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# SOI: 1.1/TAS DOI: $10.15863 / \mathrm{TAS}$ <br> International Scientific Journal <br> Theoretical \& Applied Science <br> Published: 15.05.2021 http://T-Science.org 



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# ON THE ACTION OF A MOVING PRESSURE WAVE ON A VISCOELASTIC CYLINDRICAL SHELL INTERACTING WITH AN IDEAL LIQUID 


#### Abstract

In this paper, we present statements, develop methods for solving and obtain numerical results for new problems of stationary deformation of infinitely long viscoelastic cylindrical shells on a deformable base when a non-axisymmetric wave of normal pressure moves along the axis of the shell with up to a resonant velocity. The solution methods are based on the joint application of the integral Fourier transform (or the method of fundamental solutions) with respect to the axial coordinate and the decomposition of all the given $n$ desired quantities into Fourier series with respect to the angular coordinate. An efficient algorithm for the joint calculation of integrals and Fourier series is developed and implemented on a computer. The mixed shell is estimated as a function of the pressure wave velocity and the viscosity of the materials.


Key words: pressure waves, cylindrical shell, viscoelasticity, resonant velocity, Fourier transform, deformable base.

Language: English
Citation: Akhmedov, M. S., Juraev, T. O., \& Kulmuratov, N. R. (2021). On the action of a moving pressure wave on a viscoelastic cylindrical shell interacting with an ideal liquid. ISJ Theoretical \& Applied Science, 05 (97), 213-218

Soi: http://s-o-i.org/1.1/TAS-05-97-42
Doi: crossef https://dx.doi.org/10.15863/TAS.2021.05.97.42
Scopus ASCC: 2200.

## Introductions

Natural oscillations and propagation of free waves in cylindrical shells interacting with a liquid have been studied by many authors, in particular in [1,2]. In this case, axisymmetric and non-axisymmetric problems were considered, and various models for the liquid and shell were used. The question of the action of a moving pressure wave on a cylindrical shell filled or surrounded by a liquid is less studied, and only axisymmetric loading was considered [3,4]. In this paper, using the integral transformation with respect to
the axial coordinate and the Fourier series with respect to the angle, the solution of the problem of motion along an infinitely long cylindrical shell interacting with an ideal compressible fluid of normal pressure, arbitrary in length and circumference, but unchanged in time, is obtained. The speed of movement of the load is constant and, in the subsection, it is considered in the case when it is less than the speed of sound in a liquid. The liquid fills the cavity

## 2. Problem statement and basic equations

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Consider the action of a non-axisymmetric moving pressure wave on a viscoelastic cylindrical shell interacting with an ideal compressible fluid The equations of motion of a viscoelastic cylindrical shell and an ideal fluid are introduced in the coordinate system $r, \theta, z$ [5].

The equations of motion of the bearing layers (shells) in displacements, in symbolic vector-matrix form, are written in the form

$$
\begin{array}{r}
L_{i j} \vec{U}_{k}-\int_{0}^{t} L_{i j} R_{E \kappa}(t-\tau) \vec{U}_{k}(\vec{r}, \tau) d \tau= \\
\frac{\left(1-v_{0 \kappa}{ }^{2}\right)}{G_{0 \kappa} h_{0 \kappa}} \vec{q}_{\kappa}++\rho_{0 \kappa} \frac{\left(1-v_{0 \kappa}{ }^{2}\right)}{G_{0 \kappa}} \frac{\partial^{2} \vec{U}_{k}}{\partial t^{2}} . \quad(k=1,2) \tag{1}
\end{array}
$$

Here, the index $k=1$ refers to the inner carrier layer, and $k=2$-to the outer layer, $U_{k}$ is the displacement vector of the points of the middle surface of the bearing layer, and for Timoshenko-type shells the dimension of the vector $U_{k}$.

$$
\begin{aligned}
\left(U_{1 k}=u_{k} ; U_{2 k}\right. & =\vartheta_{k} ; U_{3 k}=w_{k} ; \quad U_{4 k}=\psi_{x k} ; \quad U_{5 k} \\
& \left.=\psi_{y k}\right)
\end{aligned}
$$

Here, respectively, axial, circumferential and normal displacements are also added the angles of rotation of the normal to the middle surface in the axial and circumferential directions: $P_{k}$ is the vector of loads on the shell, the dimension of which also depends on the chosen theory of shells. $L_{i j}$ is a matrix of differential operators of the theory of shells, including in problems of dynamics and time differentiation (terms with damping and inertial terms in expanded form). If, when writing the equations of motion of the bearing layers, shear deformations and inertia of rotation (Timoshenko -type shell) are taken into account, then the differential operators have the form

$$
\begin{gather*}
\begin{array}{c}
L_{11}=\frac{\partial^{2}}{\partial z^{2}}+\frac{1-v_{k}}{2 a_{k}^{2}} \frac{\partial^{2}}{\partial \theta^{2}}--\rho_{k} \frac{1-v_{k}}{2 G_{k 0}} \frac{\partial^{2}}{\partial t^{2}} ; L_{12} \\
=L_{21}=\frac{1+v_{k}}{2 a_{k}} \frac{\partial^{2}}{\partial z \partial \theta} ; \\
L_{13}=L_{31}=\frac{v_{k}}{a_{k}} \frac{\partial}{\partial z} ; L_{14}=L_{15}=0 ; \\
L_{22}=\frac{1-v_{k}}{2 a_{k}^{2}} \frac{\partial^{2}}{\partial z^{2}}+\frac{1}{a_{k}^{2}} \frac{\partial^{2}}{\partial \theta^{2}}-\rho_{k} \frac{1-v_{k}}{2 G_{k 0}} \frac{\partial^{2}}{\partial t^{2}} ; L_{23} \\
=\frac{1}{a_{k}^{2}}\left[1+\frac{\left(1-v_{k}\right) k_{0}^{2}}{2}\right] \frac{\partial}{\partial \theta} ; \\
L_{33}=-\frac{1-v_{k}}{2} k_{0}^{2}\left(\frac{\partial^{2}}{\partial z^{2} k}+\frac{1}{a_{k}^{2}} \frac{\partial^{2}}{\partial \theta^{2}}\right)+\frac{1}{a_{k}^{2}}+\rho_{k} \frac{1-v_{k}}{2 G_{k 0}} \frac{\partial^{2}}{\partial t^{2}} ; \\
L_{34}=\frac{\left(1-v_{k}\right)}{2 a_{k}} k_{0}^{2} \frac{\partial}{\partial z} ; L_{35}=-\frac{\left(1-v_{k}\right)}{2 a_{k}} k_{0}^{2} \frac{\partial}{\partial \theta} ; L_{41}= \\
L_{42}=0 ;
\end{array} \\
L_{43}=-6 k_{0}^{2} \frac{1-v_{k}}{h_{k}^{2}} \frac{\partial}{\partial z} ; L_{45}=L_{54}=\frac{1+v_{k}}{2 a_{k}} \frac{\partial^{2}}{\partial z \partial \theta} ;
\end{gather*}
$$

$$
\begin{aligned}
& L_{44}=\frac{\partial^{2}}{\partial x^{2}}+\frac{1-v_{k}}{2 a_{k}^{2}} \frac{\partial^{2}}{\partial \theta^{2}}-6 k_{0}^{2} \frac{1-v_{k}}{h_{k}^{2}} \\
& \quad-\rho_{k} \frac{1-v_{k}}{2 G_{k}} \frac{\partial^{2}}{\partial t^{2}} ; \\
& L_{51}=L_{52}=0 ; L_{53}=-6 k_{0}^{2} \frac{1-v_{k}}{a_{k} h_{k}^{2}} \frac{\partial}{\partial \theta} ; \\
& L_{55}=\frac{1-v_{k}}{2} \frac{\partial^{2}}{\partial z^{2}}+\frac{1}{a_{k}^{2}} \frac{\partial^{2}}{\partial \theta^{2}}-6 k_{0}^{2} \frac{1-v_{k}}{h_{k}^{2}} \\
&-\rho_{k} \frac{1-v_{k}}{2 G_{k 0}} \frac{\partial^{2}}{\partial t^{2}} ; \\
& \lambda_{0 s}=\frac{2 v_{s} G_{s 0}}{1-2 v_{s}} ; \mu_{0 s}=G_{s 0} .
\end{aligned}
$$

Here $k_{0}^{2}$-is the Timoshenko coefficient; $h_{k}, a_{k}$ thickness and radius of the middle surface of the bearing layer; nu $v_{k}$-Poisson's ratio; $R_{\mathrm{EK}}(t-\tau)$ relaxation core; $G_{k 0^{-}}$instant modulus of elasticity.

The components of the load vector, respectively, has the form

$$
\begin{gather*}
\left\{P_{1 k}, P_{2 k}, P_{3 k}\right\}=-\frac{1-v_{k}}{2 G_{k 0} h_{k}}\left\{p_{z k} \pm q_{z k}, p_{\theta k} \pm q_{\theta k}, p_{r k}\right. \\
\left. \pm q_{r k}\right\}
\end{gathered} \begin{gathered}
P_{4 k}=-\frac{3\left(1-v_{k}\right)}{G_{k 0} h_{k}^{2}}\left(p_{z k} \pm q_{z k}\right) ; \\
P_{5 k}=-\frac{3\left(1-v_{k}\right)}{G_{k} h_{k}^{2}}\left(p_{\theta k} \pm\right.
\end{gather*}
$$

where the minus sign corresponds to the inner shell, and the plus sign to the outer shell: $q_{z k}, q_{\theta k}, q_{r k}$ are the reaction components from the filler side: $p_{z k}, p_{\theta k}, p_{r k^{-}}$ intensity of the given load in the corresponding direction.

In the problem of free wave propagation, the components of a given load, $\psi(t)$ are taken to be zero. We accept the integral terms in (1) small. Then the function $\phi(t)=\psi(t) e^{-i \omega_{R} t}$, where is a slowly varying function of time, $\omega_{R}$ - is real constant. Further, applying the freezing procedure [6,7], we replace relations (1) with approximate ones of the form

$$
\begin{equation*}
\bar{L}_{i i}^{k} \vec{U}_{k}=\frac{\left(1-v_{0 \mathrm{~K}}\right)}{G_{0 \mathrm{~K}} h_{0 \mathrm{~K}}} \vec{q}_{\mathrm{K}}+\rho_{0 \mathrm{~K}} \frac{\left(1-v_{o k^{2}}\right)}{G_{0 \mathrm{~K}}} \frac{\partial^{2} \vec{U}_{k}}{\partial t^{2}} . \tag{4}
\end{equation*}
$$

Here $\quad \bar{L}_{i i}^{k}[\varphi(t)]=L_{i j}^{k}\left(1-\left(R_{i j}^{k} \delta_{i j}\right)^{-1}\right)[\varphi(t)]$, $\delta_{i j}$-Kronecker symbols, $\quad R_{i j}^{k}\left(R_{11}^{k}=R_{22}^{k}=R_{33}^{k}=\right.$ $\left.=G_{k}[\varphi(t)]\right)$ is the third order diagonal matrix for the Kirchhoff - Love conjecture, and the fifth order for the Timoshenko conjecture. The system of differential equations (4) is solved under boundary conditions. The non-axisymmetric motion of a Timoshenko-type shell is described by equations (1), and (4) and in the components of the load vector only the term [8] $p_{3 k}=$ $-\frac{1-v_{k}}{2 G_{k} h_{k}}\left(q_{r k} \mp p_{r k}\right)$, the motion of an ideal compressible fluid is described by the wave equation

$$
\begin{equation*}
\frac{\partial^{2} \varphi}{\partial r^{2}}+\frac{1}{r} \frac{\partial \varphi}{\partial r}+\frac{1}{r^{2}} \frac{\partial^{2} \varphi}{\partial \theta^{2}}+\frac{\partial^{2} \varphi}{\partial x^{2}}=\frac{1}{c_{1}^{2}} \frac{\partial^{2} \varphi}{\partial t^{2}} . \tag{5}
\end{equation*}
$$

where $\varphi$ - potential of fluid velocities; $c_{1}$ - acoustic speed of sound in liquid; $\rho_{0}$ - density of liquid.

The problem is reduced to the joint integration of equations (1), (4), and (5) when the boundary

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conditions of impermeability of the shell and rigid wall are satisfied

$$
\begin{equation*}
\frac{d \varphi}{d r} \mathrm{I}_{r=a}=\frac{\partial w_{k}}{\partial t} ; \frac{d \varphi}{d r} I_{r=b}=0 . \tag{6}
\end{equation*}
$$

in this case, the pressure entering into (3) from the liquid side is expressed through the velocity potential by the formula

$$
\begin{equation*}
q_{r k}=-\rho_{0} \frac{\partial \varphi}{\partial t} I_{r=a} . \tag{7}
\end{equation*}
$$

Considering the steady-state process, we pass in the equations of motion of the shell and fluid to the coordinate system.

## 3.Methods of solution

When considering the establishing process, the Galilean transformation is applied [9]

$$
\begin{equation*}
\eta=(x-c t) / H, \tag{8}
\end{equation*}
$$

where $c$ - is the speed of movement of the load, H is some characteristic value in the problem under consideration, which has the dimension of length
( $H=a$ ). We apply the Fourier transform according to $\eta$ [10]:

$$
\begin{gather*}
\varphi^{(0)}(\zeta)=\int_{-\infty}^{\infty} \varphi(\eta) e^{-i \zeta \eta} d \eta ; \quad \varphi(\eta)= \\
\int_{-\infty}^{\infty} \varphi^{(0)}(\zeta) \varphi(\eta) e^{i \zeta \eta} d \zeta . \tag{9}
\end{gather*}
$$

Here $\zeta$-is the Fourier transform parameter.
In the image space, the solution of the transformed equations is sought in the form of Fourier series in the angular coordinate $\theta$. Assuming that the transformants of a given normal load and fluid pressure are expandable in Fourier series in $\theta$

$$
\begin{align*}
\left\{u_{k}^{(0)}, w_{k}^{(0)}, \psi_{x k}^{(0)}, p_{r k}^{(0)}, q_{r k}^{(0)}\right\} & =\sum_{n=0}^{\infty}\left\{u_{n k}^{(0)}, w_{n k}^{(0)}, \psi_{x n k}^{(0)}, p_{r n k}^{(0)}, q_{r n k}^{(0)}\right\} \cos (n \theta) \\
\left\{v_{k}^{(0)}, \psi_{y k}^{(0)}\right\} & =\sum_{n=1}^{\infty}\left\{v_{n k}^{(0)}, \psi_{y n k}^{(0)}\right\} \sin (n \theta) \tag{10}
\end{align*}
$$

where $n$ - is the number of harmonics along the angular coordinate.

Substituting (10) into the transformed equations of motion of the shell, we obtain a system of algebraic equations for the Fourier coefficients of the transformants of the displacements of the middle surface. In this system, the unknown are the expansion coefficients of the liquid pressure, which should be expressed through the coefficients of normal displacement

$$
\frac{\partial^{2} \varphi_{n}^{(0)}}{\partial r_{*}^{2}}+\frac{1}{r_{*}} \frac{\partial \varphi_{n}^{(0)}}{\partial r_{*}}-\left[\frac{n^{2}}{r_{*}^{2}}+\left[1-M^{2}\right] \zeta^{2}\right] \varphi_{n}^{(0)}=0
$$

shell. Representing the transformant of the velocity potential in the form (10) and substituting it into the transformed equation (1), we arrive at the equation $\frac{\partial^{2} \varphi_{n}^{(0)}}{\partial r_{*}^{2}}+\frac{1}{r_{*}} \frac{\partial \varphi_{n}^{(0)}}{\partial r_{*}}-\left[\frac{n^{2}}{r_{*}^{2}}+\left[1-M^{2}\right] \zeta^{2}\right] \varphi_{n}^{(0)}=0$,
where $M=\frac{c}{c_{1}}$ is the Mach number, $\mathrm{c}_{1}$ is the acoustic speed of sound in the fluid. The solution to equation (11) in the subsonic regime of motion $c<c_{1}$ has the form [11]:

$$
\varphi_{n}^{(0)}=A_{n}(\xi) K_{n}\left(\beta \xi r_{*}\right)+B_{n}(\xi) I_{n}\left(\beta \xi r_{*}\right)
$$

where $\beta=\sqrt{1-M^{2}}$.
Substituting (11) into (2), (3), we find the relationship between the liquid reaction and the normal displacement of the shell:

$$
\begin{equation*}
q_{r . n k}^{0}=\rho_{0} c^{2} k \xi^{2} f_{k}(\xi, n, c) \frac{w_{n k}^{0}}{h_{k}}, \tag{12}
\end{equation*}
$$

where for $c<c_{1}$

$$
\begin{gathered}
f_{k}(\xi, n, c)=\frac{n s_{4}-\beta \xi \varepsilon-\left(n s_{2}+\beta \xi \varepsilon s_{3}\right) s_{5}}{\left(n+\beta \xi s_{1}\right)\left(n s_{4}-\beta \xi \varepsilon\right)-\left(n s_{2}+\beta \xi \varepsilon s_{3}\right)\left(n s_{5}-\beta \xi s_{6}\right)} ; \\
s_{1}=\frac{I_{n+1}(\beta \xi)}{I_{n}(\beta \xi)} ; \quad s_{2}=\frac{I_{n}(\beta \xi \varepsilon)}{I_{n}(\beta \xi)} ; \\
s_{3}=\frac{I_{n+1}(\beta \xi \varepsilon)}{I_{n}(\beta \xi)} ; \quad s_{4}=\frac{I_{n}(\beta \xi \varepsilon)}{I_{n+1}(\beta \xi \varepsilon)} ; \\
s_{5}=\frac{K_{n}(\beta \xi)}{K_{n+1}(\beta \xi \varepsilon)} ; \quad s_{6}=\frac{K_{n+1}(\beta \xi \varepsilon)}{K_{n+1}(\beta \xi \varepsilon)},
\end{gathered}
$$

where $\varepsilon=b / a, I_{n+1}(\beta \xi)$ and $K_{n}(\beta \xi)$ - modification of the Bessel function of the 1st and 2nd kind, $f_{k}(\xi, n, c)$ for $\mathrm{k}=1,2$, it differs in meaning with opposite signs.

If the shell is completely filled with liquid, then formula (13) takes the form

$$
\begin{gather*}
f_{k}(\xi, n, c)=\delta_{k}\left(n+\beta \xi s_{1}\right)^{-1}, \delta_{k}=\left\{\kappa=1, \delta_{1}=\right. \\
\left.1 ; \kappa=2, \delta_{1}=-1\right\} . \tag{14}
\end{gather*}
$$

Substituting the found relation (12) into the system of algebraic equations for determining the expansion coefficients of the transformants of the displacements of the shell, we find

$$
\begin{array}{r}
\left\{u_{n k}^{(0)}, v_{n k}^{(0)}, w_{n k}^{(0)}, \psi_{x n k}^{(0)}, \psi_{y n k}^{(0)}\right\}= \\
-\frac{1-v_{k}}{2 G_{k} k^{2}} p_{z, n k} \frac{\left\{\Delta_{1 k}, \Delta_{2 k}, \Delta_{3 k}, \Delta_{4 k}, \Delta_{5 k}\right\}}{\Gamma_{E 1} \operatorname{det}_{n}\left\|a_{\bar{k}}\right\|},(\bar{k}, l=1, \ldots, 5) \tag{15}
\end{array}
$$

Determinant elements $\operatorname{det}_{n}\left\|a_{k l}\right\|$ calculated by the formulas

$$
\begin{gathered}
a_{11 k}=-\left(1-\frac{1-v_{k}}{3} c_{0 k}^{2}\right) \xi^{2}-\frac{1-v_{k}}{3} n^{2} \\
a_{12 k}=-a_{21 k}=a_{45 k}=-a_{54 k}=i \xi \frac{1+v_{k}}{2} n \\
a_{13 k}=a_{31 k}=i \xi v_{k}
\end{gathered}
$$

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$$
a_{22 k}=-\frac{1-v_{k}}{2}\left(1-\frac{2}{3} c_{0 k}^{2}\right) \xi^{2}-n^{2} ; a 23 k=-2+1+v k k 0 k 22 n ; a_{25 k}=k^{-1} ; a_{32 k}=n ; a_{33 k}=1+
$$

$$
\begin{equation*}
k_{0 k}^{2} \frac{1-v_{k}}{2}\left(n^{2}+\xi^{2}\right)-\frac{1-v_{k}}{3} c_{0 k}^{2} \xi^{2}\left[1+\frac{\rho_{0 k}^{*}}{k} f_{k}(\xi, n, c)\right] ; \tag{16}
\end{equation*}
$$

$$
\begin{gathered}
a_{34 k}=-i \xi k_{0 k}^{2} \frac{1-v_{k}}{2 k} ; \quad a_{35 k}=-k_{0 k}^{2} \frac{1-v_{k}}{2 k} \frac{n}{k} ; \\
a_{43 k}=12 a_{34 k} ; \quad a_{44 k}=a_{11 k}-6\left(1-v_{k}\right) \frac{k_{0 k}^{2}}{k^{2}} ; \\
a_{53 k}=-12 a_{35 k} ; \quad a_{55 k}=a_{22 k}-6(1-v) \frac{k_{0 k}^{2}}{k^{2}} ; \\
a_{14 k}=a_{15 k}=a_{24 k}=a_{41 k}=a_{12 k}=a_{51 k}=a_{52 k}=0 ; \\
\rho_{0 k}^{*}=\frac{\rho_{0}}{\rho_{k}} ; \quad c_{0 k}=c\left({ }^{\frac{33_{k}}{\sigma_{k}}}\right. \\
\Gamma_{E 1}=1-\Gamma_{E 1}^{\mathrm{C}}\left(\omega_{R}\right)-i \Gamma_{E 1}^{S}\left(\omega_{R}\right), \\
\Gamma_{\mathrm{E}}^{\mathrm{C}}\left(\omega_{R}\right)=\int_{0}^{\infty} R_{E}(\tau) \cos \omega_{R} \tau d \tau ; \Gamma_{\mathrm{E}}^{S}\left(\omega_{R}\right)= \\
\int_{0}^{\infty} R_{E}(\tau) \sin \omega_{R} \tau d \tau,
\end{gathered}
$$

Where $R_{E}(\tau)$ - the core of relaxation
Determinants $\Delta_{j k}(j=1, \ldots, 5)$ are obtained from $\operatorname{det}_{n}\left\|a_{k l}\right\| \quad$ by replacing the j -th column with elements $\{0,0,1,0,0\}$.

Substituting (15) into formula (12), we find the Fourier coefficients of the transformants of fluid pressure

Substituting (15) into formula (12), we find the Fourier coefficients of the transformants of fluid pressure
$q_{r, n k}^{0}=-\frac{1-v}{3} \frac{\rho_{0 k}^{*} c_{0}^{2}}{k} \xi^{2} f_{k}(\xi, n, c) \frac{\Delta_{3 k}}{\Gamma_{E 1} \operatorname{det}_{n}\left\|a_{k l}\right\|} p_{r . n k}^{0}(17)$
For the bending moment and shear force in the shell, we obtain

$$
\begin{gather*}
M_{x, n k}^{0}=-\frac{h_{k} a}{12} p_{r, n k}^{0} \frac{i \xi \Delta_{4 k} n v_{k} \Delta_{5 k}}{\Gamma_{E 1} \operatorname{det}_{n}\left\|a_{\bar{k} l}\right\|}  \tag{18}\\
Q_{x, n k}^{0}=\frac{\left(1-v_{k}\right) k_{0 k}^{5}}{2 k} a p_{r, n k}^{0} \frac{i \xi \Delta_{4 k} n v_{k} \Delta_{5 k}}{\Gamma_{E 1} \operatorname{det}_{n}\left\|a_{\bar{k} l}\right\|} . \tag{19}
\end{gather*}
$$

Now let's look at some examples. The final solution is obtained by substituting (15) - (19) in the Fourier series and applying the inverse Fourier transform. As an example, it is considered when the liquid is between a shell of radius $b$ and a coaxial rigid cylindrical wall of radius $a$

External loads are taken in the form [12 $p_{r}(\eta, \theta)=p_{2} \exp (a \eta) H(-\eta) \sum_{k=1}^{l}\left(\theta-\theta_{k}\right)$,
$H(x)$-Heaviside functions. In this case

$$
p_{r, n k}^{0}=\frac{p_{2} a_{n k}}{a-i \xi} .
$$

Here $a_{n}$ are the Fourier coefficients of the function $\sum_{k=1}^{l}\left(\theta-\theta_{k}\right)$ If you accept $p_{2}=2 \pi p_{1} / l$. Where $p_{1}$ - intensity of moving loads, $q^{*}=q_{r} / p_{1}$

$$
\begin{gather*}
w_{1}^{*}=\frac{w G_{1} \Gamma_{E 1}}{p_{1} a}--\frac{1-v_{1}}{k l} \sum_{n=0}^{\infty}\left\{\int_{-\infty}^{\infty} \frac{\Delta_{31}[a \cos (\xi \eta)-\xi \sin (\xi \eta)]}{\Gamma_{E 1}\left(a^{2}+\xi^{2}\right) \operatorname{det}\left\|a_{\bar{k}}\right\|} d \zeta\right\} \times a_{n} \cos (n \theta) ;  \tag{22}\\
q^{*}=-\frac{2(1-v) p_{01}^{*} c_{0}^{2}}{3 k l} \sum_{n=0}^{\infty}\left\{\int_{-\infty}^{\infty} \frac{f(\xi, n, c) \xi^{2} \Delta_{31}[a \cos (\xi \eta)-\xi \sin (\xi \eta)]}{\Gamma_{E 1}\left(a^{2}+\xi^{2}\right) \operatorname{det}\left\|a_{\bar{k} l}\right\|} d \zeta\right\} \times \times a_{n} \cos (n \theta) . \tag{23}
\end{gather*}
$$

Similarly, using (22) and (23), one can write the formulas for $M x, Q x$. The calculation of the nonconforming integrals (22) and (23) uses the following algorithm based on the Romberg method [13,14]

## 4. Calculation algorithm

The value $w_{-} 1^{\wedge}$ last and $q^{\wedge} \backslash$ ast of (22) and (23) is calculated on a computer as follows. All numeric parameters required for calculations are set. To calculate the integral (22) under the improper integral function is denoted by $\chi_{1}\left(r_{0}, \Omega, t\right)=$ $\frac{\Delta_{31}[a \cos (\xi \eta)-\xi \sin (\xi \eta)]}{\left(a^{2}+\xi^{2}\right) \Gamma_{E 1} \operatorname{det}\left\|a_{\bar{k} l}\right\|}$ The following elementary transformations are performed on this function $\chi_{1}\left(r_{0}, \Omega, t\right)=\left(\Delta_{1}\left(r_{0},\right) / \Omega\left(\Delta_{2} \Delta_{3}+\Delta_{4} \Delta_{5}\right)\right) e^{i \Omega t}$ you can numerically integrate it by writing it as

$$
\begin{equation*}
\chi_{1}\left(r_{0}, \Omega, t\right)=x_{1}\left(r_{0}, \Omega, t\right)-i \mathrm{x}_{2}\left(r_{0}, \Omega, t\right) \tag{24}
\end{equation*}
$$

The incident pulse w_1^\ast is described by the expression

$$
w_{1}^{*}(\Omega, t)=f_{1}(\Omega, t)-i f_{2}(\Omega, t),
$$

Where $f_{1}(\Omega, t), f_{2}(\Omega, t)$ real functions. Using Euler's formula for $\exp (i \Omega t)$, dividing into real and imaginary (25) parts, after some transformations we get

$$
\begin{equation*}
w_{1}^{*}=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{\infty}\left[x_{1}(\Omega, t)-i x_{2}(\Omega, t)\right] d \Omega \tag{26}
\end{equation*}
$$

Dividing the integral (16) into two terms

$$
\begin{gather*}
w_{1}^{*}=\frac{1}{\sqrt{2 \pi}} \int_{-\infty}^{0}\left[x_{1}(\Omega, t)-i x_{2}(\Omega, t)\right] d \Omega+ \\
\frac{1}{\sqrt{2 \pi}} \int_{0}^{\infty}\left[x_{1}(\Omega, t)-i x_{2}(\Omega, t)\right] d \Omega \tag{27}
\end{gather*}
$$

and replacing the $\Omega$ variable in the first integral with $\Omega$, we have

$$
\begin{align*}
& w_{1}^{*}=\frac{1}{\sqrt{2 \pi}} \int_{0}^{\infty}\left[x_{1}(\Omega, t)-x_{1}(-\Omega, t)\right]-i\left[x_{2}(\Omega, t)-\right. \\
& \left.x_{2}(-\Omega, t)\right] d \Omega \tag{28}
\end{align*}
$$

Since (28) is the inverse Fourier transform and contains a real value in the left part [13], the relation is vale

$$
\begin{equation*}
x_{1}(\Omega, t)=-x_{1}(-\Omega, t) ; x_{2}(\Omega, t)=-x_{2}(-\Omega, t) . \tag{29}
\end{equation*}
$$

- Given it, from (29) we finally obtain

$$
\begin{equation*}
w_{1}^{*}=\frac{\sqrt{2}}{\pi} \int_{\omega_{a}}^{\omega_{b}}\left[x_{1}(\Omega, t)+i x_{2}(\Omega, t)\right] d \Omega . \tag{30}
\end{equation*}
$$

|  | ISRA (India) | $=6.317$ | SIS (USA) | $=0.912$ | ICV (Poland) | $=6.630$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Impact Factor: | ISI (Dubai, UAE) $=1.582$ | PИHL (Russia) $=\mathbf{0 . 1 2 6}$ | PIF (India) | $=1.940$ |  |  |
|  | GIF (Australia) | $=0.564$ | ESJI (KZ) | $=9.035$ | IBI (India) | $=4.260$ |
|  | JIF | $=1.500$ | SJIF (Morocco) $=7.184$ | OAJI (USA) | $=0.350$ |  |

the integral using the Romberg method, you have to repeatedly calculate the integrand. The inverse Fourier transform for some image, the original of which is known in advance, showed that with the integration step length of 0.01 , the error of the procedure does not exceed $0.3-0.5 \%$. For a system without damping, the first resonant velocity must first be determined by constructing dispersion curves for a different number of waves in the circumferential direction.

## 5. Numerical results.

Calculations are carried out for a steel shell interacting with a water layer.

The following parameter values were assumed:

$$
\begin{gathered}
k=\sqrt{2 / 3}, k=0.005, \epsilon=0.45, v_{1}=0.25, a=1.0 \\
\rho_{0}^{*}=0.13, \mathrm{c}_{0}=0.1, \mathrm{M}=1.66
\end{gathered}
$$

Table 1

| $\theta$ |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $l$ |  |  |  |  |  |  |  |  |  |  |  |
|  | 0 | $\frac{\pi}{10 l}$ | $\frac{\pi}{5 l}$ | $\frac{3 \pi}{10 l}$ | $\frac{2 \pi}{5 l}$ | $\frac{\pi}{2 l}$ | $\frac{3 \pi}{5 l}$ | $\frac{7 \pi}{10 l}$ | $\frac{4 \pi}{5 l}$ | $\frac{9 \pi}{10 l}$ | $\frac{\pi}{l}$ |
| 2 | -9,96 | -7,64 | 3,53 | -3,43 | 1,92 | -1,63 | 0,41 | 0,019 | $-1,00$ | 1,12 | -1,54 |
| 4 | -6,12 | -6,19 | -3,04 | -0,39 | 1,29 | 0,08 | -1,47 | -0,61 | 1,21 | -0,11 | -1,70 |
| 6 | -4,75 | -4,18 | -2,91 | -1,72 | -0,92 | -0,31 | 0,25 | 0,47 | 0,04 | -0,74 | -1,13 |
| 8 | -2,95 | -2,44 | -2,74 | -1,78 | -1,03 | -0,59 | -0,53 | -0,42 | -0,21 | -0,02 | -0,07 |

As an example of a viscoelastic material, we take the three parametric relaxation kernels: $R_{\lambda}(t)=R_{\mu}(t)=A e^{-\beta t} / t^{1-\alpha}, v=0.25$,
$\mathrm{A}=\mathbf{0}, \mathbf{0 4 8} ; \boldsymbol{\beta}=\mathbf{0}, \mathbf{0 5} ; \boldsymbol{\alpha}=\mathbf{0}, \mathbf{1}$. All results are obtained in dimensionless parameters. In the table.1.the distribution over the angular coordinate of the liquid pressure on the shell $\mathbf{q}^{*}$ in the cross section $\boldsymbol{\eta}=\mathbf{0} .5$ for a different number of self-balanced forces is given.


Figure 1. The change in the imaginary part of the radial displacement as a function of the velocity at different viscosity parametrizes. $\mathrm{A}=0.01,2$. $\mathrm{A}=0.005,3$. $\mathrm{A}=0.02$, 4. $\mathrm{A}=0.015$. 5. $\mathrm{A}=0.017$

The table shows that the greatest fluid pressure is at $\boldsymbol{\theta}=\mathbf{0}^{\mathbf{0}}$. With increasing angles of pressure, the moving load exerted on the shell fits. The change in the imaginary part $0 \leq \theta \leq \pi / l$ of the radial displacement depending on the speed, the moving pressure at different viscosity parameters is shown in Figure 1. It can be seen that the mixed shells decrease exponentially with increasing speed.

|  | ISRA (India) $=\mathbf{6 . 3 1 7}$ | SIS (USA) | $=\mathbf{0 . 9 1 2}$ | ICV (Poland) | $=\mathbf{6 . 6 3 0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Impact Factor: | ISI (Dubai, UAE) $=\mathbf{1 . 5 8 2}$ | PИHL (Russia) $=\mathbf{0 . 1 2 6}$ | PIF (India) | $=\mathbf{1 . 9 4 0}$ |  |
| GIF (Australia) | $=\mathbf{0 . 5 6 4}$ | ESJI (KZ) | $=9.035$ | IBI (India) | $=\mathbf{4 . 2 6 0}$ |
|  | $=1.500$ | SJIF (Morocco) $=\mathbf{7 . 1 8 4}$ | OAJI (USA) | $=\mathbf{0 . 3 5 0}$ |  |

## 6. Conclusion

1. A mathematical formulation and methods for solving the problem are proposed, when a driving fluid with a constant velocity is between a viscoelastic shell and a coaxial rigid cylindrical wall.
2. An algorithm has been developed for calculating improper integrals with high accuracy.
3. Taking into account the viscous properties of the shell does not increase the interleaving to $12-16 \%$. The mixed and force factories of the shell smoothly decrease with increasing fluid velocity.

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