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# Certain Transformation Formulae for Basic Hypergeometric Series

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**Abstract:** In present paper we have taken certain known summation formulae due to Verma and Jain and by making use of Bailey's transformation an attempt has made to establish certain beautiful and interesting transformation formulae for q-Hypergeometric series.

**Keywords:** Basic Hypergeometric series, transformation, summation formulae.



## 1. Introduction

In 1947 W.N. Bailey [1] established the following result. If

$$\beta_n = \sum_{r=0}^{n} \alpha_r U_{n-r} V_{n+r} \tag{1}$$

$$\textit{and} \quad \gamma_n = \sum_{r=0}^{n} \delta_r U_{r-n} V_{r+n} = \sum_{r=0}^{n} \delta_{r+n} U_r V_{r+2n} \tag{2}$$

Then under suitable convergence conditions

$$\sum_{r=0}^{n} \alpha_n \gamma_n = \sum_{r=0}^{n} \beta_n \delta_n \tag{3}$$

Where $\alpha_r$, $\delta_r$, $U_r$ and $V_r$ are any functions of r only, such that the series $\gamma_r$ exists.

Making use of [3], Bailey developed a technique to obtain various transformation formulae for ordinary and q-series which play an important role in the number theory and transformation theory of hypergeometric series. Recently Singh [2], has obtained many transformation formulae for q-series by using baileys transformation and certain known results due to Verma and Jain [3]. In present paper, we have made to establish certain transformation formulae for q-hypergeometric series by using Baileys transformation and some known summation formulae due to Verma and Jain [3] and also by Verma [5].

## 2. Notation and Definitions

A generalized basic hypergeometric function in defined by L. J. Stater [4]; and Exton [6]; also by Srivastava and Karlson [7] is as under.

$${}_r\phi_s \left[ \begin{matrix} a_1, a_2, \ldots, a_r; q; z \\ b_1, b_2, \ldots, b_s; \quad q^i \end{matrix} \right] = \sum_{n=0}^{\infty} q^{i\left(\frac{m}{2}\right)} \frac{(a_1)_n (a_2)_n \ldots (a_r)_n Z^n}{(b_1)_n (b_2)_n \ldots (b_s)_n (q)_n} \tag{4}$$

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Valid for $|z| < 1$ provided no zeroes appears in denominator. Here $a_1, a_2, a_3, \ldots, a_r$ and $b_1, b_2, \ldots, b_s$ and Z are assumed to be complex numbers. The shifted factorial in defined by

$$(a;q)_n = \left\{ \begin{array}{ll} 1, & \text{if } n = 0 \\ (1-a)(1-aq)\ldots(1-aqn-1), & \text{if } n = 1, 2, \ldots \end{array} \right\} \tag{5}$$

And for real or complex q, $|q| < 1$, we have

$$(a;q)_\infty = \sum_{n=0}^{\infty}(1-aq^n)$$
$$\textit{and} \quad (a;q)_\infty = \frac{(a;q)_\infty}{(aq^\alpha;q)_\infty} \tag{6}$$

and

$${}_A\phi_B \left[ \begin{array}{l} (a); q; z \\ (b); i \end{array} \right] = \sum_{n=0}^{\infty} q^{in(n-1)/2} \frac{\prod_{J=1}^{A} (a_J;q)_n Z^n}{\prod_{J=1}^{B} (b_J;q)_n (q;q)_n} \tag{7}$$

in the special case when $i = 0$ the first member of (7) will be written simply as

$${}_A\phi_B \left[ \begin{array}{l} (a); q; z \\ (b) \end{array} \right]$$

We shall use the following known results to establish our transformations.

$${}_2\phi_1 \left[ \begin{array}{l} a, b; q; c/ab \\ c \end{array} \right] = \frac{(c/a;q)_\infty (c/b;q)_\infty}{(c/ab;q)_\infty (c;q)_\infty} \qquad \text{Slater [4];} \tag{8}$$

$${}_2\phi_1 \left[ \begin{array}{l} a, b; q; c/ab \\ cq \end{array} \right] = \frac{(cq/a;q)_\infty (cq/b;q)_\infty}{(cq;q)_\infty (cq/ab;q)_\infty} \times \left\{ \frac{ab(1+c)-(a+b)c}{ab-c} \right\} \qquad \text{Verma [5];} \tag{9}$$

$${}_4\phi_3 \left[ \begin{array}{l} a, q\sqrt{a}, -q\sqrt{a}; q - q^{n+1/2} \\ \sqrt{a}, -\sqrt{a}, aq^n \end{array} \right] = \left\{ \frac{(a;q)_n (q\sqrt{a};q)_n (-q^{-1/2};q)_n}{2(\sqrt{a};\sqrt{q})_{2n} (-\sqrt{a};q)_n} \right\} + \left\{ \frac{(a;q)_n (-q\sqrt{a};q)_n (-q^{-1/2};q)_n}{2(-\sqrt{a};\sqrt{q})_{2n} (\sqrt{a};q)_n} \right\} \text{Verma [3];} \tag{10}$$

$${}_3\phi_2 \left[ \begin{array}{l} a, q\sqrt{a}, q^{-n}; q; -q^n \\ \sqrt{a}, aq^n \end{array} \right] = \frac{1}{2} \left\{ \frac{(a;q)_n (-1;q)_n (q\sqrt{a};q)_n}{(\sqrt{a};\sqrt{q})_{2n} (-\sqrt{a}q;q)_n} \right\} + \left\{ \frac{(a;q)_n (-1;q)_n}{(a;q^2)_n} \right\} \qquad \text{Verma [3];} \tag{11}$$

## 3. Main Results

(i). Let us suppose

$$U_n = \frac{1}{(q;q)_n}, \qquad V_n = \frac{1}{(a;q)_n}, \qquad \alpha_n = \frac{(a, q\sqrt{a};q)_n}{(q, \sqrt{a};q)_n (q)^n}$$

and $\delta_n = (b,c;q) \left(\frac{a}{bc}\right)^n$ in (1) and (2), we get

$$\beta_n = \left[ \frac{1}{2} \left\{ \frac{(-1;q)_n (q\sqrt{a};q)_n}{\sqrt{a};\sqrt{q})_{2n} (-\sqrt{a}q;q)_n (q;q)_n} + \frac{(-1;q)_n}{(a;q^2)_n (q;q)_n} \right\} \right] \qquad \text{by using (10)}$$

$$\text{and } \gamma_n = \frac{\left(\frac{a}{b}, \frac{a}{c};q\right)_\infty}{\left(a, \frac{a}{bc};q\right)_\infty} \frac{(b,c;q)_n}{\left(\frac{a}{b}, \frac{a}{c};q\right)_n} \left(\frac{a}{bc}\right)^n \qquad \text{by using (8)}$$

Now putting these values of $\alpha_n$, $\beta_n$, $\gamma_n$ and $\delta_n$ in (3), we get the transformation.

$$ {}_4\phi_3\left[\begin{matrix} a, q\sqrt{a}, b, c; q; \frac{a}{bc} \\ \sqrt{a}, \frac{a}{b}, \frac{a}{c}; q \end{matrix}\right] = \frac{\left(a, \frac{a}{bc}; q\right)_\infty}{\left(\frac{a}{b}, \frac{a}{c}; q\right)_\infty} \times \left\{ \frac{1}{2}\, {}_4\phi_3\left[\begin{matrix} b, c, -1, q\sqrt{a}; q; \frac{a}{bc} \\ \sqrt{a}, \sqrt{a}q, -\sqrt{a}q \end{matrix}\right] + \frac{1}{2}\, {}_3\phi_2\left[\begin{matrix} b, c, -1; q; \frac{a}{bc} \\ \sqrt{a}, -\sqrt{a} \end{matrix}\right] \right\} $$

(ii). Choosing

$$ U_n = \frac{1}{(q;q)_n}, \qquad V_n = \frac{1}{(a;q)_n}, \qquad \alpha_n = \frac{(a, q\sqrt{a}; q)_n}{(q, \sqrt{a}; q)_n} q^{n\frac{(n+1)}{2}} $$

and $\delta_n = (b, c; q)\left(\frac{a}{bc}\right)^n$ in (1) and (2), we get

$$ \beta_n = \left[\frac{1}{2}\left\{ \frac{(-1;q)_n (q\sqrt{a};q)_n}{\sqrt{a};\sqrt{q})_{2n}(-\sqrt{a}q;q)_n(q;q)_n} + \frac{(-1;q)_n}{(a;q^2)_n(q;q)_n} \right\}\right] \qquad \text{by using (11)} $$

$$ \text{and } \gamma_n = \frac{\left(\frac{a}{b}, \frac{a}{c}; q\right)_\infty}{\left(a, \frac{a}{bc}; q\right)_\infty} \frac{(b, c; q)_n}{\left(\frac{a}{b}, \frac{a}{c}; q\right)_n} \left(\frac{a}{bcq}\right)^n \times \frac{bcq\,(aq^{2n-1}) - aq^n(b+c)}{bcq - a} \qquad \text{by using (9)} $$

Now putting these values of $\alpha_n$, $\beta_n$, $\gamma_n$, $\nu_n$ and $\delta_n$ in (3), we get the transformation.

$$ \frac{\left(\frac{a}{c}, \frac{a}{b}; q\right)_\infty}{\left(a, \frac{a}{bc}; q\right)_\infty} \sum_{n=0}^{\infty} \frac{(a, q\sqrt{a}, b, c; q)_n}{(q, \sqrt{a}, \frac{a}{b}, \frac{a}{c}, q; q)_n} \left(\frac{a}{bc}\right)^n q^{n(n-3)} \times \frac{bcq\,(1 + aq^{2n-1}) - aq^n(b+c)}{bcq - a} $$
$$ = \frac{1}{2}{}_4\phi_3\left[\begin{matrix} b, c, -1, q\sqrt{a}; q; \frac{a}{bcq} \\ \sqrt{a}, \sqrt{a}q, -\sqrt{a}q \end{matrix}\right] + \frac{1}{2}{}_3\phi_2\left[\begin{matrix} b, c, -1; q; \frac{a}{bcq} \\ \sqrt{a}, -\sqrt{a} \end{matrix}\right] $$

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