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ON MULTIPLICATIVE K BANHATTI INDICES OF LINE GRAPHS

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Abstract

Let G = (V,E) be a connected graph. The multiplicative K Banhatti indices of G are defined as $B\Pi_*(G) = {}^{Q}{}_{ue}[d_G(u) * d_G(e)]$*, where* $*$ *is usual addition or multiplication and ue means that the vertex u and edge e are incident in G. In this paper, we compute the multiplicative K Banhatti indices of line graphs..*
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1 Introduction

By a graph, we mean a finite, undirected without loops and multiple edges. Let G be a connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(v)$ of a vertex v is the number of vertices adjacent to v. The edge connecting the vertices u and v will be denoted by uv. Let $d_G(e)$ denotes the degree of an edge e in G, which is defined by $d_G(e) = d_G(u) + d_G(v) - 2$ with $e = uv$. For definitions and notions, the reader may refer to [7].

A molecular graph is a graph such that its vertices correspond to the atoms and the edges to the bonds. In Chemical Science, the physico- chemical properties of chemical compounds are often modeled by means of molecular graph based structure descriptors, which are also referred to as topological indices, see [3].

The K Banhatti indices of G are defined as $B_*(G) = {}^{P}{}_{ue}[d_G(u) * d_G(e)]$, where $*$ is usual addition or multiplication and ue means that the vertex u and edge e are incident in G. If $*$ is addition, then first K Banhatti index $B_+(G) = B_1(G) = {}^{P}{}_{ue}[d_G(u) + d_G(e)]$, and if $*$ is multiplication, then the second K Banhatti index $B_\times(G) = B_2(G) = {}^{P}{}_{ue}[d_G(u) \times d_G(e)]$. The K Banhatti indices were introduced by Kulli in [8].

Todeschini et al. [12] proposed that multiplicative variants of molecular structure descriptors be considered. When this idea is applied to Zagreb indices, for example, in [2]. The multiplicative version of first and second K Banhatti indices were introduced by Kulli in [10] and [11] as follows.

The multiplicative K Banhatti indices of G are defined as $B\Pi_*(G) = \prod_{ue}[d_G(u) * d_G(e)]$, where $*$ is usual addition or multiplication and ue means that the vertex u and edge e are incident in G. If $*$ is addition, then first multiplicative K Banhatti index $B\Pi_+(G) = B\Pi_1(G) = \prod_{ue}[d_G(u) + d_G(e)]$, and if $*$ is multiplication, then the second multiplicative K Banhatti index $B\Pi_\times(G) = B\Pi_2(G) = \prod_{ue}[d_G(u)d_G(e)]$. Recently many other indices were studied, for example, in [4] and [9].

The Line graph $L(G)$ is the graph with vertex set $V(L) = E(G)$ and whose vertices correspond to the edges of G with two vertices being adjacent if and only if the corresponding edges in G have a vertex in common two. For more details, we refer to [5].

2 Results

Theorem 2.1 *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

(i) $B\Pi_1(L(G)) = [2(3r-4)]^{nr(r-1)}$,

(ii) $B\Pi_1(L(G)) = [4(r-1)(2r-3)]^{nr(r-1)}$.

Proof.

Let G be a r- regular graph with $n \geq 2$ vertices. By algebraic method, we have $|V(L(G))| = \frac{nr}{2}$ and $|E(L(G))| = \frac{nr}{2}(r-1)$. Since line graph of a r - regular graph is $(2r-2)$ - regular and $B\Pi_*(L(G)) = \prod_{ue}[d_{L(G)}(u) * d_{L(G)}(e)]$. Hence, we have the following cases:

Case 1. $B\Pi_+(L(G)) = B\Pi_1(L(G)) = \prod_{ue}[d_{L(G)}(u) + d_{L(G)}(e)]$

$$= \left[(2r-2+4r-6)^2\right]^{\frac{1}{2}nr(r-1)}$$

$$= [2(3r-4)]^{nr(r-1)}.$$

Case 2. $B\Pi_\times(L(G)) = B\Pi_2(L(G)) = \prod_{ue}[d_{L(G)}(u) \times d_{L(G)}(e)]$

$$= \left[[(2r-2)(4r-6)]^2\right]^{\frac{1}{2}nr(r-1)}$$

$$= [4(r-1)(2r-3)]^{nr(r-1)}.$$

Thus the result follows.

By above Theorem, we have the following result without proof.

Theorem 2.2 *Let G be a r- regular graph with $n \geq 2$ vertices. Then*

$$B\Pi_2(G)(L(G)) = \left[\frac{2(r-1)(2r-3)}{(3r-4)}\right]^{nr(r-1)} B\Pi_1(G)(L(G))$$
.

Corollary 2.3 *Let C_n be a cycle with $n \geq 3$ vertices. Then*

$B\Pi_1(L(C_n)) = B\Pi_2(L(C_n)) = 4^{2n}$.

Corollary 2.4 *Let K_n be a complete graph with $n \geq 3$ vertices. Then*

(i) $B\Pi 1(L(Kn)) = (6n - 14)n(n-1)(n-2)$,

(ii) $B\Pi_1(L(K_n)) = [4(n-2)(2n-5)]^{n(n-1)(n-2)}$.

Theorem 2.5 *Let P_n be a path with $n \geq 4$ vertices. Then*

(i) $B\Pi_1(L(P_n)) = 9 \times 2^{4n-14}$, *(ii)*

$B\Pi_1(L(P_n)) = 2^{4n-14}$.

Proof.

Let P_n be a path with $n \geq 4$ vertices. Since $L(P_n) \sim= P_{n-1}$. By algebraic method, we have $|V(L(P_n))| = n - 1$ and $|E(L(P_n))| = n - 2$. We have two partitions of the vertex set $V(L(P_n))$ as follows: $V_1 = \{v \in V(L(P_n)) : d_{L(Pn)}(v) = 1\}; |V_1| = 2$, and

$V_2 = \{v \in V(L(P_n)) : d_{L(Pn)}(v) = 2\}; |V_2| = n - 3$.

Also we have two partitions of the edge set $E(L(P_n))$ as follows:

$E_1 = \{uv \in E(L(P_n)) : d_{L(Pn)}(u) = 1, d_{L(Pn)}(v) = 2\}; |E_1| = 2$, and

$E_2 = \{uv \in E(L(P_n)) : d_{L(Pn)}(u) = d_{L(Pn)}(v) = 2\}; |E_2| = n - 4$.

Then $B\Pi*(L(Pn)) = \text{Q}ue[dL(Pn)(u) * dL(Pn)(e)]$

$$= \prod_{uv \in E_1} [dL(Pn)(u) * dL(Pn)(e)] + \prod_{uv \in E_2} [dL(Pn)(u) * dL(Pn)(e)]$$

We have the following two cases are arise:

Case 1. $B\Pi_+(L(P_n)) = B\Pi_1(L(P_n))$

$$= \prod_{uv \in E_1} [(1 + 1) \times (2 + 1)] \times \prod_{uv \in E_2} [(2 + 2) \times (2 + 2)]$$

$$= (2 \times 3)^2 \times (4 \times 4)^{n-4} = 9 \times 24n-14.$$

Case 2. $B\Pi_\times(L(P_n)) = B\Pi_2(L(P_n))$

$$= \prod_{uv \in E_1} [(1 \times 1) \times (2 \times 1)] \times \prod_{uv \in E_2} [(2 \times 2) \times (2 \times 2)]$$

$= (1 \times 2)^2 \times (4 \times 4)^{n-4} =$

$24n-14.$

Thus the result follows.

By above Theorem, we have the following result without proof.

Theorem 2.6 *Let P_n be a path with $n \geq 4$ vertices. Then*

$B\Pi_1(L(P_n)) = 9\, B\Pi_2(L(P_n))$.

Corollary 2.7 *Let C_n be a cycle and P_n be a path with $n \geq 4$ vertices.*

Then

(i) $B\Pi_1(L(P_n)) = 9 \times 2^{-14} B\Pi_1(L(C_n))$, *(ii)*

$B\Pi_2(L(P_n)) = 2^{-14}\, B\Pi_2(L(C_n))$.

Theorem 2.8 *Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. Then*

(i) $B\Pi 1(L(Kr,s)) = [3r + 3s - 8]rs(r+s-2)$,

(ii) $B\Pi_2(L(K_{r,s})) = [(r + s - 2)(2r + 2s - 6)]^{rs(r+s-2)}$.

Proof.

Let $K_{r,s}$ be a complete bipartite graph with $1 \leq r \leq s$ vertices. By algebraic method, we have $|V (L(K_{r,s}))| = rs$, and $|E(L(K_{r,s}))|$ =. Since line graph of complete bipartite graph $K_{r,s}$ is a $(r+s-2)$-regular graph and $B\Pi_*(L(K_{r,s})) =$

$Que[dL(Kr,s)(u) * dL(Kr,s)(e)]$. We have

(i) $B\Pi_+(L(K_{r,s})) = B\Pi_1(S(K_{r,s}))$

$$= \left[[(r+s-2) + (r+s-2+r+s-2-2)]^2\right]^{\frac{1}{2}rs(r+s-2)}$$

$$= [3r+3s-8]^{rs(r+s-2)}.$$

(ii) $B\Pi_\times(L(K_{r,s})) = B\Pi_2(L(K_{r,s}))$

$$= \left[[(r+s-2) \times (r+s-2+r+s-2-2)]^2\right]^{\frac{1}{2}rs(r+s-2)}$$

$$= [(r+s-2)(2r+2s-6)]^{rs(r+s-2)}.$$

The following results are immediate from above theorem.

Corollary 2.9 *Let $K_{1,s}$ be a star graph with $s \geq 1$ vertices. Then*

(i) $B\Pi_*(L(K_{1,s})) = B\Pi_*(K_s)$,

(ii) $B\Pi_1(L(K_{1,s})) = (3s - 5)^{s(s-1)}$,

(iii) $B\Pi_2(L(K_{1,s})) = [2(s - 1)(s - 2)]^{s(s-1)}$,

Corollary 2.10 *Let $K_{r,r}$ be a regular complete bipartite graph with $r \geq 2$ vertices. Then*

$$B\Pi_2(L(K_{r,r})) = \left[\frac{2(r-1)(2r-3)}{3r-4}\right]^{2r^2(r-1)} B\Pi_1(L(K_{r,r})).$$

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