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Quadripartitioned Neutrosophic Mappings with its Relations and Quadripartitioned Neutrosophic Topology

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Abstract: The purpose of this paper is to construct a quadripartitioned neutrosophic mappings with its relations which is a generalization of single valued neutrosophic mapping. And also studied the properties of quadripartitioned single valued neutrosophic mappings. Further we introduce the concept of quadripartitioned single valued neutrosophic topological spaces and define the basic concepts with examples in detail.

MSC: 47A63.

Keywords: Quadripartitioned single valued neutrosophic mappings, Quadripartitioned single valued neutrosophic relations, Quadripartitioned single valued neutrosophic topology.

1. Introduction

In recent years many real life problems includes in the field of engineering, economics are consist of incomplete knowledge and it is necessary to solve those problems effectively based on the knowledge of Fuzzy set theory [23], Intuitionstic fuzzy set theory [2], Neutrosophic set theory [21]. These concept helps to deal the imperfect knowledge efficiently. Wang [22] introduced the concept of single valued neutrosophic set which is a generalization of classic set, fuzzy set, intuitionistic fuzzy set etc. Later Quadripartitioned Single valued neutrosophic set was introduced by Chatterjee [15] and it consist of four components namely truth, contradiction, unknown and falsity membership function in the real unit interval of [0,1].

The idea of a mapping is very common and it recently started to apply in many areas. Mapping plays an important role in computer science [19], formal logic [9], graph theory [8] and group theory [4] etc. Many authors applied the concept of mapping to fuzzy set [3, 7, 11, 12], intutionstic fuzzy set [20], single valued neutrosophic set [10] and studied its behaviour in their respective sets. Equivalence relations and mappings of fuzzy set was introduced by Lim [11]. Recently Single valued neutrosophic mappings defined by single valued neutrosophic relations studied by Abdelkrim Latreche [1] and the author focused to the concept related to its identity and composition of mappings. Further they also discussed about the continuity property in single valued neutrosophic topological space. Many authors studied the topology on fuzzy set [5, 13, 14], intuitionstic fuzzy set [6, 16], neutrosophic set [17, 18] and single valued neutrosophic set and defined its topological properties.

This motivates to study the concept of Quadripartitioned Neutrosophic mappings with its relations and its topology. In this

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paper Section 2 deals the basic definitions which helps to study the rest of the paper. In Section 3 we introduce the concept of quadripartitioned neutrosophic mappings with its relations and studied its properties with examples in detail. Section 4 introduces the concept of quadripartitioned single valued neutrosophic topology with its basic properties of open, closed, interior and closure sets. Section 5 concludes the paper.

2. Preliminaries

2.1. Quadripartitioned single valued neutrosophic sets

Definition 2.1 ([15]). *Let X be a non-empty set. A quadripartitioned single valued neutrosophic set (QSVNS) A over X characterizes each element in X by a truth-membership function $T_A(x)$, a contradiction membership function $C_A(x)$, an ignorance membership function $U_A(x)$ and a falsity membership function $F_A(x)$ such that for each $x \in X$, $T_A, C_A, U_A, F_A \in [0,1]$ and $0 \leq T_A(x) + C_A(x) + U_A(x) + F_A(x) \leq 4$ when X is discrete, A is represented as $A = \sum_{i=1}^{n} \langle T_A(x_i), C_A(x_i), U_A(x_i), F_A(x_i)\rangle / x_i, x_i \in X$.*

Definition 2.2 ([15]). *The complement of a QSVNS A is denoted by A^C and is defined as, $A^C = \sum\limits_{i=1}^{n} \langle F_A(x_i), U_A(x_i), C_A(x_i), T_A(x_i)\rangle / x_i, x_i \in X$ i.e., $T_{A^C}(x_i) = F_A(x_i)$, $C_{A^C}(x_i) = U_A(x_i), U_{A^C}(x_i) = C_A(x_i), F_{A^C}(x_i) = T_A(x_i), x_i \in X$.*

Definition 2.3 ([15]). *Consider two QSVNS A and B, over X. A is said to be contained in B, denoted by $A \subseteq B$ iff $T_A(x) \leq T_B(x), C_A(x) \leq C_B(x), U_A(x) \geq U_B(x)$, and $F_A(x) \geq F_B(x)$.*

Definition 2.4 ([15]). *The union of two QSVNS A and B is denoted by $A \cup B$ and is defined as, $A \cup B = \sum\limits_{i=1}^{n} \langle T_A(x_i) \vee T_B(x_i), C_A(x_i) \vee C_B(x_i), U_A(x_i) \wedge U_B(x_i), F_A(x_i) \wedge F_B(x_i)\rangle / x_i, x_i \in X$.*

Definition 2.5 ([15]). *The intersection of two QSVNS A and B is denoted by $A \cap B$ and is defined as, $A \cap B = \sum\limits_{i=1}^{n} \langle T_A(x_i) \wedge T_B(x_i), C_A(x_i) \wedge C_B(x_i), U_A(x_i) \vee U_B(x_i), F_A(x_i) \vee F_B(x_i)\rangle / x_i, x_i \in X$.*

Definition 2.6 ([1]). *Let A be a single valued neutrosophic set on X and B be a single valued neutrosophic set on Y, let $f : Supp\ A \to Supp\ B$ be an ordinary mapping and R be a single valued neutrosophic relation on $X \times Y$. Then f_R is called a single valued neutrosophic mapping if for all $(x, y) \in Supp\ A \times Supp\ B$ with the following conditions are satisfied:*

$$\mu_R(x,y) = \begin{cases} min(\mu_A(x), \mu_B(f(x)), & \text{if } y = f(x). \\ 0, & \text{otherwise.} \end{cases}$$

$$\sigma_R(x,y) = \begin{cases} min(\sigma_A(x), \sigma_B(f(x)), & \text{if } y = f(x). \\ 0, & \text{otherwise.} \end{cases}$$

$$\nu_R(x,y) = \begin{cases} max(\nu_A(x), \nu_B(f(x)), & \text{if } y = f(x). \\ 1, & \text{otherwise.} \end{cases}$$

Definition 2.7 ([1]). *Let A, B and C are a single valued neutrosophic sets on $X, Y$and Z respectively, let $f : Supp\ A \to Supp\ B$ and $g : Supp\ B \to Supp\ C$ are an ordinary mappings and R, S are a single valued neutrosophic relations on $X \times Y$ and $Y \times Z$ respectively. Then $(g \circ f)_T$ is called the composition of single valued neutrosophic mappings f_R and g_R such that $g \circ f : Supp\ A \to Supp\ C$ and the single valued neutrosophic relation T is defined by*

$$\mu_T(x,z) = sup_y(min(\mu_R(x,y), \mu_s(y,z)))$$

$$\sigma_T(x,z) = sup_y(min(\sigma_R(x,y), \sigma_s(y,z)))$$

$$\nu_T(x,z) = inf_y(max(\nu_R(x,y), \nu_s(y,z)))$$

for any $(x,z) \in Supp\ A \times Supp\ C$.

3. Quadripartitioned Neutrosophic Mappings with its Relations

This section deals about the concept of Quadripartitioned single valued neutrosophic mapping which is a generalization of single valued neutrosophic mapping. And also we discussed the properties of Quadripartitioned single valued neutrosophic mapping.

Definition 3.1. *Consider two QSVN sets* X, Y *on* U, V *respectively. Let* ψ *be a mapping from* $Supp\ X$ *to* $Supp\ Y$ *i.e.,* $\psi : Supp\ X \rightarrow Supp\ Y$ *and QSVN relation on* $U \times V$ *is denoted by* R*. Then* ψ_R *is known as QSVN mapping for each* $(u,v) \in Supp\ X \times Supp\ Y$ *with the following conditions.*

$$T_R(u,v) = \begin{cases} min(T_X(u), T_Y(\psi(u)), & if\ v = \psi(u). \\ 0, & otherwise. \end{cases}$$

$$C_R(u,v) = \begin{cases} min(C_X(u), C_Y(\psi(u)), & if\ v = \psi(u). \\ 0, & otherwise. \end{cases}$$

$$U_R(u,v) = \begin{cases} max(U_X(u), U_Y(\psi(u)), & if\ v = \psi(u). \\ 1, & otherwise. \end{cases}$$

$$F_R(u,v) = \begin{cases} max(F_X(u), F_Y(\psi(u)), & if\ v = \psi(u). \\ 1, & otherwise. \end{cases}$$

Example 3.2. *Let the universal sets* U *and* V *be* $U = \{p,q,r,s\}, V = \{a,b,c,d\}$*. Consider two QSVN sets* X, Y *belongs to* U, V *respectively.*

$$X = \{\langle p, 0.6, 0.7, 0.3, 0.2\rangle, \langle q, 0, 0.3, 0.2, 0.9\rangle, \langle r, 0.5, 0.3, 0.1, 0.7\rangle, \langle s, 0.2, 0.3, 0.4, 0.9\rangle\}$$

$$Y = \{\langle a, 0, 0.3, 0.7, 0.1\rangle, \langle b, 0.5, 0.2, 0.3, 0.4\rangle, \langle c, 0.1, 0.6, 0.7, 0.8\rangle, \langle d, 0.5, 0.6, 0.7, 0.3\rangle\}$$

Define QSVN mapping ψ_R *by*

$$\psi : \{p,r,s\} \rightarrow \{b,c,d\} \quad such\ that \quad \psi(p) = b, \psi(r) = c, \psi(s) = d.$$

Now QSVN relation R *defined by,*

$$T_R(p, \psi_{(p)}) = T_R(p,b) = T_X(p) \wedge T_Y(b) = 0.5$$

$$T_R(r, \psi_{(r)}) = T_R(r,c) = T_X(r) \wedge T_Y(c) = 0.1$$

$$T_R(s, \psi_{(s)}) = T_R(s,d) = T_X(s) \wedge T_Y(d) = 0.2$$

$$T_R(p,a) = T_R(p,c) = T_R(p,d) = 0$$

$$T_R(q,a) = T_R(q,b) = T_R(q,c) = T_R(q,d) = 0$$
$$T_R(r,a) = T_R(r,b) = T_R(r,d) = 0$$
$$T_R(s,a) = T_R(s,b) = T_R(s,c) = 0$$

In Similar manner we can find,

$$C_R(p,\psi_{(p)}) = 0.2, C_R(r,\psi_{(r)}) = 0.3, C_R(s,\psi_{(s)}) = 0.3$$
$$C_R(p,a) = C_R(p,c) = C_R(p,d) = 0$$
$$C_R(q,a) = C_R(q,b) = C_R(q,c) = C_R(q,d) = 0$$
$$C_R(r,a) = C_R(r,b) = C_R(r,d) = 0$$
$$C_R(s,a) = C_R(s,b) = C_R(s,c) = 0$$
$$U_R(p,\psi_{(p)}) = 0.3, U_R(r,\psi_{(r)}) = 0.7, U_R(s,\psi_{(s)}) = 0.7$$
$$U_R(p,a) = U_R(p,c) = U_R(p,d) = 1$$
$$U_R(q,a) = U_R(q,b) = U_R(q,c) = U_R(q,d) = 1$$
$$U_R(r,a) = U_R(r,b) = U_R(r,d) = 1$$
$$U_R(s,a) = U_R(s,b) = U_R(s,c) = 1$$
$$F_R(p,\psi_{(p)}) = 0.4, F_R(r,\psi_{(r)}) = 0.8, F_R(s,\psi_{(s)}) = 0.9$$
$$F_R(p,a) = F_R(p,c) = F_R(p,d) = 1$$
$$F_R(q,a) = F_R(q,b) = F_R(q,c) = F_R(q,d) = 1$$
$$F_R(r,a) = F_R(r,b) = F_R(r,d) = 1$$
$$F_R(s,a) = F_R(s,b) = F_R(s,c) = 1$$

Hence the relation,

$$\psi_R = \{\langle(p,\psi(p)),0.5,0.2,0.3,0.4\rangle, \langle(r,\psi(r)),0.1,0.3,0.7,0.8\rangle, \langle(s,\psi(s)),0.2,0.3,0.7,0.9\rangle, \langle(p,a),0,0,1,1\rangle, \langle(p,c),0,0,1,1\rangle,$$
$$\langle(p,d),0,0,1,1\rangle, \langle(q,a),0,0,1,1\rangle, \langle(q,b),0,0,1,1\rangle, \langle(q,c),0,0,1,1\rangle, \langle(q,d),0,0,1,1\rangle, \langle(r,a),0,0,1,1\rangle, \langle(r,b),0,0,1,1\rangle,$$
$$\langle(r,d),0,0,1,1\rangle, \langle(s,a),0,0,1,1\rangle, \langle(s,b),0,0,1,1\rangle, \langle(s,c),0,0,1,1\rangle\}$$

Hence ψ_R is a QSVN mapping.

Note: The following steps are needed to construct a QSVN mapping.

(i). Find Supp X and Supp Y.

(ii). Find ordinary mapping from Supp X to Supp Y.

(iii). Using Truth, Contradiction, Unknown and Falsity membership function find the QSVN relation.

(iv). Finally construct a QSVN mapping.

Definition 3.3. *Two QSVN mappings ψ_R and ϕ_T are equal if and only if $\psi = \phi$ and $R = T$ i.e.,*

$$(T_R(u,\psi(u)) = T_T(u,\phi(u)),$$

$$(C_R(u,\psi(u)) = C_T(u,\phi(u)),$$
$$(U_R(u,\psi(u)) = U_T(u,\phi(u)),$$
$$(F_R(u,\psi(u)) = F_T(u,\phi(u))$$

Definition 3.4. *Consider a QSVN set X on U and define an ordinary mapping $\psi : Supp\ X \to Supp\ Y$ such that $\psi(u) = u$ and R is a QSVNR on $U \times U$. Then the mapping ψ_R is known as QSVN identity mapping if $\forall\ u, v \in Supp\ X$ with the following conditions are satisfied.*

$$T_R(u,v) = \begin{cases} T_X(u), & \text{if } u = v\ . \\ 0, & \text{otherwise.} \end{cases}$$
$$C_R(u,v) = \begin{cases} C_X(u), & \text{if } u = v\ . \\ 0, & \text{otherwise.} \end{cases}$$
$$U_R(u,v) = \begin{cases} U_X(u), & \text{if } u = v\ . \\ 1, & \text{otherwise.} \end{cases}$$
$$F_R(u,v) = \begin{cases} F_X(u), & \text{if } u = v\ . \\ 1, & \text{otherwise.} \end{cases}$$

Definition 3.5. *Let X, Y, Z be QSVN sets on U, V and W respectively. Let ψ, ϕ be an ordinary mapping i.e., $\psi : Supp\ X \to Supp\ Y$, $\phi : Supp\ Y \to Supp\ Z$ and R, T be a QSVN relation on $U \times V$ and $V \times W$ respectively. Then the composition of QSVN mappings ψ_R and ϕ_T be $(\phi \circ \psi)_S$ such that $\phi \circ \psi : Supp\ X \to Supp\ Z$ and also QSVN relation S is defined by,*

$$T_S(u,w) = \bigvee_v (\wedge(T_R(u,v), T_T(v,w)))$$
$$C_S(u,w) = \bigvee_v (\wedge(C_R(u,v), C_T(v,w)))$$
$$U_S(u,w) = \bigwedge_v (\vee(U_R(u,v), U_T(v,w)))$$
$$F_S(u,w) = \bigwedge_v (\vee(F_R(u,v), F_T(v,w)))$$

for any $(u,w) \in Supp\ X \times Supp\ Z$.

Example 3.6. *Let the universal sets $U = \{p,q,r,s\}, V = \{a,b,c,d\}$ and $W = \{\alpha,\beta,\gamma,\delta\}$. Consider the QSVN sets X, Y, Z which belongs to U, V, W respectively i.e., $X \in QSVNS(U)$, $Y \in QSVNS(V)$, $Z \in QSVNS(W)$ and also it is given by,*

$$X = \{\langle p, 0.6, 0.7, 0.3, 0.2\rangle, \langle q, 0, 0.3, 0.2, 0.9\rangle, \langle r, 0.5, 0.3, 0.1, 0.7\rangle, \langle s, 0.2, 0.3, 0.4, 0.9\rangle\}$$
$$Y = \{\langle a, 0, 0.3, 0.7, 0.1\rangle, \langle b, 0.5, 0.2, 0.3, 0.4\rangle, \langle c, 0.1, 0.6, 0.7, 0.8\rangle, \langle d, 0.5, 0.6, 0.7, 0.3\rangle\}$$
$$Z = \{\langle \alpha, 0.2, 0.5, 0.3, 0.6\rangle, \langle \beta, 0.1, 0.4, 0.7, 0.5\rangle, \langle \gamma, 0.6, 0.4, 0.3, 0.1\rangle, \langle \delta, 0.2, 0.1, 0, 0.5\rangle\}$$

(i). Define QSVN ordinary mappings $\psi_R : X \to Y$ and $\phi_T : Y \to Z$ by, $\psi : Supp\ X \to Supp\ Y$ i.e. $\psi : \{p, r, s\} \to \{b, c, d\}$ such that $\psi(p) = b, \psi(r) = c, \psi(s) = d$ and $\phi : Supp\ Y \to Supp\ Z$ i.e., $\phi : \{b, c, d\} \to \{\alpha, \beta, \gamma\}$ such that $\phi(b) = \alpha, \phi(c) = \beta, \phi(d) = \gamma$.

ψ_R	**b**	**c**	**d**
p	(0.5,0.2,0.3,0.4)	(0,0,1,1)	(0,0,1,1)
r	(0,0,1,1)	(0.1,0.3,0.7,0.8)	(0,0,1,1)
s	(0,0,1,1)	(0,0,1,1)	(0.2,0.3,0.7,0.9)

(ii). Now QSVN relations R and T defined as follows:

The relation R defined here is same as in the previous Example 3.1 and hence we get,

In the same manner we get QSVN relation T by,

ϕ_R	α	β	γ
b	(0.2,0.2,0.3,0.6)	(0,0,1,1)	(0,0,1,1)
c	(0,0,1,1)	(0.1,0.4,0.7,0.8)	(0,0,1,1)
d	(0,0,1,1)	(0,0,1,1)	(0.5,0.4,0.7,0.3)

Now the composition $(\phi \circ \psi)_S$ is defined by an ordinary mapping $S : Supp\ X \to Supp\ Z$ and hence we get,

$(\phi \circ \psi)_S$	α	β	γ
p	(0.2,0.2,0.3,0.6)	(0,0,1,1)	(0,0,1,1)
r	(0,0,1,1)	(0.1,0.3,0.7,0.8)	(0,0,1,1)
s	(0,0,1,1)	(0,0,1,1)	(0.2,0.3,0.7,0.9)

Definition 3.7. *Let X and Y be two QSVN sets and $Z \subseteq X$. Consider ψ_R be a QSVN mapping from X to Y i.e., $\psi_R : X \to Y$. Then the image of Z by ψ_R is defined as,*

$$\psi_R(Z) = \{\langle v, T_{\psi_R(Z)}(v), C_{\psi_R(Z)}(v), U_{\psi_R(Z)}(v), F_{\psi_R(Z)}(v)\rangle | v \in V\},$$

where,

$$T_{\psi_R(Z)}(v) = \begin{cases} T_Y(v), & \text{if } v \in \psi(Supp(Z)) \ . \\ 0, & \text{otherwise.} \end{cases}$$

$$C_{\psi_R(Z)}(v) = \begin{cases} C_Y(v), & \text{if } v \in \psi(Supp(Z)) \ . \\ 0, & \text{otherwise.} \end{cases}$$

$$U_{\psi_R(Z)}(v) = \begin{cases} U_Y(v), & \text{if } v \in \psi(Supp(Z)) \ . \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\psi_R(Z)}(v) = \begin{cases} F_Y(v), & \text{if } v \in \psi(Supp(Z)) \ . \\ 1, & \text{otherwise.} \end{cases}$$

Suppose if $Z' \subseteq Y$ then the inverse image of Z' by ψ is defined as, $\psi_R^{-1}(Z) = \{\langle u, T_{\psi_R^{-1}(Z')}(u), C_{\psi_R^{-1}(Z')}(u), U_{\psi_R^{-1}(Z')}(u), F_{\psi_R^{-1}(Z')}(u), \rangle | u \in U\}$, where,

$$T_{\psi_R^{-1}(Z')}(u) = \begin{cases} T_X(u), & \text{if } u \in \psi^{-1}(Supp(Z')) \ . \\ 0, & \text{otherwise.} \end{cases}$$

$$C_{\psi_R^{-1}(Z')}(u) = \begin{cases} C_X(u), & \text{if } u \in \psi^{-1}(Supp(Z')) \ . \\ 0, & \text{otherwise.} \end{cases}$$

$$U_{\psi_R^{-1}(Z')}(u) = \begin{cases} U_X(u), & \text{if } u \in \psi^{-1}(Supp(Z^{'})) \ . \\ 1, & \text{otherwise.} \end{cases}$$

$$F_{\psi_R^{-1}(Z')}(u) = \begin{cases} F_X(u), & \text{if } u \in \psi^{-1}(Supp(Z^{'})) \ . \\ 1, & \text{otherwise.} \end{cases}$$

Definition 3.8. *Let X, Y be a QSVN set on U and V respectively. Then the product of two QSVN sets X and Y denoted by $X \times Y$ which is again a QSVN set on $U \times V$ and also it is defined by,*

$$T_{U \times V}(u, v) = \bigwedge\{T_X(u), T_Y(v)\}$$
$$C_{U \times V}(u, v) = \bigwedge\{C_X(u), C_Y(v)\}$$
$$U_{U \times V}(u, v) = \bigvee\{U_X(u), U_Y(v)\}$$
$$F_{U \times V}(u, v) = \bigvee\{F_X(u), F_Y(v)\}$$

Definition 3.9. *A QSVN i^{th} projection map written by $(proj_i)_R : X \times Y \to X$ by defining an ordinary mapping $proj_i$: $Supp(X \times Y) \to Supp(X)$ such that*

(i). $proj_i(u, v) = u$ for $(u, v) \in Supp(X \times Y)$,

(ii). A QSVN relation R defined by,

$$\begin{aligned} T_R((u, v), proj_i(u, v)) &= \bigwedge\{T_{X \times Y}(u, v), T_X(proj_i(u, v))\} \\ &= \bigwedge\{T_X(u), T_Y(v), T_X(u)\} \\ &= \bigwedge\{T_X(u), T_Y(v)\} \\ C_R((u, v), proj_i(u, v)) &= \bigwedge\{C_{X \times Y}(u, v), C_X(proj_i(u, v))\} \\ &= \bigwedge\{C_X(u), C_Y(v), C_X(u)\} \\ &= \bigwedge\{C_X(u), C_Y(v)\} \\ U_R((u, v), proj_i(u, v)) &= \bigvee\{U_{X \times Y}(u, v), U_X(proj_i(u, v))\} \\ &= \bigvee\{U_X(u), U_Y(v), U_X(u)\} \\ &= \bigvee\{U_X(u), U_Y(v)\} \\ F_R((u, v), proj_i(u, v)) &= \bigvee\{F_{X \times Y}(u, v), F_X(proj_i(u, v))\} \\ &= \bigvee\{F_X(u), F_Y(v), F_X(u)\} \\ &= \bigvee\{F_X(u), F_Y(v)\} \end{aligned}$$

Note: It is similar to prove the j^{th} QSVN projection.

4. Quadripartitioned Single Valued Neutrosophic Topology

In this section we define a topology on Quadripartitioned Single valued neutrosophic set which is in known as QSVNT and also discuss its properties. Before to define first we will discuss about Quadripartitioned single valued neutrosophic sets 0 and 1 which is defined as follows.

$$0_Q = \{\langle x, 0, 0, 1, 1\rangle : x \in X\}$$

$$1_Q = \{\langle x, 1, 1, 0, 0\rangle : x \in X\}$$

Definition 4.1. *A Quadripartitioned single valued neutrosophic topology(QSVNT) on a non empty set X is a family τ_Q of QSVN sets in X which should satisfy the following conditions.*
(QT_1) $0_Q, 1_Q \in \tau_Q$
(QT_2) $F_1 \cap F_2 \in \tau_Q$ *for any* $F_1, F_2 \in \tau_Q$,
(QT_3) $\cup F_i \in \tau_Q$ *for every* $\{F_i : i \in J\} \subseteq \tau_Q$
Here the pair (X, τ_Q) is known as quadripartitioned single valued neutrosophic topological space(QSVNTS). All the elements of τ_Q are quadripartitioned neutrosophic open set(QNOS) in X. A QSVN set G is quadripartitioned neutrosophic closed set(QNCS) if and only if the complement of G is quadripartitioned neutrosophic open.

Example 4.2. *Let $X = \{u, v, w\}$ and $A, B, C, D \in QSVN(X)$*

$$A = \{\langle u, 0.6, 0.5, 0.3, 0.2\rangle, \langle v, 0.6, 0.5, 0.4, 0.2\rangle, \langle w, 0.3, 0.2, 0.1, 0.7\rangle\}$$
$$B = \{\langle u, 0.3, 0.8, 0.2, 0.7\rangle, \langle v, 0.4, 0.5, 0.6, 0.7\rangle, \langle w, 0.4, 0.5, 0.2, 0.1\rangle\}$$
$$C = \{\langle u, 0.6, 0.8, 0.2, 0.2\rangle, \langle v, 0.6, 0.5, 0.4, 0.2\rangle, \langle w, 0.4, 0.5, 0.1, 0.1\rangle\}$$
$$D = \{\langle u, 0.3, 0.5, 0.3, 0.7\rangle, \langle v, 0.4, 0.5, 0.6, 0.7\rangle, \langle w, 0.3, 0.2, 0.2, 0.7\rangle\}$$

Then the family $\tau_Q = \{0_Q, A, B, C, D, 1_Q\}$ in X is QSVNT on X.

Note: If any QSVN set is both open and closed then it is known as clopen sets. Here $0_Q, 1_Q$ are clopen sets.

Definition 4.3. *Let (X, τ_Q) be a QSVNTS and $A = \langle x, T_A(u), C_A(u), U_A(u), F_A(u)\rangle$ be QSVN set in X. Then quadripartitioned single valued neutrosophic closure(QCl(A) in short) and quadripartitioned single valued neutrosophic interior(QInt(A) in short) of A are defined by,*

$$QCl(A) = \cap\{E \;:\; E \text{ is a QNCS in } X \text{ and } A \;\subseteq\; E\}$$
$$QInt(A) = \cup\{H \;:\; H \text{ is a QNOS in } X \text{ and } H \;\subseteq\; A\}$$

Here $QCl(A)$, $QInt(A)$ are QNCS and QNOS respectively in X. Further,

$$A \text{ is QNOS if and only if } A = QInt(A)$$
$$A \text{ is QNCS if and only if } A = QCl(A)$$

Example 4.4. *Consider the Example 4.2 and its QSVNTS τ_Q and if*

$$G = \{\langle u, 0.6, 0.9, 0.2, 0.1\rangle, \langle v, 0.7, 0.5, 0.3, 0.1\rangle, \langle w, 0.5, 0.6, 0.1, 0.1\rangle\}$$

Then,

$$QInt(G) = \{\langle u, 0.6, 0.8, 0.2, 0.2\rangle, \langle v, 0.6, 0.5, 0.4, 0.2\rangle, \langle w, 0.4, 0.5, 0.1, 0.1\rangle\}; \quad QCl(G) = 1_Q,$$

Here $QInt(G)$, $QCl(G)$ are QNOS and QNCS respectively in X.

Proposition 4.5. *For any QSVN set A in (X, τ_Q) we have,* *(a).* $QCl(\overline{\mathrm{A}}) = \overline{\mathrm{QInt(A)}}$

(b). $QInt(\overline{\mathrm{A}}) = \overline{\mathrm{QCl(A)}}$

Proof. (a). Consider a QSVN set A denoted by, $A = \{\langle x, T_A, C_A, U_A, F_A\rangle : x \in X\}$ and denote the family of QSVN subsets contained in A are indexed by the family,

$$A = \{\langle x, T_{F_i}, C_{F_i}, U_{F_i}, F_{F_i}\rangle : x \in X\}$$

Then we get,

$$QInt(A) = \{\langle x, \vee T_{F_i}, \vee C_{F_i}, \wedge U_{F_i}, \wedge F_{F_i}\rangle : x \in X\}$$

and hence

$$\overline{\mathrm{QInt(A)}} = \{\langle x, \wedge F_{F_i}, \wedge U_{F_i}, \vee C_{F_i}, \vee T_{F_i}\rangle : x \in X\}$$

Since, $T_{F_i} \leq T_A, C_{F_i} \leq C_A, U_{F_i} \geq U_A$ and $F_{F_i} \geq F_A$ for each $i \in J$ and the complement of family of QSVN sets A denoted by,

$$\overline{\mathrm{A}} = \{\langle x, F_{F_i}, U_{F_i}, C_{F_i}, T_{F_i}\rangle : x \in X\}$$

we find that,

$$QCl(\overline{\mathrm{A}}) = \{\langle x, \wedge F_{F_i}, \wedge U_{F_i}, \vee C_{F_i}, \vee T_{F_i}\rangle : x \in \mathrm{X}\}$$

Hence we get, $\overline{\mathrm{QInt(A)}} = QCl(\overline{\mathrm{A}})$.

(b). It is similar to the proof of (a). □

Definition 4.6. *Let (X, τ_Q) be a QSVNTS and A is a QNOS and B is a QSVN set on QSVNTS i.e.,*

$$A = \{\langle x, T_A(x), C_A(x), U_A(x), F_A(x)\rangle : x \in X\}$$

$$B = \{\langle x, T_B(x), C_B(x), U_B(x), F_B(x)\rangle : x \in X\}$$

Then,

(a). A is known to be quadripartitioned neutrosophic regular open if and only if $A = QInt(QCl(A))$.

(b). If $B \in QNCS(X)$ then B is known as quadripartitioned neutrosophic regular close if and only if $A = QCl(QInt(A))$.

Definition 4.7. *A QSVN set A in a $QSVNTS(X, \tau_Q)$ is called,*

(i). Quadripartitioned neutrosophic semi-open set(QNSOS) if $A \subseteq QCl(QInt(A))$.

(ii). Quadripartitioned neutrosophic pre-open set(QNPOS) if $A \subseteq QInt(QCl(A))$.

(iii). Quadripartitioned neutrosophic α-open set(QNαOS) if $A \subseteq QInt(QCl(QInt(A)))$.

(iv). Quadripartitioned neutrosophic β-open set(QNβOS) if $A \subseteq QCl(QInt(QCl(A)))$.

A QSVN set A is called quadripartitioned neutrosophic semi-closed set(QNSCS), quadripartitioned neutrosophic pre-closed set(QNPCS), quadripartitioned neutrosophic α-closed set(QNαCS), quadripartitioned neutrosophic β-closed set(QNβCS) if its complement of A is QNSOS, QNPOS, QNαOS, QNβOS respectively.

Proposition 4.8. *Let (X, τ_Q) be a QSVNTS and A, B be a QSVN sets in X. Then the following properties hold:*

(a). $QInt(A) \subseteq A$.

(b). $A \subseteq QCl(A)$.

(c). $A \subseteq B \Rightarrow QInt(A) \subseteq QInt(B)$.

(d). $A \subseteq B \to QCl(A) \subseteq QCl(B)$.

(e). $QInt(QInt(A)) = QInt(A)$.

(f). $QCl(QCl(A)) = QCl(A)$.

(g). $QInt(A \cap B) = QInt(A) \cap QInt(B)$.

(h). $QCl(A \cup B) = QCl(A) \cup QCl(B)$.

(i). $QInt(1_Q) = 1_Q$.

(j). $QCl(0_Q) = 0_Q$.

Proof. The proof of $(a), (b)$ and (i) are straightforward. It is easy to prove the result (d) from (a) and Definition 4.2.
(g) From $QInt(A \cap B) \subseteq QInt(A)$ and $QInt(A \cap B) \subseteq QInt(B)$ we get, $QInt(A \cap B) \subseteq QInt(A) \cap QInt(B)$ by the result of $A \subseteq B, A \subseteq C \Rightarrow A \subseteq B \cap C$ where A, B, C are QSVN sets in X. Now from the fact of $QInt(A) \subseteq A$ and $QInt(B) \subseteq B$. We see that, $QInt(A) \cap QInt(B) \subseteq A \cap B$ and also $QInt(A) \cap QInt(B) \in \tau_Q$ we get, $QInt(A) \cap QInt(B) \subseteq QInt(A \cap B)$ which shows the required proof.
The rest can be proved easily from the previous results and the Proposition 4.1. □

5. Conclusion

In this paper we introduced the concept of quadripartitioned single valued neutrosophic mappings with its relations which is a generalization of single valued neutrosophic mappings. And also studied the definitions of quadripartitioned neutrosophic mappings with examples in detail. Finally we introduced the concept of quadripartitioned single valued neutroophic topological spaces and defined some of its main properties with examples.

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