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Pendant Edge Domination in Graphs

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Abstract: A dominating set of a graph G is said to be pendant dominating set if the induced subgraph contains at least one pendant vertex. A subset F of edges of G is said to be an edge dominating set if every edge not in F is incident to at least one edge in F. An edge dominating set is said to be pendant edge dominating set if an edge induced subgraph $\langle F\rangle$ contains an edge of degree one. In this article, we initiate the study of pendant edge domination of graph and compute exact values for some standard graphs.

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Keywords: Pendant Domination, Edge Domination, Pendant edge domination.

1. Introduction

The domination parameter in graph was one of the fundamental graph invariant that was extensively studied by many research scholars from various domains, since it was introduced. Many new parameters were defined and studied combining domination with other graph theoretical parameters. For a graph G, its line graph denoted by $L(G)$ is defined to be a graph whose vertex set will be the edge set of G and two vertices in $L(G)$ are adjacent if corresponding edges are adjacent in G. The dominating set of G and its line graph $L(G)$ are related in the sense that vertices dominated in $L(G)$ represents edges dominated in G. With this connection, the edge domination parameter was defined and studied for trees by Mitchell and Hedetniemi [1] in 1977. Later, the concept was well studied by S. R. Jayaram [2]. In a graph G, the number of edges incident to a vertex is called the degree of that vertex, denoted by $\deg v$.

Any vertex of degree zero is called an isolated vertex and a vertex of degree one is called a pendant vertex. An edge incident to a pendant vertex is called pendant edge. Suppose $e = uv \in E(G)$ be any edge in G, then $\deg(e) = \deg u + \deg v - 2$. $\delta'(G), \Delta'(G)$ respectively denotes minimum and maximum degree of an edge in G. An edge of degree zero is called an isolated edge. The set of all isolated edges in G is called edge independent set in G. The maximum cardinality of a an edge independent set is called edge independent edge domination number of G. $N(e)$, the set of all edges incident to e is called open neighborhood of e. The set $N(e)$ along with e is called closed neighborhood of e, denoted by $N[e]$. A subset of vertices is said to be dominating set if each vertex not in S is adjacent to some vertex in S. The minimum cardinality of a dominating set is called the domination number of G, denoted by $\gamma(G)$. A dominating set S is called a pendant dominating set if the induced subgraph of S contains at least one pendant vertex. The minimum cardinality of a pendant dominating set is called the pendant domination number, denoted by $\gamma_{pe}(G)$.

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A subset F of edges is said to be edge dominating set if any edge not in F is adjacent some edge in F. The minimum cardinality of an edge dominating set is called the edge domination number, denoted by $\gamma'(G)$. An edge dominating set F is said to be an independent edge dominating set if $\langle F\rangle$ is independent. The minimum cardinality of an independent edge dominating set is called the independent edge domination number, denoted by $i'(G)$.

Theorem 1.1 ([2]). *For a path of length k,*

$$\gamma'(P_k) = \begin{cases} \frac{k}{3}, & \text{if } k \equiv 0\,(mod\ 3); \\ \lfloor \frac{k+2}{3} \rfloor, & \text{if } k \equiv 1\,(mod\ 3); \\ \lfloor \frac{k+1}{3} \rfloor, & \text{if } k \equiv 2\,(mod\ 3); \end{cases}$$

Theorem 1.2 ([3]). *A dominating set S is a minimal pendant dominating set if and only if for each vertex $u \in S$ one of the following condition holds.*

(1). u is either an isolate or a pendant vertex of S.

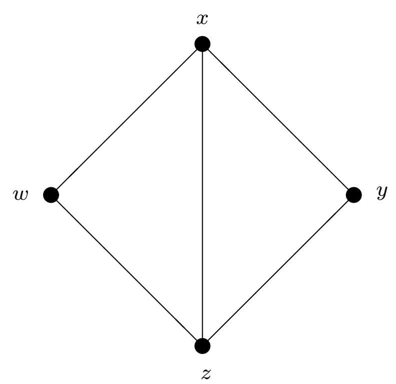
(2). each vertex of $S - \{u\}$ belongs to some cycle in G.

(3). there exists a vertex $v \in V - S$ for which $N(v) \cap S = \{u\}$.

2. Pendant Edge Domination number of a Graph

Consider a graph $G = (V, E)$ of order n and size m. A subset F of an edge set of G is called a pendant edge dominating set (PEDS, in short) if each edge not in F is incident to some edge in F and edge induced subgraph $\langle F\rangle$ contains an edge of degree one. The minimum number of edges in a PEDS of G is called the pendant edge domination number, denoted by $\gamma'_{pe}(G)$.

Example 2.1. *Consider a diamond graph as shown in figure 2. Clearly $F = \{(xy), (xz)\}$ will be an edge dominating set of G. Let $e = xy$, then degree of e in F is $\deg e = \deg x + \deg y = 1$. Hence, $\gamma'_{pe}(G) = 2$.*



Remark 1. *From the definition, it is clear that any PEDS must have cardinality at least two and so the parameter is defined only for graphs containing P_3. In other words, $\gamma'_{pe}(G)$ is defined only for connected graphs of order at least 3. For other graphs, we define $\gamma'_{pe}(G) = 0$.*
Further, it is easy to note that $E(G)$ is a unique maximum PEDS if graph contains a pendant edge or else there may be more than one PEDS for a given graph of cardinality $k < m$. A complete graph K_n contains m such PEDS of maximum cardinality. Throughout this article, for our convenience, by a graph we mean a connected graph of order at least 3, unless otherwise stated.

For any path graph on n vertices, it contains a path of length $m = n - 1$ and so we may re-write 1.1 as below. We also determine $\gamma'_{pe}(G)$ for standard family of graphs.

Proposition 2.2. *Let P_n be a path of order n, then $\gamma'(P_n) = \lceil \frac{n-1}{3} \rceil$.*

Proposition 2.3.

(1). Let G be a cycle of order $n \geq 3$, then $\gamma'_{pe}(G) = \lceil \frac{n+2}{3} \rceil$.

(2). Let G be a path of order $n \geq 2$, then $\gamma'_{pe}(G) = \lceil \frac{n+1}{3} \rceil$.

(3). Let $W_{1,n}$ be a wheel graph of order $n+1$, then $\gamma'_{pe}(W_{1,n}) = 1 + \lceil \frac{n}{3} \rceil$.

(4). For a complete bipartite graph $K_{m,n}, (m, n \geq 2)$, then $\gamma'_{pe}(G) = \min\{m, n\} + 1$.

(5). For a complete multi-partite graph $G \cong K_{m_1,m_2,\ldots m_k}$ such that $2 \leq m_1 \leq m_2 \leq \ldots \leq m_k$, then we have $\gamma'_{pe}(G) = \sum_{i=1}^{k-1} m_i$.

(6). For any two connected graphs G and H, $\gamma'_{pe}(G \cup H) = \gamma'_{pe}(G) + \gamma'_{pe}(H)$.

From the definition of line graph of a graph, it is clear that the pendant dominating set of $L(G)$ will be the pendant edge dominating set of G. Therefore we have the following characterization for minimal edge dominating set of G.

Theorem 2.4. *An edge dominating set F is a minimal pendant edge dominating set if and only if for each edge $e \in F$ one of the following condition holds.*

(1). e is either an isolate edge or a pendant edge of F.

(2). Each edge of $F - \{e\}$ belongs to some cycle in G.

(3). There exists an edge $f \in E - F$ for which $N(f) \cap F = \{e\}$.

Let F be an edge dominating set of G, then $E - F$ is also an edge dominating set, the same is not true for PEDS. For instance, consider a path P_5 of order 5. Then for any PEDS F of P_5, its complement $E(P_5) - F$ will be a disconnected graph which having no pendant edge. Hence, edge complement PEDS is not a PEDS.
We shall think of characterizing trees for which pedant edge domination number coincides with the edge domination number. Following theorem gives condition under which $\gamma'_{pe}(T) = \gamma'(T)$.

Theorem 2.5. *For any tree T, $\gamma'_{pe}(T) = \gamma'(T)$ if and only if there is a $\gamma'-$set which is edge independent in T.*

Theorem 2.6. *For any graph G of size m, we have $\gamma'_{pe}(G) \leq \lfloor \frac{m}{2} \rfloor + 1$. Equality holds if G is either a path P_3, P_4 or C_4 or union of these graphs.*

We shall have Nardhaus-Gaddum type results from the above theorem which stated in the next result.

Theorem 2.7. *For any connected graph of size m, we have*

(1). $\gamma'_{pe}(G) + \gamma'_{pe}(\overline{G}) \leq 2(\lfloor \frac{m}{2} \rfloor + 1)$.

(2). $\gamma'_{pe}(G).\gamma'_{pe}(\overline{G}) \leq 2(\lfloor \frac{m}{2} \rfloor + 1)^2 + m + 1$.

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