars, and neither counted the number of wheels for the first car nor the wheel in the boot, and 22 learners did not add the wheel in the boot. As exemplars to this question, Learner 1 responded “fourty (sic) eight” and Learner 2 responded “9”.

Analysis of responses: It is believed that Learner 1 simply multiplied $12 \times 4 = 48$ and simply forgot about the wheel in the boot. However, this pupil may have read the question as not caring about the wheel in the boot, but singularly about the wheels on the car. If so, his/her answer may not be incorrect. Learner 2 wrote a response of “9.” This may mean that the student was

merely guessing because it is difficult to imagine any mathematical reasoning which would produce this response.

However, it must be wondered how many pupil responses may have been in error due to the uncomfortable sentence structure of the question. It may have been difficult to interpret, “How many wheels of 12 cars?” This question could have been stated, “How many total wheels will the 12 cars have?” or some other form. It is difficult to assess student understanding of a question which is poorly written.

Question 2. Students were provided with the calendar in Table 1. They were asked, “How many days are there in May?” The learners were expected to say the last number shown in this Table, which was part of the data handling (reading the calendar), but only 70 learners were able to answer the question correctly; ten learners were not able to read the sentence and thus, could not comprehend the sentence. As exemplars of answers to this question, Learners 2, 39, 17 and 47 responded respectively, “4 day”, “13”, “5” and “30”.

Table 1. Analysis of Table, the calendar for May

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

Analysis of responses: While 70 of 80 learners answered this question correctly, others struggled. Among others, Learners 2, 47, 17 and 39 gave the incorrect answers. However, again, from what might their errors have originated? Could the responses of Learners 2 and 17 have been a misreading of the question as, “How many weeks are there in May?” If so, they may have correctly answered the wrong question. Without more information, it is impossible to know why Learner 47 had a response of 30 rather than 31. It may be that s/he attempted to answer the question by the memory of the number of days in the month rather than look at the Table. Learner 39’s response of “13” may not be a mathematical error at all. S/he may have dyslexia or dyscalculia.

Question 3. From the same calendar, students were asked, “How many Sundays are there in May?” The learners were expected to count the number of Sundays in the calendar. Only 50 learners managed to count the four Sundays, whilst it is believed that 30 learners did not have sufficient reading skills to correctly interpret the question. The following Learners provided the respective responses: Learner 11 wrote “20”, Learner 4 wrote “5”, Learner 39 wrote “12”, and Learner 29 wrote “10”.

Analysis of responses: As reflected in the sample work, several learners (e.g., 11, 4, 39, and 29) struggled to correctly answer this question. While it is easy to simply state that their responses are incorrect, this does not do the learners’ work justice. The respective teachers reported that these students had difficulty reading numbers and words, specifically relating to mathematical language. Thus, the responses may not be mathematically erroneous as much as the result of poor reading skills. It is possible that Learner 4 simply miscounted the number of Sundays. However, it is unsure how the other students derived their responses. If their reading skills were sufficiently poor, their responses might have been little more than guesses.

Question 4. Again employing the previous calendar, this was the open-ended prompt, “June starts on _____?” The anticipated response was “Friday”. The participating learners were expected to recognise that June starts the first day after Thursday May 31. Only 40

learners were able to answer the question correctly, stating “Friday”, whilst 25 learners wrote “1” on which the month starts; due to limited reading skills, ten learners did not comprehend the question and could not provide a correct answer; and five learners decided not to answer the question, maybe also not being able to read or understand the question. The following responses were provided: Learners 30 and 2 wrote “7” and Learner 23 wrote “27”.

Analysis of responses: While 40 students provided incorrect responses, the question may again be asked why. For instance, it is conceivable that poor reading skills could lead a student to incorrectly respond that June starts on “1”, denoting the first day of the month. If so, this may not have been a mathematical error. However, the responses of “7” and “27” may have been the result of even poorer reading skills and guesses regarding the meaning of the question. These incorrect responses may not have been the result of lacking reasoning skills.

Question 5. As reflected in Figure 1, from the prompt “The number of netball players”, the participating learners were expected to count the number of players in the graph. Only 55 (68%) learners were able to answer the question correctly that there are seven players, whilst 15 learners were unable to respond correctly or to evaluate and interpret the Figure, since they could not comprehend the question. An additional ten (13%) did not learners did not answer the question. A sample of responses to this question includes: Learners 1 and 60 wrote “Nana” and Learner 19 wrote “120”.

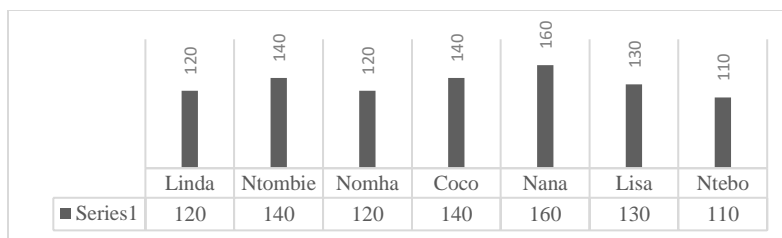


Figure 1. Number of Netball Players

Analysis of responses: Again, we see an example question which may be inadequately defined. It can be questioned how many ways students can interpret the prompt “The number of netball players”. The values for Series 1 may lead to even greater confusion than the ill-formed prompt. Thus, it remains mysterious precisely why 32% of the respondents provided incorrect responses. As in previous cases, it may be in part that some students lacked sufficient reading skill to decipher the question. However, since the ratio of students with incorrect answers is higher than for previous questions, it seems that the question stymied some students with adequate reading skills. This is more likely a product of the ill-posed prompt.

Question 6. Returning to Figure 1, the students were to respond to the prompt, “The tallest netball player?_____”. The learners were expected to read from the graph and interpret which player was taller than the other players by looking at the length of the players (each rectangular shape) or by the measurements provided. Only 60 (75%) learners managed to read from the graph that the tallest player was Nana. Even after the teacher provided some assistance, about 16 (20%) of learners did not understand the question and gave the wrong answer, and 4 (6%) learners did not answer the question. Some responses included: Learner 26, “140”; Learner 23, “Ntombie”; and Learner 71, “Lisa”.

Analysis of responses: The fact that 75% of the students correctly responded to this question may not be an indication that the question was understood. The prompt and Figure provide no indication that the numbers presented are to be interpreted as heights. Also, bar graphs are most often used as a representation of counting (cardinality). If students previously encountered similar looking bar graph representing the cardinality of represented sets, this Figure may have little meaning, particularly in association with a height (an ordinal number). In a similar

manner, pupils who produced an incorrect response may have made a mathematical error, may have made a reading error, or may simply have not been able to interpret the figure which appears to be a bar graph as an indication of height.

Question 7. Returning to Figure 1, the students are to respond to the prompt, “The length of Coco is _____ cm.” the anticipated response was “140” cm. Only 30 (38%) learners correctly responded to the prompt, whilst 43 (54%) learners were unable to answer the question – assumedly because they could not read and understand the question or the Table, and a further 7 (9%) learners did not answer the question. Some responses included: Learner 5 wrote “120” and Learner 7 wrote “7”.

Analysis of responses: This, again, is a poorly worded prompt. While, again, there is no indication on the figure that the numbers represent measurements, this may be even more problematic in that the students may confuse the meaning of length and height. The prompt should probably have been written, “The height of Coco is _____ cm.” This may have been less confusing. The fact that 62% of the student did not correctly answer this prompt and far less than that number were recognised as students who had notable reading difficulties, may indicate that the phrasing of the question is far more problematic than immediately recognized. However, it is hoped that students would recognize the connected context among questions 6-8.

Question 8. Returning to Figure 1, students are posed the prompt, “The player who has the same length/height as Coco is _____. The expected response is “Ntombie”. Only 58 (73%) learners were able to correctly respond to this prompt. However, 13 (16%) learners were unable to give the correct answer and 9 (11%) learners decided not to answer the question and leave it blank. For a response, Learner 2 wrote “140”.

Analysis of responses: While Learner 2’s response of “140” may have more clearly indicated an inability to correctly interpret the prompt, the fact 27% provided incorrectly (or no) responses may not have been the result of reading skills alone. This prompt now uses the expression “length/height”. While this may help to clarify previous ideas, there yet remains no indication in Figure 1 that the numbers represent measurements.

Further analysis connecting questions 5-8: If students indeed had difficulty interpreting Figure 1, regardless of the reason for such (including poorly written prompts and a figure lacking sufficient explanation), having four questions all connected to the same poorly defined figure may not provide a good measurement of student understanding.

6. Discussion

The current section discuss’ among others the results taking in account the aim of the study which was to examine how to model the role of the learner’s (1) understanding in the context of numerical processing as well as (2) misunderstandings sourced in weak information processing skills and (3) the different types of math stimuli (words and gestures) within the mathematics classroom. At the beginning of the current paper, it was indicated that due to schools closing because of the COVID-19, learning generally and parents are taking on the role of teaching their children at home. Which suggests that parents ought to recognize mathematical errors made by their children during these uncertain times of default home-schooling caused by the COVID 19. From the findings above, shed light on the fact that many of these mathematical errors could be caused from reading difficulties and ill-posed problems. Such results may assist non-academicians and parents.

In order to understand student skills in numerical processing and acquisition of arithmetic competence, it is essential to consider the mathematics in the form which they have been studying and being assessed, including word problems (Question 1), tables (Questions 2-4),

and graphs (Questions 5-8). However, in all these cases, some participants seemed to have more difficulty with reading and interpreting the table and graph and the prompt. Moreover, as seen in the findings, the assessment tools were questionably designed. In almost every case, the table, graph, and prompt itself was lacking sufficient detail to ensure that the student was answering the intended question. Thus, the assessment tasks lacked validity. Student weaknesses in reading together with problems and prompts which were difficult to interpret made a reliable assessment of student understanding difficult. While pattern recognition is an essential ability needed in understanding figures and shapes, most of the participants did not attempt to develop and record patterns to investigate. In almost all cases, responses were devoid of any accompanying work. Thus, there was no evidence of typical problem-solving strategies such as making a table of results, searching for a pattern among the data, or trying different approaches, or testing results.

In summary, the results of this study should remind the entire educational system of the need for mathematics instructions and questions to be posed correctly. Without such, it is difficult to determine the origin of student mathematical errors.

(1) understanding in the context of numerical processing

As with previous research (Zheng et al., 2011) and the embodied cognition theory, understanding in the context of numerical processing has been characterized as working memory being the predictor of learners' mathematical word problem solving. That is to say the development of executive functions and early mathematics tend to have dynamic relationship (Van der Ven et al., 2012). In current study, it was revealed that for instance, learning to measure through action enables learners to improve upon their prior knowledge matters. The results are consistent with the work of Congdon et al. (2018) as well as Broaders et al. (2007). The consistency hinges on the fact that, making learners use for instance gesture could bring out implicit knowledge thereby leading to learning as explained by the embodied cognition theory (Gordon & Ramani, 2021).

Number of implications could be drawn from the results and the previous studies. For instance, as with the work of Cotton (2010), it could be implied that understanding and teaching primary mathematics is predicated upon identifying and rectifying persistent and pernicious errors in in early mathematics learning (Booth et al., 2014). In effect, the use of erroneous examples for instance could improve mathematics learning as advocated by Adams et al. (2014). In support of the studies, another implication is directly linked with the idea that spatial abilities through adequately presented instructions improve mathematics achievement.

(2) misunderstandings sourced in weak information processing skills

On the other hand, and as reflected in the results, misunderstandings sourced in weak information processing skills could be improved by error pattern recognition of learners with limited English proficiency. As reflected by the results, this is rooted in number of activities such as ability to diagnose the misconceptions, it also includes being able to reveal changing type of knowledge applied by the learner and consequently the learning instruction. Durkin and Rittle-Johnson (2015) and Gordard (2005) thus suggest that learners' number difficulties could thus be improved through error analysis by identifying errors in learner's mathematical thinking and methods for remediation (Pardesi, 2008). Consequently, this could assist as suggested by Jarvin (2009) in developing knowledge and understanding of numbers characterized by thought-provoking activities, which defines learning to understand arithmetic. However, as demonstrated by the current study, fine-tuning a language of description for mathematics items which incorporate everyday activities as supported by Sethole et al. (2006) is not only key but as noted by Shalem and Sapire (2012), it simultaneously improves teachers' knowledge of error analysis.

6.1 Implication

In addressing foundation phase learners' numerical error, which is as consequence of poor comprehension and instructions, we narrowed our understanding to information processing but keenly on embodied cognition model. The model involved in mathematics learning though well-modelled by the information processing approach, however such a model is not able to fully explain the underlying mechanisms of how to model the role of the learner's (1) understanding in the context of numerical processing as well as (2) misunderstandings sourced in weak information processing skills. The main tenets as suggested from embodied cognition particularly is "...within the body, and by extending the surrounding of mathematics environment. The use of embodied cognition thus allows for the consideration of gestures as a form of mathematics input from the environment, as well as a form of mathematics-output from..." learners' (Gordon & Ramani, 2021). What is key too is that embodied cognition is primarily informed by instruction. The implication leads to future research within this domain for instance how individual differences in learners' embodied cognition impact their use of gestures during mathematics tasks. For instance, how learners' level of mathematics knowledge influences their embodied cognition or the interaction between the two remain to be established. For instance, it remains to be established as to how the nature of such relations change as learners' mathematics knowledge improves, and the specific content they are learning concurrently changes.

7. Conclusion

The evidence shows that mathematical errors is predicated upon weak reading skills or ill-presented problems. What that means is that assessing learners' understanding and errors is multidimensional. It is not simply determined by whether learners' responses to mathematical questions are correct. Keenly, the instructions associated with mathematics and tools for assessment need to be correctly provided. The need to be presented correctly in order to avoid errors. This is because, such errors may not necessarily be mathematical in nature. As a result, it is hoped that the current study could inform both educators and parents home-schooling their children due to the COVID-19 pandemic challenges as alluded in both the introductory as well as background of the study.

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