A model for thermal conductivity of carbon nanotubes with ethylene glycol/water based nanofluids

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Received 25 April 2017; accepted 2 June 2017

Abstract:

Due to its unique thermal properties, carbon nanotubes (CNTs) have been used as additives in order to increase thermal conductivity and other mechanical properties of nanofluids. There have been many studies of thermal conductivity for single phase fluids containing CNTs; however, most commercial coolants are two-phase fluids, such as the mixture of ethylene glycol and water (E/W). Similarly, there are some models that can be used to predict thermal conductivity of single phase fluids containing CNTs but not yet as a model for thermal conductivity of the E/W solution containing CNTs. In this paper, we present a model to predict the thermal conductivity of CNTs nanofluids based on an E/W solution. The model is found to correctly predict trends observed in experimental data of V. Kumaresan, et al. with varying concentrations of CNTs in nanofluids.

Keywords: ***carbon nanotube, ethylene glycol, nanofluids, thermal conductivity, water.***

Classification number: ***2.1, 5.1***

Introduction

Research into thermal dissipation materials of high power electronic devices has been receiving special interest from scientists and technologists. Besides finding new materials and technologies to increase component density and processing speed of electronic and optoelectronic devices, it is very important to find new materials and appropriate configuration to accelerate the thermal dissipation [1].

In recent years, there are many approaches that can improve the cooling system's performance; the most feasible one being to enhance the heat transfer (dissipation) performance through a working fluid without modifying either its mechanical designs or its key components. Researchers have recently shown a lot of interest in the issue of nanofluid thermal properties [2]. The heat transfer performance of nanofluids has been found to be enhanced by adding solid nanoparticles, including metals (Cu, Au, Ag, Ni), metal oxides (Al_2O_3, CuO, Fe_2O_3, SiO_2, TiO_2), or ceramics (SiC, AlN, SiN) [3-6].

CNTs are one of the most valuable materials with high thermal conductivity (above 1,400 W/m.K compared to the thermal conductivity of Ag 419 W/m.K) [7-9]. Owing to their unique thermal properties, CNTs have been used as additives to increase the thermal conductivity and other mechanical properties of nanofluids [10-13].

So far, there have been many studies into the thermal conductivity of single phase fluids containing CNTs. However, most commercial coolants are two-phase fluids, such as the mixture of E/W. Similarly, there are some models used to predict the thermal conductivity of single phase fluids containing CNTs [14-31], but not yet a model for thermal conductivity of E/W solution containing CNTs.

In this work, we present a model for predicting the thermal conductivity of the CNT nanofluids based E/W solution, which takes into consideration the effects of size, volume fraction, and thermal conductivity of CNTs, as well as the properties of the base liquid. This model is found to correctly predict trends observed in the experimental data of V. Kumaresan, et al., with varying concentrations of CNTs in nanofluids.

The model

As we already know, CNT is a very good thermal conductor to be used in tubes, but also is a good insulator laterally for tube axis. On the other hand, CNT disperses nanofluids in all direction, randomly. Therefore, we need to replace the thermal conductivity property of CNT (k_{CNT}) with an effective thermal conductivity of CNT ($k_{eff-CNT}$) for all calculations. In the report [31], we calculated effective thermal conductivity of CNT ($k_{eff-CNT}$) as follows:

$$k_{eff-CNT} = \frac{1}{2} k_{CNT} \qquad (1)$$

This model considers three paths for heat to flow in an E/W solution containing CNTs, one through which the E molecules allows one through the W molecules and the other through

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the CNTs. The total heat transfer through nanofluid can be expressed as:

$$q = q_E + q_W + q_{CNT} \tag{2}$$

$$q = -k_E A_E \left(\frac{dT}{dx}\right)_E - k_W A_W \left(\frac{dT}{dx}\right)_W - k_{eff-CNT} A_{CNT} \left(\frac{dT}{dx}\right)_{CNT} \tag{3}$$

Where A, k, and (dT/dx) denote the heat transfer area, thermal conductivity, and temperature gradient of the respective media. Subscripts "*E*", "*W*" and "*CNT*" denote quantities corresponding to ethylene glycol, water and carbon nanotubes, respectively. The liquid medium and the CNTs are assumed to be in local thermal equilibrium at each location, which gives:

$$\left(\frac{dT}{dx}\right)_E = \left(\frac{dT}{dx}\right)_W = \left(\frac{dT}{dx}\right)_{CNT} = \left(\frac{dT}{dx}\right) \tag{4}$$

Thus, the equation (3) can be written as:

$$q = -\left(k_E A_E + k_W A_W + k_{eff-CNT} A_{CNT}\right)\left(\frac{dT}{dx}\right) \tag{5}$$

$$-k\left(A_E + A_W + A_{CNT}\right)\left(\frac{dT}{dx}\right) = -\left(k_E A_E + k_W A_W + k_{eff-CNT} A_{CNT}\right)\left(\frac{dT}{dx}\right) \tag{6}$$

$$k\left(A_E + A_W + A_{CNT}\right) = k_E A_E + k_W A_W + k_{eff-CNT} A_{CNT} \tag{7}$$

It is proposed that the ratio of heat transfer areas $A_E{:}A_W{:}A_{CNT}$ could be taken in proportion to the total surface areas of E molecules (S_E), W molecules (S_W), and nanotubes (S_{CNT}) per unit volume of the suspension. We take the E molecules, and W molecules to be spheres with radii of r_E, r_W and the CNTs to be cylinders with radii r_{CNT} and length L, respectively. The surface area and volume of the individual liquid molecules can be respectively calculated as:

$$s_E = 4\pi r_E^2 \tag{8}$$

$$v_E = \frac{4}{3}\pi r_E^3 \tag{9}$$

$$s_W = 4\pi r_W^2 \tag{10}$$

$$v_W = \frac{4}{3}\pi r_W^3 \tag{11}$$

Note that the two ends of the CNTs are hemispherical, and therefore the surface area and volume of the individual CNTs can be respectively calculated as:

$$s_{CNT} = 4\pi r_{CNT}^2 + 2\pi r_{CNT} L \tag{12}$$

$$v_{CNT} = \frac{4}{3}.\pi r_{CNT}^3 + \pi r_{CNT}^2 L \tag{13}$$

Total surface area can be calculated as the product of the number of particles and the surface area of those particles for each constituent. Denoting the fraction of the volume of the CNTs as ε_{CNT}, so that the volume fraction of the liquid is $(1 - \varepsilon_{CNT})$. Denoting the volume fraction of the E in based solution as ε_E, so the volume fraction of E in nanofluids as $(1 - \varepsilon_{CNT}).\varepsilon_E$ and the volume fraction of W in nanofluids as $(1 - \varepsilon_{CNT})(1 - \varepsilon_E)$. The number of particles for the three constituents can be calculated as, respectively:

$$n_E = \frac{(1-\varepsilon_{CNT})\varepsilon_E}{v_E} = \frac{(1-\varepsilon_{CNT})\varepsilon_E}{\frac{4}{3}\pi r_E^3} \tag{14}$$

$$n_W = \frac{(1-\varepsilon_{CNT})(1-\varepsilon_E)}{v_W} = \frac{(1-\varepsilon_{CNT})(1-\varepsilon_E)}{\frac{4}{3}\pi r_W^3} \tag{15}$$

$$n_{CNT} = \frac{\varepsilon_{CNT}}{v_{CNT}} = \frac{\varepsilon_{CNT}}{\frac{4}{3}.\pi r_{CNT}^3 + \pi r_{CNT}^2 L} \tag{16}$$

The corresponding surface areas of the E molecules are given by:

$$S_E = n_E.s_E = \frac{(1-\varepsilon_{CNT})\varepsilon_E}{\frac{4}{3}\pi r_E^3}.4\pi r_E^2 = 3\frac{(1-\varepsilon_{CNT})\varepsilon_E}{r_E} \tag{17}$$

The corresponding surface areas of the W molecules are given by:

$$S_W = n_W.s_W = \frac{(1-\varepsilon_{CNT})(1-\varepsilon_E)}{\frac{4}{3}\pi r_W^3}.4\pi r_W^2 = 3\frac{(1-\varepsilon_{CNT})(1-\varepsilon_E)}{r_W} \tag{18}$$

The corresponding surface areas of the CNT phase are given by:

$$S_{CNT} = n_{CNT}.s_{CNT} \tag{19}$$

$$S_{CNT} = 3\varepsilon_{CNT}\frac{4r_{CNT} + 2L}{4r_{CNT}^2 + 3r_{CNT}L} \tag{20}$$

$$S_{CNT} = 3\varepsilon_{CNT}\frac{4\frac{r_{CNT}}{L} + 2}{\left(4\frac{r_{CNT}}{L} + 3\right)r_{CNT}} \tag{21}$$

Note that the CNT length is very large compared to the CNT radii, thus:

$$\frac{r_{CNT}}{L} \approx 0 \tag{22}$$

From (21) and (22), S_{CNT} is expressed as:

$$S_{CNT} = \frac{2\varepsilon_{CNT}}{r_{CNT}} \tag{23}$$

Taking $A_E : A_W : A_{CNT} = S_E : S_W : S_{CNT}$, we obtain:

$$k\left(S_E + S_W + S_{CNT}\right) = k_E S_E + k_W S_W + k_{eff-CNT} S_{CNT} \qquad (24)$$

Substituting from equation (17), (18) and (23) into the expression for heat transfer rate in equation (24), we obtain:

$$k = \frac{k_E 3\frac{(1-\varepsilon_{CNT})\varepsilon_E}{r_E} + k_W 3\frac{(1-\varepsilon_{CNT})(1-\varepsilon_E)}{r_W} + k_{eff-CNT}\frac{2\varepsilon_{CNT}}{r_{CNT}}}{3\frac{(1-\varepsilon_{CNT})\varepsilon_E}{r_E} + 3\frac{(1-\varepsilon_{CNT})(1-\varepsilon_E)}{r_W} + \frac{2\varepsilon_{CNT}}{r_{CNT}}} \qquad (25)$$

$$k = \frac{\frac{\varepsilon_E k_E}{r_E} + \frac{(1-\varepsilon_E)k_W}{r_W} + \frac{2\varepsilon_{CNT}k_{eff-CNT}}{3(1-\varepsilon_{CNT})r_{CNT}}}{\frac{\varepsilon_E}{r_E} + \frac{(1-\varepsilon_E)}{r_W} + \frac{2\varepsilon_{CNT}}{3(1-\varepsilon_{CNT})r_{CNT}}} \qquad (26)$$

Note that ε < 10% in all experiments, $r_E << r_{CNT}$, $r_W << r_{CNT}$, from (26) we have:

$$k = \frac{\frac{\varepsilon_E k_E}{r_E} + \frac{(1-\varepsilon_E)k_W}{r_W} + \frac{2\varepsilon_{CNT}k_{eff-CNT}}{3(1-\varepsilon_{CNT})r_{CNT}}}{\frac{\varepsilon_E}{r_E} + \frac{(1-\varepsilon_E)}{r_W}} \qquad (27)$$

From (1) and (27), the effective thermal conductivity of CNT-nanofluids is expressed as:

$$k = \frac{\frac{\varepsilon_E .k_E}{r_E} + \frac{(1-\varepsilon_E).k_W}{r_W} + \frac{\varepsilon_{CNT}.k_{CNT}}{3(1-\varepsilon_{CNT})r_{CNT}}}{\frac{\varepsilon_E}{r_E} + \frac{(1-\varepsilon_E)}{r_W}} \qquad (28)$$

From (28), the enhancement of thermal conductivity of CNT-nanofluids is expressed as:

$$\Delta k = k - k_0 = \frac{\frac{\varepsilon_{CNT}.k_{CNT}}{3(1-\varepsilon_{CNT})r_{CNT}}}{\frac{\varepsilon_E}{r_E} + \frac{(1-\varepsilon_E)}{r_W}} \qquad (29)$$

From (29), the percent enhancement of thermal conductivity of CNT-nanofluids is expressed as:

$$\%\Delta k = 100\%.\frac{k-k_0}{k_0} = 100\%.\frac{\frac{\varepsilon_{CNT}.k_{CNT}}{3(1-\varepsilon_{CNT})r_{CNT}}}{\frac{\varepsilon_E .k_E}{r_E} + \frac{(1-\varepsilon_E).k_W}{r_W}} \qquad (30)$$

Validation of the model

In order to validate the model, we compared the experimental data of V. Kumaresan, et al. (2012) [32] with calculation data from our model. In the Kumaresan experiment, the average diameter of dispersed nanotubes is found to be 42.6 nm [32], therefore, the average radius of CNTs is r_{CNT} = 21.3 nm. In calculation from this, the radius of E molecule and W molecule are 0.12 nm and 0.1 nm, respectively; and the thermal conductivity of E and W are 0.26 W/mK and 0.6 W/mK, respectively [31].

In ref. [33], Li, et al. reported thermal conductivity of single-walled carbon nanotubes (SWCNTs) and multi-walled carbon nanotubes (MWCNTs) are 2,400 W/mK and 1,400 W/mK, respectively, measured using the non-contact Raman spectra shift method. So we choose the thermal conductivity of CNTs in this calculation is k_{CNT} = 1,400 W/mK. Fig. 1 shows that the model has correctly predicted the trends observed in the experimental data of V. Kumaresan, et al. [32].

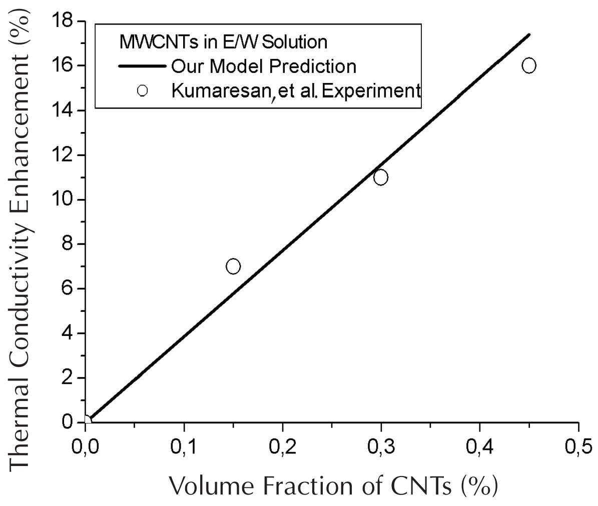

Fig. 1. The comparison between our model and the experimental data of V. Kumaresan, et al. in the case of dispersing MWCNTs in E/W solution.

Conclusions

We have developed a model for predicting the thermal conductivity of CNT nanofluids based E/W solution. Calculation results showed that the thermal conductivity of CNT nanofluids increased linearly with low volume concentration. The model was compared to experimental data of Kumaresan et al., and the result shows that the model correctly predicted the trends observed in experimental data. This model is close to the commercial coolant as well as has important implications for predicting the thermal conductivity of coolants based E/W containing CNTs for industrial applications.

ACKNOWLEDGEMENTS

The authors acknowledge financial support from Vietnam Academy of Science and Technology (Project No. VAST. TD.QP.03/17-19) and the Vietnam National Foundation for Science and Technology Development (Project No. 103.99-2015.70).

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