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A Study on Complementedness in the Subgroup Lattices of 2×2 Matrices Over Z_{11}

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Abstract: In this paper, we verify the complementedness in the subgroup lattices of the group of 2×2 matrices over Z_{11}.

Keywords: Matrix group, subgroups, Lagrange's theorem, Lattice.

1. Introduction

Let $\mathcal{G} = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} : \ a, b, c, d \in Z_p, ad - bc \neq 0 \right\}$. Then $\mathcal{G}$ is a group under matrix multiplication modulo p. Let $G = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathcal{G} : ad - bc = 1 \right\}$. Then G is a subgroup of $\mathcal{G}$. We have, $o(\mathcal{G}) = p(p^2 - 1)(p - 1)$ [6] and $o(G) = p(p^2 - 1)$ [6]. In this paper we are going to the study about the complementedness in the subgroup lattice of the group of 2×2 matrices over Z_{11}.

2. Preliminaries

In this section we give the definition needed for the development of the paper.

Definition 2.1. *A partial order on a non-empty set P is a binary relation $\leq$ on P that is reflexive, anti-symmetric and transitive. The pair $(P, \leq)$ is called a partially ordered set or poset. A poset $(P, \leq)$ is totally ordered if every $x, y \in P$ are comparable, that is either $x \leq y$ or $y \leq x$. A non-empty subset S of P is a chain in P if S is totally ordered by $\leq$.*

Definition 2.2. *Let $(P, \leq)$ be a poset and let $S \subseteq P$. An upper bound of S is an element $x \in P$ for which $s \leq x$ for all $s \in S$. The least upper bound of S is called the supremum or join of S.A lower bound for S is an element $x \in P$ for which $x \leq s$ for all $s \in S$. The greatest lower bound of S is called the infimum or meet of S.*

Definition 2.3. *Poset $(P, \leq)$ is called a lattice if every pair x, y elements of P has a supremum and an infimum, which are denoted by $x \vee y$ and $x \wedge y$ respectively.*

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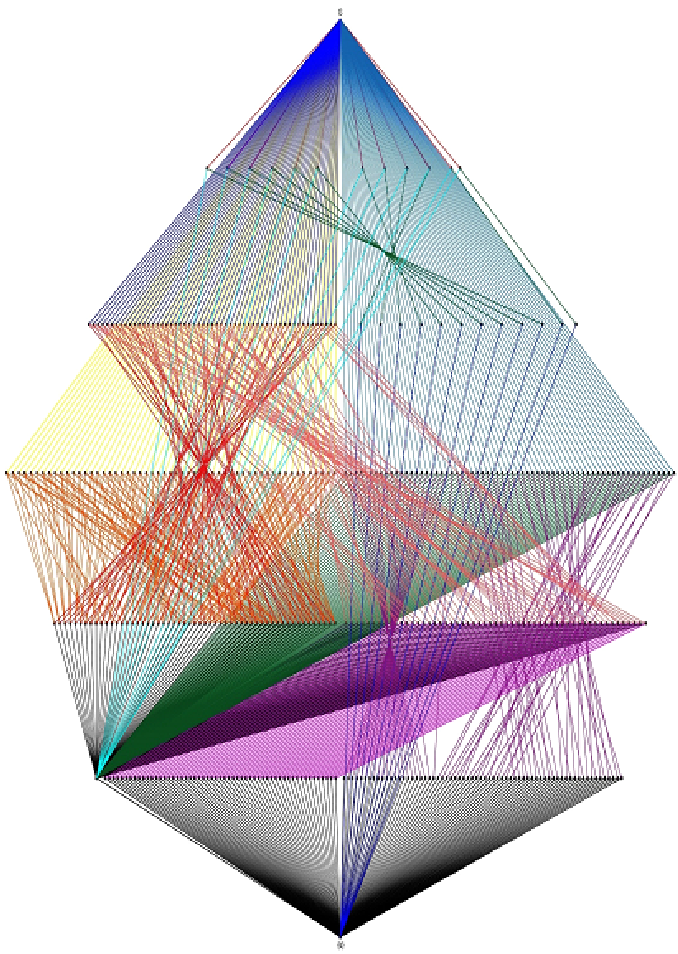
Definition 2.4. *A poset is said to be complete lattice if all its subsets have both join and meet. In particular, every complete lattice is a bounded lattice.*

Definition 2.5. *Let L be a bounded lattice with greatest element 1 and least element 0. Two elements x and y of L are said to be complements of each other if $x \vee y = 1$ and $x \wedge y = 0$. If every element of L has a complement, then L is called a complemented lattice.*

We give below the diagram of $L(G)$ when $p = 11$ [9].



Row I (Left to right): L_1 to L_{12}.

Row II (Left to right): J_1 to J_{55} and I_1 to I_{12}.

Row III (Left to right): F_1 to F_{55} and H_1 to H_{66}.

Row IV (Left to right): C_1 to C_{55} and E_1 to E_{55}.

Row V (Left to right): A_1B_1 to B_{55} and D_1 to D_{66}.

3. Subgroups of G of Different Orders in L(G) Over Z_{11} [9]

Let A be an arbitrary subgroup of G of order 2. Then the number of subgroups of order 2 is 1. Let B be an arbitrary subgroup of G of order 3. Then the number of subgroups of order 3 is 55. Let C be an arbitrary subgroup of G of order 4. Then the number of subgroups of order 4 is 55. Let D be an arbitrary subgroup of G of order 5. Then the number of subgroups of order 5 is 66. Let E be an arbitrary subgroup of G of order 6. Then the number of subgroups of order 6 is 55. Let F be an arbitrary subgroup of G of order 8. Then the number of subgroups of order 8 is 55. Let H be an arbitrary subgroup of G of order 10. Then the number of subgroups of order 10 is 66. Let I be an arbitrary subgroup of G of order 11. Then the number of subgroups of order 11 is 12. Let J be an arbitrary subgroup of G of order 12. Then the number of subgroups of order 12 is 55. Let L be an arbitrary subgroup of G of order 22. Then the number of subgroups of order 22 is 12.

4. Complementedness in the Lattice of Subgroups of the Group of 2×2 Matrices Over Z_{11}

Lemma 4.1. *For $p = 11$, the two-element subgroup A_1 does not have a complement in $L(G)$.*

Proof. An even order subgroup cannot be a complement of A_1. So, if X were a complement of A_1, then in $L(G)$, $X \vee A_1 = G$ and $X \wedge A_1 = \{e\}$ where $o(X)$ is odd.

If X is odd order which is a prime number, say k. Now, $k - 1 \equiv 0 \ (mod\ 2)$. So, there exists a subgroup of order $2k$.

$$O(X \vee A_1) = 2k.$$

Therefore, $X \vee A_1 \neq G$.

If $o(X) = st$, where s and t are odd primes and $s - 1 \equiv 0 \ (mod\ t)$, $(s > t)$. Then $o(X \vee A_1) = 2st \neq (p-1)p(p+1) = o(G)$. Therefore, $X \vee A_1 \neq G$. So, A_1 has no complement in $L(G)$. □

Theorem 4.2. *$L(G)$ is not complemented if $p = 11$.*

Proof. Follows from the above Lemma 4.1 □

5. Conclusion

In this paper, we proved that $L(G)$ is not complemented when $p = 11$.

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