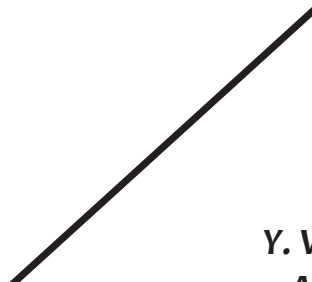


UDC classification: 330.3: 330.4

JEL Classification: C22, C51, C59, C61, D21

## Logistics system model optimization of an enterprise producing one type of goods



Y. V. Sherstenikov<sup>†</sup>,  
A. G. Yakovenko<sup>#</sup>,  
Y. S. Bortulov<sup>##</sup>

**Purpose** – to develop the logistics system (LS) model of an enterprise that allows performing joint model optimization of production capacity, retail network, and advertising campaign using J. Forrester's approach.

**Findings.** The paper elaborates on a numerical method for solving the optimization problem based on a system of equations in discrete time form. There we had the time behavior computations of all logistics system rates (production and supply), including the stocks' level in the wholesale and retail sales networks. The optimization problem of determining the maximum economic efficiency is formulated and solved.

**Theoretical implications.** The paper's research performed in the paper forms a methodological basis for the mathematical model development of various logistics systems.

**Practical implications.** The created model is applicable for optimizing or determining the logistics systems' optimal parameters.

**Originality/Value.** The authors' logistics system model is original. The model has no analogs in the scientific literature.

**Research limitations/Future research.** The main limitation is producing one type of product by an enterprise. Further research should reflect the product diversification possibility.

**Paper type** – theoretical.

**Keywords:** Logistics System, Production Capacity, Economic, and Mathematical Model.

**Paper type** – theoretical.

**Keywords:** logistics system; production capacity; economic and mathematical model.

<sup>†</sup>Yuriy V. Sherstenikov,  
doctor of economic science, the associated professor, the associated  
professor of chair of economic cybernetics  
Oles Honchar Dnipro National University, Dnipro, Ukraine,  
e-mail: [hm001@ukr.net](mailto:hm001@ukr.net),  
<https://orcid.org/0000-0003-0919-2013>

<sup>#</sup>Aleksandr G. Yakovenko,  
doctor of technical science, professor, professor of chair of economic cybernetics,  
Oles Honchar Dnipro National University, Dnipro, Ukraine,  
e-mail: [yakovenkoa@i.ua](mailto:yakovenkoa@i.ua),  
<https://orcid.org/0000-0003-1315-6322>

<sup>##</sup>Bortulov Yevhen S.,  
master of the first year education of the chair of economic cybernetics,  
Oles Honchar Dnipro National University, Dnipro, Ukraine,  
e-mail: [jenya.bortulyov@gmail.com](mailto:jenya.bortulyov@gmail.com),  
<https://orcid.org/0000-0001-8910-5113>

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## Модельна оптимізація логістичної системи підприємства, що випускає один вид продукції

Юрій Всеволодович Шерстенников<sup>‡</sup>,  
Олександр Григорович Яковенко<sup>‡</sup>,  
Євген Сергійович Бортульов<sup>‡</sup>

<sup>‡</sup>Дніпровський національний університет  
імені Олеся Гончара, Дніпро, Україна

**Мета роботи** – розробити модель логістичної системи (ЛС) підприємства, на підставі якої можна виконувати спільну модельну оптимізацію виробничої потужності, мережі роздрібної торгівлі й рекламної кампанії, застосовуючи підхід Дж. Форрестера

**Результати дослідження.** Розбудовано чисельний метод розв'язку проблеми оптимізації, заснований на системі рівнянь, записаних у формі з дискретним часом. Виконані обчислення часової поведінки всіх темпів логістичної системи (темпу виробництва, темпу поставки), включаючи поведінку рівня запасів в оптовому складі й у роздрібному продажі. Оптимізаційна проблема визначення максимальної економічної ефективності сформульована й вирішена.

**Теоретичне значення дослідження.** Цим дослідженням створено методологічну базу для розробки математичних моделей різноманітних логістичних систем.

**Практичне значення дослідження.** Створена модель може бути застосована для оптимізації або визначення оптимальних параметрів логістичних систем.

**Оригінальність/Цінність/Наукова новизна дослідження.** Модель логістичної системи, яка запропонована авторами, є оригінальною. У цієї моделі немає аналогів у науковій літературі.

**Обмеження дослідження/Перспективи подальших досліджень.** Основне обмеження – це випуск підприємством одного виду продукції. В подальших дослідженнях має бути відображена можливість диверсифікації видів продукції.

**Тип статті** – теоретичний.

**Ключові слова:** логістична система; виробничі потужності; економіко-математична модель.

## Модельная оптимизация логистической системы предприятия, выпускающего один вид продукции

Юрий Всеволодович Шерстенников<sup>‡</sup>  
Александр Григорьевич Яковенко<sup>‡</sup>,  
Евгений Сергеевич Бортульов<sup>‡</sup>

<sup>‡</sup>Днепропетровский национальный университет  
имени Олеся Гончара, Днепр, Украина

**Цель работы** – разработать модель логистической системы (ЛС) предприятия, позволяющей проводить совместную оптимизацию модели производственных мощностей, розничной сети и рекламной кампании, используя подход Дж. Форрестера.

**Результаты исследования.** Разработан численный метод решения задачи оптимизации, основанный на системе уравнений в дискретной временной форме. Выполнены расчеты динамики всех показателей логистической системы (производства и поставок), включая поведение уровня запасов в оптовых и розничных торговых сетях. Сформулирована и решена оптимизационная задача определения максимальной экономической эффективности.

**Теоретическое значение исследования.** Этим исследованием создана методологическая основа для разработки математических моделей различных логистических систем.

**Практическое значение исследования.** Созданная модель применима для оптимизации или определения оптимальных параметров логистических систем.

**Оригинальность/Ценность/Научная новизна исследования.** Авторская модель логистической системы оригинальна. Модель не имеет аналогов в научной литературе.

**Ограничения исследований/Перспективы дальнейших исследований.** Основное ограничение – производство одного типа продукции на предприятии. Дальнейшие исследования должны отражать возможность диверсификации продукции.

**Тип статьи** – теоретический.

**Ключевые слова:** логистическая система; производственные мощности; экономико-математическая модель.

## 1. Introduction

This paper developed the interdependent model optimization system of production capacity, retail sales network, and product advertising. The presented algorithm of model computations allows performing coordinated production capacity optimization, retail sales network, and the enterprise advertising depending on the logistics system's main parameters. The main focus is on the retail sales network as an essential link that largely determines goods sales.

The papers (Wang, Chen, Chen, 2008; Kelly, Kerr, Drennan, 2010) presented "the fuzzy structure ranking method of preferences to evaluate enrichment" (PROMETHEE fuzzy method) that allowed potential suppliers evaluation by several criteria.

Several papers offer some models of advertising campaigns. The paper (Xiuli, Ashutosh, Suresh, 2009) deals with joint advertising for the manufacturer and retail sales and prices in a dynamic stochastic supply chain. The authors developed the supply chain problem model as a stochastic differential Stackelberg game.

The paper (Du, Xu, 2012) builds a hybrid pricing strategy based on CPM and CPC variables. Subsequently, a new multi-purpose model for solving web advertising resources based on a hybrid pricing strategy is put forward.

Recently, the model approach has actively been applied in the economics of sociology. Notably, there is an influence modeling of some behavioral factors on the frequency of consumer purchases of green goods (Marques, Almeida, 2013). The next direction of optimization modeling refers to the problem solution of joint (cooperative) advertising at the manufacturer of a supply chain - retail sales. The papers (Xie, Neyret, 2009; Ching-Shih, Hsiao-Hua, Hui-Chiung, Chih-Ho, 2009) devoted to the joint strategy of the advertising campaign and the pricing; as the authors chose such areas as production and retail sales for the research.

The papers (Wei, & Choi, 2010; Kogler, Rauch, 2018) consider the mean-variance analysis. The average variance method allows the authors to solve the supply chains' coordination problem according to the wholesale pricing and profit distribution.

The paper aims to create a reference model of business processes for the supply chain, which would include the special conditions when moving perishable goods.

## 2. Objectives

The study aims to develop an economic and mathematical model of an enterprise's production activities, including the retail sales network, developed model application for joint production capacity optimization, retail sales network, and the enterprise advertising on the daily demanded goods.

## 3. The research methods and information sources

Many aspects of planning the current activities of enterprises are studied. For instance, Pedchenko (2011) proposed a dynamic model of market pricing and production, which allows determining the general patterns of production and technological specifics for the economic system evolution. The theoretical basis for creating a model is the balance relations that

combine the approaches of L. Walras and A. Marshall to describe the dynamics of prices and volumes of industrial goods in the one product market. The synthesized mathematical model is a system of two linear differential equations for determining the price and volume of goods in discrete time. For this dynamic system, the equilibrium position stability conditions we obtained with the corresponding parametric analysis.

In his paper, Voronov (1997) built and analyzed models of consumers' behavior of the one type of goods by simulation modeling methods, namely, agent modeling methods. Ferber's reactive agent model is used to determine the agents' behavior within the simulation model. In the reactive agents' models, a specific feature is the states and transitions' concept and behavior tools such as "stimulus-response."

Gvozdetska (2011): applies the system of quantitative and qualitative research methods in the management process (adaptive approach). The modeling of a real economic enterprise, including the outside environment, is performed. Probabilistic decision-making methods in an unstable market environment are widely used, as well as information systems, models, and methods with a single information base and capacity to adapt to changing conditions.

Meanwhile, the logistics systems models are actively developed and discussed (Mushunje, 2019). Many papers (Du et al., 2012; Guha et al., 2010; Marques et al., 2013;) offer different models of advertising campaigns.

Various aspects of logistics systems are actively studied in the modern scientific literature (Bortolinietal., 2015; Mihalovič, 2016; Morozova et al., 2013). The authors (Velychko & Velychko, 2017) presented a methodology for building logistics models in the individual market systems' management of enterprises according to the minimization and maximization criteria.

Those papers do not sufficiently disclose the quantitative relationship between the logistics system parameters of an enterprise and the consumer market's current features: the potential demand for goods and the goods consumption rate. This modern theory deficiency complicates the study of the advertising campaigns' impact on the enterprise's economic efficiency. Gorsky (1998) proposes the model that meets the formulated requirements from a fundamental perspective. This model allows considering the detailed features of the market. Nevertheless, the model has a significant deficiency, leading to unstable solutions in a wide range of parameters. Sherstennikov (2013) proposed the method to eliminate that deficiency. The method is based on averaging the sales rate and goods delivery over a while. Today, there are no effective methods to plan a real-time advertising campaign for an enterprise covering the enterprise logistics and market demand for goods. Some approach principles were introduced by Sherstennikov and Yakovenko (2019).

## 4. Results

Fig. 1 reflects the enterprise logistics system. Working in a competitive market requires the enterprise manager to expand the enterprise's market niche or maintain it at some acceptable level. One of the effective means is to conduct a periodic or permanent advertising campaign. Therefore, the model creation begins with a model description of the potential demand Q's advertising campaign's impact.

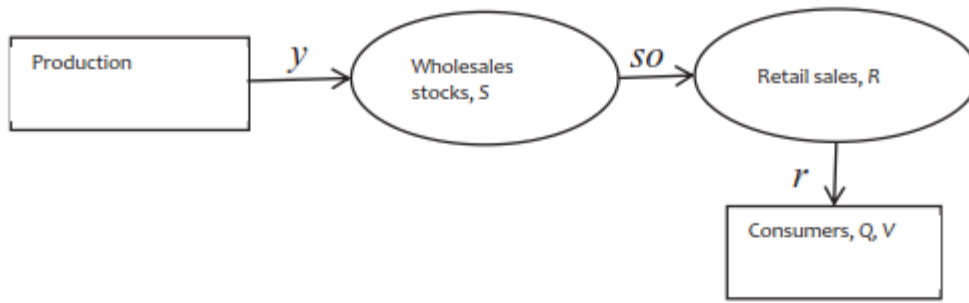


Fig. 1. The enterprise logistics system\*

\*Source: developed by the authors.

We assume that in each period  $t$  the costs of the for enterprise advertising are constant and equal to  $Zr$ . With such costs, some value of potential demand  $Qpr$  is achieved. We assume that the model of the first-order delay (Sherstennikov, 2013) describes the advertising campaign impact on the current potential demand  $Q$ :

$$dQ \frac{Qp(Zr) - Q(t)}{tr} \tag{1}$$

The potential demand maximum value  $Qpr$  depends on the advertising campaign cost. The potential demand maximum value  $Qp$  as the costs' function  $Zr$  reaches enrichment - this value at any cost can not exceed some maximum value

$$Qmax(Zr) = Qmax * (1 - \exp\{-\alpha * Zr\}), \tag{2}$$

wherein  $\alpha$  is a constant that depends on the market and the product in question.

The paper summarizes the approach of J. Forrester (Forrester, 1958; Forrester, 1971) that allows evaluating the market demand for goods.

The equation (1) means that the contribution to potential demand caused by the advertising campaign  $Q_i$  is described by the first-order delay model (Sherstennikov, 2013).

Let us formulate a system of equations that describe the enterprise's logistics system shown in Fig. 1. We believe that the enterprise is wholly provided with running costs. The system of equations determines the defining performance of the logistics system in the finite-difference form. That means that we consider the time to be discrete. Yakovenko (2017) considers general principles of economic dynamics applied below to a separate enterprise.

1. Demand change  $Q_i$  on the goods at the market is the enterprise's input impact, which aims to align its output with demand. Sales rate

$$r_{i+1} = rR_i * (Q_i - V_i) \tag{3}$$

wherein  $r_i$  is the goods sales rate (units/period) in the  $i$ -th period;  $n$  is the parameter, which is determined by the average number of sales for the previous quarter (or year);  $R_i$  is goods level in the retail sales network (RS N) in the  $i$ -th period;  $V_i$  is quantity of goods in consumers' hands (not yet consumed).

2. The level of the goods (goods quantity) in the retail sales network  $R_i$  determined by a recurrent equation:

$$R_{i+1} = R_i + R * (s0_i - r_i), \tag{4}$$

wherein  $s0_i$  is the supply rate (units/period) from the wholesale warehouse to RS N;  $Td$  is the model sampling period, the time interval between the adoption of solutions (choose  $Td(\text{day})$ ).

3. Level  $R_i$  must be within  $0 \leq R_i \leq R_m$ , wherein  $R_m$  is the goods' maximum possible level to RSN. This requirement is met by the following equation for the supply rate from the wholesale warehouse to RSN:

$$s0_{i+1} = s0, \tag{5}$$

wherein  $S_i$  is the goods stock level in the wholesale warehouse.

The paper substantiates the need to perform averaging when computing the proposed model:

$$\bar{s0}_i = (s0)_{i-ps}, \tag{6}$$

wherein  $ps$  is averaging time interval.

4. The production pace  $y_i$  is determined by the following equations:

$$y_{i+1} = (y_i + \frac{ym - y_i}{ty}) * S_i, \tag{7}$$

$$S = \begin{cases} 1, & \text{if } S_i < qS + Sm, \\ \frac{qS + Sm}{S_i} & \text{otherwise,} \end{cases} \tag{8}$$

wherein  $y_i$  is production capacity in  $i$ -the period;  $ym$  is the of production capacity planned value;  $Sm$  is the maximum level of goods in the wholesale warehouse. The equation avoids the overflow of the wholesale warehouse.

5. The goods stock level in the wholesale warehouse  $S_i$  computed as:

$$S_{i+1} = S_i + S * (y_i - s0_i), \tag{9}$$

wherein  $y_i$  is the flow rate, which is part of the wholesale production.

6. Adopted the following equation to determine the net profit:

$$M_i = (1 - kp) * [(1 - kad) * Mr_i - rc + -k * S_i - z * Rm - z1 + (Rm)^2 - qz * Z], \tag{10}$$

wherein  $c$  is the cost-share in the production cost;  $p$  is the unit price;  $k$  is payment for storage of the good unit for one period in a wholesale warehouse;  $kp$  is the income tax rate;  $kad$  is the value-added tax rate.

The equation for net income includes a quadratic dependence on the RSN maximum capacity, which under the contract is assigned to the manufacturer's goods. This dependence can occur for several reasons. For example, as the RSN outlets increase, the delivery distance increases, etc.

### 5. Discussion

Computed by model (1) – (10) we will perform at such values of parameters:

$$Rm0, qy = 100, Q = 1200, n = 0.0001, k1 = 0.33,$$

$$k = 0,01, So = 100, Sm = 200, Ro = 50, n1 = 011, So, 80 (11)$$

$$kp = 0,25, kad = 0,06, c = 0,6, p = 100z = 0,01, Sm.$$

The enterprise's management needs to solve the following two tasks. First (I) is to determine the optimal parameters of the logistics system. The second (II) is to bring the enterprise production capacity in line with current market demand.

Before proceeding to the formal solution of problems (I) and (II), make sure that the model (1) – (10) leads to meaningful results. At the transition from continuous time to discrete, we perform designations replacement  $Mt \rightarrow M_i$  (and for other quantities). Equation (2) at  $Qmaxi \alpha 2$  leads to the maximum value dependence of potential demand  $Qp$  from costs  $Zr$  of advertising campaign depicted in Fig. 2.

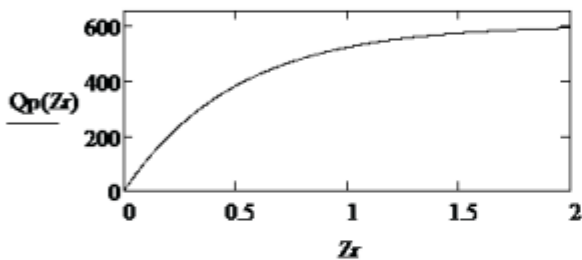


Fig. 2. The maximum value dependence of potential demand  $Qp$  from costs  $Zr$ (hrn/period) for an advertising campaign\*  
\*Source: developed by the authors.

Let the maximum enterprise productivity (see equation (7))  $ym = 4,6$ . Let us ask for some values of advertising costs -  $Zr = 0,9$ . Then  $Q(0,9) = 500,8$ , according to equation (1), we obtain the dependence shown in Fig. 3.

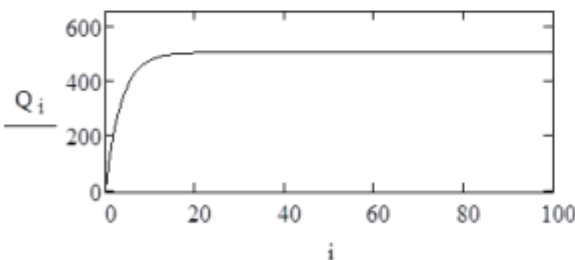


Fig.3. The current value of potential demand\*  
\*Source: developed by the authors.

Next, for the planning horizon,  $T \equiv im = 365$  using other model equations for the values of parameters (11), we obtain the results shown in Fig. 4 and Fig. 5.

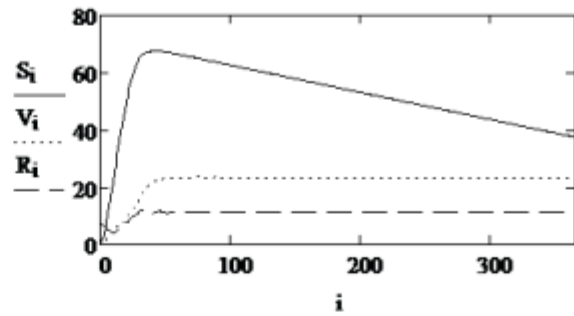


Fig. 4. Current quantities of goods in the wholesale warehouse  $S_i$ , in the retail network  $R_i$  and goods in the consumer's hands  $V_i$ \*  
\*Source: developed by the authors.

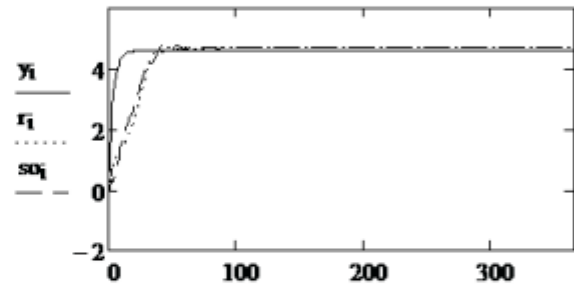


Fig. 5. Current values of the production rate  $y_i$ , sales  $r_i$ , and the delivery rate from the wholesale warehouse to retail sales  $so_i$ \*  
\*Source: developed by the authors.

Відставання темпу продажу й темпу перевезень від темпу виробництва в глині перших 20 періодів (днів) пов'язане з тим, що як видно з рис. 4 у ці періоди товар в основному надходить на оптовий склад а не в роздрібний продаж. Хоча в наступні періоди, як видно з рис. 5 темпи продажу й перевезень трохи перевершують темпи виробництва. З рис. 4 видно, що ця перевага досягається за рахунок зменшення запасів на оптовому складі. При цьому дотримується баланс товару: зроблений товар – реалізований товар = зміна кількості товару на всіх рівнях. Для нашого випадку розрахунки дають:

The sales lag rate and the delivery rate from the production rate during the first 20 periods (days) is due to the fact that, (see Fig. 4), during these periods the goods are mainly delivered to the wholesale warehouse and not to retail sales. Although in subsequent periods, (see Fig. 5), rates of sale and delivery slightly exceed the production rates. Fig. 4 demonstrates that reducing stocks in the wholesale warehouse leads to this advantage. At the same time the goods balance is observed: the produced goods - the realized goods = change of goods quantity at all levels. For our case, computations are as following

$$\sum_{i=0}^{im-1} (y_i - r_i) = 36,463, (iR)_{im} - (S - R)_0 = 36,463,$$

i.e., the goods balance is performed.

Thus, we made sure of the model adequacy and can proceed to solve the set problems.

The optimal parameters determination of the logistics system. As the optimality criterion, we chose the total profit that the enterprise receives for the year:

$$F(Rm, ym, Z) = \sum_{i=1}^{365} M_i \rightarrow \max. \quad (12)$$

The optimization problem is as follows. Find the objective function (12) maximum for the following variation parameters:  $ym$  is the planned value of production capacity;  $Rm$  is the RSN,  $Z$  advertising costs in one period (each period has the same costs).

The constraints for the optimization problem (12) are the system of equations (1) - (9).

Previous experience shows that the financial result ( $F$ ) significantly depends on the RSN's initial filling, i.e., the  $R_0$  value. For technical reasons,  $R_0$  can have the following three values  $R_0 = 10, 40, 60$ .

Therefore, the optimization problem (12) must be solved separately for three values  $R_0$ .

The optimization problem solution at  $R_0 = 10$  is as follows.

$$\begin{pmatrix} Rm_{opt} \\ y_{opt} \\ Z_{opt} \end{pmatrix} = \begin{pmatrix} 84,3 \\ 6,46 \\ 56 \end{pmatrix}. \quad (13)$$

The total profit is as follows

$$F(Rm_{opt}, y_{opt}, Z_{opt}) = 10311,7. \quad (14)$$

Fig. 6 illustrates the found optimal solution.

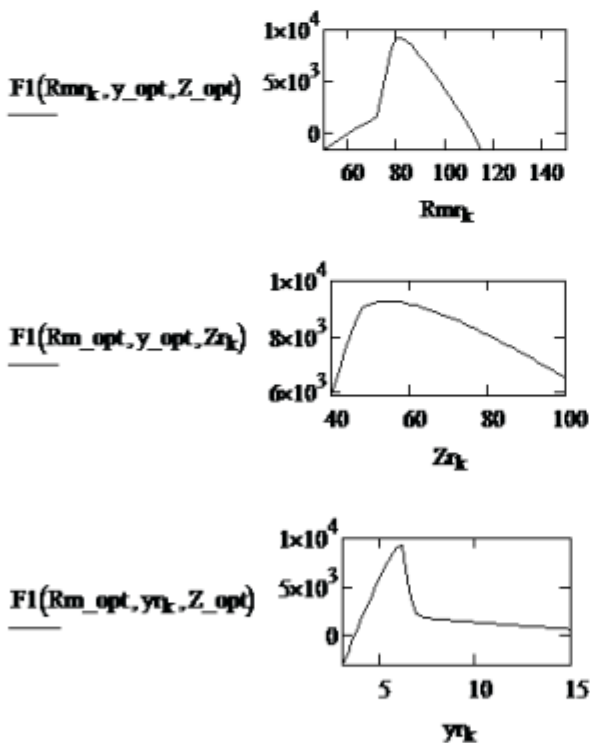


Fig. 6. Graphical demonstration of the optimal solution (12), (13) for  $R_0 = 10^*$

\*Source: developed by the authors.

Fig. 6 shows the changes in total profit on one of the parameters at fixed values of the other two in the case when the RSN initial filling is 10 (units of goods).

The solution to the optimization problem at  $R_0 = 40$  is as follows.

$$\begin{pmatrix} Rm_{opt} \\ y_{opt} \\ Z_{opt} \end{pmatrix} = \begin{pmatrix} 90 \\ 7,1 \\ 49 \end{pmatrix}.$$

with

$$F(Rm_{opt}, y_{opt}, Z_{opt}) = 14258,3. \quad (15)$$

Similarly, the optimization problem solution at  $R_0 = 60$  is as follows.

$$\begin{pmatrix} Rm_{opt} \\ y_{opt} \\ Z_{opt} \end{pmatrix} = \begin{pmatrix} 105 \\ 8,7 \\ 64 \end{pmatrix}. \quad (16)$$

with

$$F(Rm_{opt}, y_{opt}, Z_{opt}) = 16419,4. \quad (17)$$

For clarity, we present the results in tabular form:

**Table 1**

**Dependence of the enterprise's total profit for the year ( $F$ ) on the RSN initial completion ( $R_0$ )<sup>\*</sup>**

$R$	$F(Rm_{opt}, y_{opt}, Z_{opt})$
10	10311,7
40	14258,3
60	16419,4

\*Source: developed by the authors.

The results comparison from Table 1 illustrates a significant dependence of the financial result on the RSN initial filling. To understand this, we perform detailed dynamics computations of the leading indicators in the LS. Fig. 7 depicts the dynamics of the LS's main flows.

Fig. 7 shows that when  $R_0 = 60$  are the absolute values of all flows are far larger. So the total number of goods sold per year is:  $\sum r = 2286,3$  (for  $R_0 = 10$ ) and  $\sum r = 3119,9$  (for  $R_0 = 60$ ). This difference in the number of goods sold per year (see Fig. 8). We can observe the comparison of optimal solutions (13) and (17)  $R = 84,3$ , for  $R_0 = 10$ ,  $R = 105$  for  $R_0 = 60$ .

Moreover, Fig. 8 illustrates that RSN  $R_i$ 's current product values  $R_i$  are near the  $R$ 's corresponding maximum values.

However, as follows from equation (3), the current values of the sales rate  $r_i$  are determined by the  $R_i$  value (as one of the factors).

Another contribution to the total profit is due to the quantity of the goods in the wholesale warehouse  $S_i$  (Fig. 9).

Equation (10) demonstrates that the model adopted the following contractual payment form with the wholesale warehouse: the enterprise pays only for the quantity of the wholesale warehouse's actual goods. Therefore, a larger number of goods in the wholesale warehouse (for  $R_0 = 10$ ) requires higher storage costs. Let us compare the current profit of the enterprise at different values  $R_0$  (Fig. 10). The initial work period attracts our attention. At  $R_0 = 10$  at the beginning of work, there is a considerable period (of  $0 \leq i \leq 60$ ), where the current profit value takes significant negative values (up to -100). Formula (10) shows that the enterprise's current profit value is mainly determined by the sales ratio  $r_i$  and the production pace  $y_i$ . The initial rate comparison for the specified period is shown in Fig. 11. Fig. 11, including equation (10), explains the current enterprise profit behavior, demonstrated in Fig. 10.

It is bringing the enterprise production capacity under current market demand. We consider the problem of bringing the enterprise production capacity, which operates according to the

model (1) - (10) with the current market demand in the following two formulations:

A) the production rate exceeds the sales rate, but due to the advertising campaign, the demand for goods may be increased, and the sales pace becomes equal to the production pace;

B) in any (even the most intensive) advertising campaign, the sales rate cannot be increased so that it equals the production rate. In this case, one has to perform the optimal limitation of the production pace.



Fig. 7. Dynamics of main flows: left for  $R_0 = 10$ , right for  $R_0 = 60$

\*Source: developed by the authors.



Fig. 8. Dynamics of available stocks of goods in RSN: left for  $R_0 = 10$ , right for  $R_0 = 60$

\*Source: developed by the authors.



Fig. 9. Dynamics of available goods stocks in the wholesale warehouse  $S_i$ : left for  $R_0 = 10$ , right for  $R_0 = 60$

\*Source: developed by the authors.



Fig. 10. Dynamics of current enterprise profit: left for  $R_0 = 10$ , right for  $R_0 = 60$

\*Source: developed by the authors.



Fig. 11. Dynamics of sales  $r_i$  and the production pace  $y_i$ : left for  $R_0 = 10$ , right for  $R_0 = 60$

\*Source: developed by the authors..

If the advertising campaign lasts less than the duration of the project, the equation is as follows:

$$Q_i = ZR \left( QS_i - \frac{QS_i}{Zr} \right),$$

wherein  $ZR = \begin{cases} ZR & i < T_{ost} \\ 0 & \text{otherwise} \end{cases}$ ,  $T_{ost}$  is the stopping moment of the advertising campaign. The equation computations led to the result shown in Fig. 12.

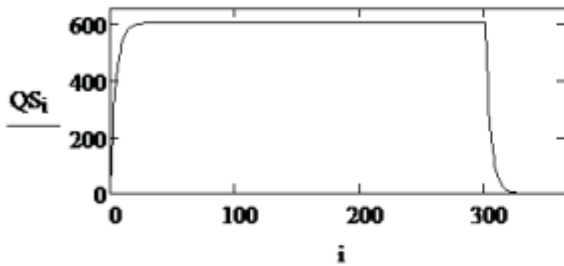


Fig. 12. Current demand dynamics, if the advertising campaign ends at  $T_{ost}$ '

\*Source: developed by the authors.

In this case, the results are shown in Fig. 13 and Fig. 14 replace the results shown in Fig. 8 and Fig. 9.

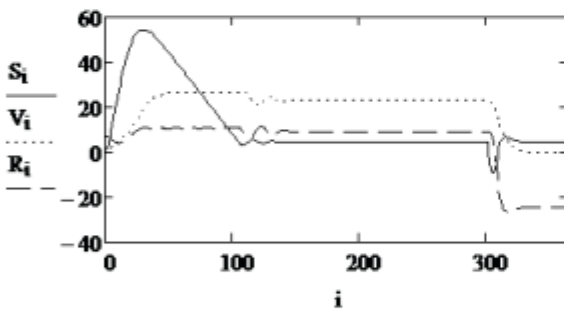


Fig. 13. Current quantities of goods in the wholesale warehouse  $S_i$ , in the retail sales network  $R_i$  and goods in the consumer's hands  $V_i$  at  $T_{ost}$ '

\*Source: developed by the authors..

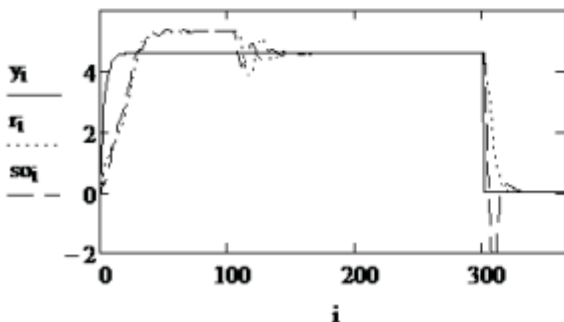


Fig. 14. Current values of production rate  $sy_i$ , sales  $r_i$ , and the delivery pace from wholesale to retail  $so_i$ '

\*Source: developed by the authors.

Task A) is reduced to the advertising campaign optimization at a given production pace. As a target function of the optimization problem, we take the profit received for the selected period  $T$  (horizon planning):

$$F_T(Zr) = \sum_{i=1}^T M_i \rightarrow \max. \quad (18)$$

The system of constraints for the optimization problem (18) is a system of model equations (1) - (10). Numerical methods must solve the optimization problem (18) under constraints (1) - (10). Given (10), we see that expression (18) for the objective function can be divided into two parts:

$$F_T(Zr) = F - (1 - kp)TZr, \quad (19)$$

wherein  $G$  in relation (2) has a similar (2) dependence on  $Zr$ , i.e.,  $G$  is the convex upwards function. We conclude that in order for the function,  $F_T(Zr)$  has a maximum at  $Zr$ ; the condition  $\left(\frac{dG(Zr)}{dZr}\right)_{Zr=0} > (1 - kp)dG$  must be met. If the condition  $\left(\frac{dG(Zr)}{dZr}\right)_{Zr=0} \leq (1 - kp)dG$  is met, then function  $F_T(Zr)$  has maximum at  $Zr$ . For  $T = 365$  the values of the parameters specified in (11) allow checking by numerical computations that the inequality is performed  $\left(\frac{dG(Zr)}{dZr}\right)_{Zr=0} > (1 - kp)dG$ ,  $5 \cdot 10^4 > 343,1$ . Thus, the function  $F_T(Zr)$  has a maximum of non-zero value  $Zr$ .

Numerical computations show that the function  $F_T(Zr)$  has a single maximum at  $(Zr)_{max} = 1,245$ , while the objective function reaches its maximum value  $[F_T(Zr)]_{max} = 1105$ .

Fig. 15 and Fig. 16 demonstrates the dynamics of economic features for an optimal solution.

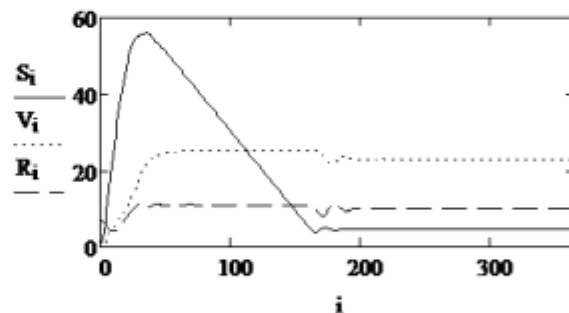


Fig. 15. The same as Fig. 13, but for an optimal solution'

\*Source: developed by the authors.

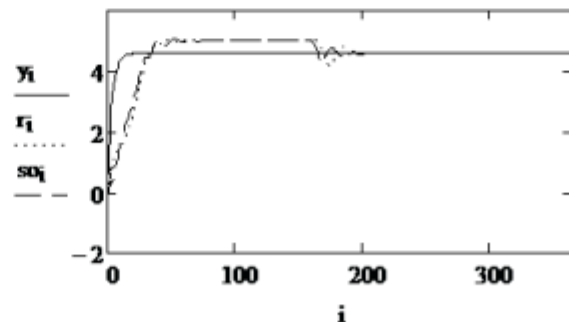


Fig. 16. The same as Fig. 14, but for an optimal solution'

\*Source: developed by the authors.



Let us move on to task B). Let now the enterprise maximum productivity  $\gamma m = 5,0$  (at constant other parameters). If all opportunities to increase demand are exhausted, then there is only a decrease in production. To do this, we apply the non-overcrowding criterion of available wholesale warehouses. In this case, it is necessary to adjust production capacity to market demand for goods. Then instead of equation (7), one needs the following equation, which has the finite-difference form:

$$y_{i+1} = \left( y_i + \frac{\gamma m - y_i}{\tau y} \right) \cdot \begin{cases} 1, & \text{if } S_i < S_e \\ \frac{S_e}{S_i}, & \text{otherwise} \end{cases} \quad (20)$$

wherein  $S_e$  permissible (close to the maximum) production level in the wholesale warehouse.

Now the model computations (1) - (10) (including the equation replacement (7) by (20)) leads to the results shown in Fig. 17 and Fig. 18.

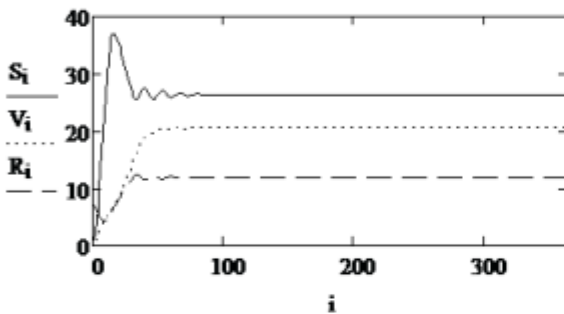


Fig. 17. The same as Fig. 13, but for enterprise maximum productivity  $\gamma m = 5,0$ \*

\*Source: developed by the authors.

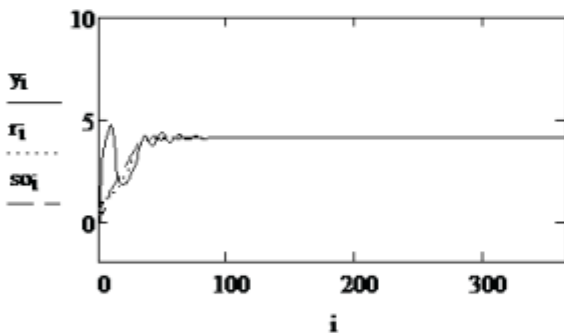


Fig. 18. The same as Fig. 14, but for enterprise maximum productivity  $\gamma m = 5,0$ \*

\*Source: developed by the authors.

Computations show that starting from the 80<sup>th</sup> period; the production rate fluctuates slightly in value  $y = 4,1$  ( $4,07 < y < 4,13$ ). In this case, the profit is 278,5 (see Table 2).

**Table 2**  
The profit value depending on the maximum enterprise productivity\*

$\gamma m$	$\sum M$	$\gamma_{cp}$ starting from the 80 <sup>th</sup> period
5,0	278,482	4,08
4,0	655,9	3,892
3,95	665,989	3,846
3,93	665,105	3,93
3,9	661,059	3,9
3,85	649,892	3,754
3,8	635,532	3,708

\*Source: developed by the authors.

The average production rate is set at 4,08 (starting from the 80<sup>th</sup> period). That means that 0,92 ( $5 - 4,08 = 0,92$ ) of enterprise capacity units are superfluous and can be involved in producing other production types. Table 2 reflects the optimal production capacity within  $3,93 < \gamma_{opt} < 3,95$ . When choosing the production capacity within those limits, the total profit for the period ( $T = 365$ ) will be around 666 (hrn per unit).

## 6. Conclusions

The developed enterprise logistics system model allows performing mutually agreed production capacity optimization, retail sales network, and advertising campaign of an enterprise, i.e., those links that directly determine the production and sale of goods, and by a closed system of equations determine the performance of all other parts of the logistics system. The proposed model allows an enterprise to plan its optimal advertising campaign. The model also allows computing the enterprise optimal production capacity when the market for this product is close to saturation, and the advertising campaign no longer completely solves the implementation problem.

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## 8. The competing interests

The authors declare that they have no competing interests.

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