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GENERALIZED HYPERBOLIC DISTRIBUTIONS: EMPIRICAL EVIDENCE ON BUCHAREST STOCK EXCHANGE

OLIVIA ANDREEA BACIU

Abstract. On five of the most liquid and important equities of the Romanian stock market together with the market index is investigated the fit of the generalized hyperbolic distributions. The parameters of the hyperbolic distribution, Variance- Gamma, Normal Inverse Gaussian, skewed t Student and generalized hyperbolic are estimated using the maximum likelihood estimation. The goodness-of-fit measures used to assess the fit of each distribution are the Kolmogorov- Smirnov distance, Akaike information criteria and the log- likelihood. Plots are also inspected. The Variance- Gamma distribution was ruled out by the Kolmogorov- Smirnov test. After inspecting the plots, a good approximation of the data was given by the Normal Inverse Gaussian distribution and the generalized hyperbolic, but based on the goodness-of-fit measures, the generalized hyperbolic distribution yield to be the best fit.

1. Introduction

The behavior of the capital markets return has been the subject of numerous studies from Mandelbrot (1963) until nowadays. Although at the beginnings it was assumed the normality in the asset returns, studies have shown that they follow a distribution with heavier tails and a longer shape than the normal curve. When analyzing the cotton price, Mandelbrot noticed that the returns exhibit heavier tails and a leptokurtic distribution.

The distribution of the asset returns play an important role both in financial models and risk management. Based on the assumptions of the behavior of the asset returns are constructed portfolios that tend to be optimal with a minimum risk of loss. Risk management theories rely only on the distribution of the tails and the choice of a distribution that underestimates the weight of the extreme values would lead to an underestimated loss. In financial modeling, most of the theories are constructed based on the distribution of the financial data.

Over the time, several distributions were identified to describe well the stock market data, but there is not known one distribution that fits perfectly. Among such distributions is the family of the generalized hyperbolic distributions (ghyd) introduced by Barndorff-Nielsen (1977).

In this paper are fit the univariate cases of the ghyd family to the daily returns of five Romanian stocks and the index of the Romanian market, BET, between 2007 and 2012.The choice between the several distributions is made using the goodness-of-fit measures, inspection of the plots and a comparison of the central moments of each distribution to the ones of the empirical distribution.

One of the main contributions of this study is that the analysis is performed not only on the Romanian market index, but also on five of its most important and liquid equities. Another contribution is that this study show the performance of the ghyd family compared to the normal

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Olivia Andreea Baciu is Ph.D. Candidate at Babes-Bolyai University, Cluj-Napoca, Romania. Email: oli_baciu@yahoo.com.
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distribution on stock market data and makes a comparison among the sub-classes of the ghyd family.

The paper is structured as follows. Second part makes a brief review of the most important studies. In the third section is introduced the ghyd family, mathematical description of the general framework and particularities of the sub-families. Section 4 describes the methodology and gives a short description of the estimation method. In section 5 is introduced the data set. Section 6 presents the results and finally, section 7 concludes the paper.

2. Literature review

Although introduced years before, the ghyd distribution was first applied to financial data by Eberlein and Keller (1995) on a portfolio of the German stock market. Several special cases were introduced around the same time.

Madan and Seneta (1990), Madan and Milne (1991) proposed and investigated a special case of the ghyd family, the Variance Gamma distribution (VG) and Seneta (2004) fits this distribution on the returns of the S&P 500 Index. Hansen (1994) introduced the skewed Student's t distribution (t Student) for modeling heavier tails, which later was improved by Aas and Haff (2006) by constructing one of the tails as a polynomial function and the other one as an exponential function.

The Normal- Inverse Gaussian distribution (NIG) was introduced by Barndorff- Nielsen (1995, 1997) and studies like the ones of Karlis (2002) or Venter and De Jongh (2002) have shown a better fit to financial data than other distributions of the ghyd family. A more recent study of Cepni et al. (2013) compares the fittings of the NIG and VG distributions on a data set of twenty emerging and developed markets and concluded that VG has a better performance. For heavy tail data, the new approach of the t Student distribution provides a better fit than NIG.

Fajardo and Farias (2004) fit the ghyd to the Brazilian stock data returns and based on the goodness of fit measures concluded that it provides a good fit. Necula (2009) used the ghyd to fit the data from ten stock markets, both developed and emerging markets, including markets from Japan, USA, France, Germany, Romania or Czech Republic. The measures used to compare the performance of ghyd with the normal distribution are the first four central moments, the Kolmogorov- Smirnov and Anderson- Darling statistics, the q-q plots and concluded a good fit of the ghyd to financial data. Studies like the ones of Prause (1997) and Behr and Potter (2009) identified the ghyd as the best fit to financial data.

Socgnia and Wilcox (2014) compared the fit on a data set of the Johannesburg Stock Exchange of the ghyd, hyperbolic distribution (hyp), VG, NIG and t Student. Based on the log-likelihood and Akaike information criteria, the best fit was given by the ghyd.

3. Mathematical description of the Generalized Hyperbolic Distributions

As introduced by Barndorff-Nielsen (1977), a univariate generalized hyperbolic distribution is described by five parameters $(\lambda, \alpha, \beta, \delta, \mu)$, which correspond to kurtosis, shape, symmetry, scale and location.

The probability density function of the univariate ghyd, as in Prause (1999) is given by:

$$\begin{aligned} f_{GHD}\left(x;\alpha,\beta,\delta,\mu,\lambda\right) &= a\left(\lambda,\alpha,\beta,\delta\right)\left(d^2+(x-\mu)^2\right)^{\frac{\lambda-\frac{1}{2}}{2}} \times \\ &\times K_{\lambda-\frac{1}{2}}(\alpha\sqrt{d^2+(x-\mu)^2}\exp\left(\,(x-\mu)\right), \end{aligned} \tag{3.1}$$

where,

$$a(\lambda,\alpha,\beta,\delta) = \frac{(\alpha^2 - {}^2)^{\frac{\lambda}{2}}}{\sqrt{2\pi}\alpha^{\left(\lambda-\frac{1}{2}\right)}\delta^\lambda K_\lambda(\delta\sqrt{\alpha^2 - {}^2})}, \tag{3.2}$$

and

$$K_{\lambda}(x) = \frac{1}{2}\int_{0}^{\infty} y^{\lambda-1}\exp\left(-\frac{1}{2}x\left(y+y^{-1}\right)\right) dy. \tag{3.3}$$

The function $a(\lambda, \alpha, \beta, \delta)$ represents the norming factor and $K_{\lambda}(x)$ is the third type of the modified Bessel function with index λ, introduced by Abramowitz and Stegun (1968). For the above, the domains of the parameters are $\mu, \lambda \in R$, $-\alpha < \beta < \alpha$, $\delta > 0$, $\alpha > 0$.

Using simplified Bessel functions leads to special cases of the ghyd family. When the parameter λ takes the value 1, it is the case of the hyp with a simplified probability density function, given by:

$$f_{Hyp}(x|\alpha, \beta, \delta, \mu) = \frac{\sqrt{\alpha^2-\beta^2}}{2\alpha\beta K_1\left(\delta\sqrt{\alpha^2-\beta^2}\right)} \times \exp\left[-\sqrt{\delta^2+(x-\mu)^2}+\beta(x-\mu)\right], \tag{3.4}$$

and with the domain of the parameters: $\alpha > 0$, $-\alpha < \beta < \alpha$, $\delta > 0$, $\mu \in R$.

For $\lambda = -1/2$ it is obtained the NIG, a distribution with heavier tails than the hyp one. NIG has the following probability density function:

$$\begin{aligned} f_{NIG}(x|\alpha, \beta, \delta, \mu) &= a(\alpha, \beta, \delta, \mu)[\delta^2+(x-\mu)^2)^{-\frac{1}{2}} \times \\ &\times K_1\left(\alpha\sqrt{\delta^2+(x-\mu)^2}\right)\exp[\beta(x-\mu)], \end{aligned} \tag{3.5}$$

where,

$$a(\alpha, \beta, \delta, \mu) = \pi^{-1}\delta\alpha\exp(\delta\sqrt{\alpha^2-\beta^2}), \tag{3.6}$$

and the Bessel has the properties $K_{\frac{1}{2}}(x) = 2^{-1/2}\sqrt{\pi}x^{-1/2}\exp(-x)$ and $K_{-\lambda}(x) = K_{\lambda}(x)$.

Another special case is the VG introduced by Madan and Seneta (1990), for which $\lambda > 0$ and the probability density function is:

$$\begin{aligned} f_{VG}(x|c, \sigma, \theta, \nu) &= a(c, \sigma, \theta, \nu)\exp\frac{\theta(x-c)}{\sigma^2}|x-c|^{\frac{1}{\nu}-\frac{1}{2}} \times \\ &\times K_{\frac{1}{\nu}-\frac{1}{2}}\left(\sigma^{-2}|x-c|\sqrt{2\frac{\sigma^2}{\nu}+\theta^2}\right), \end{aligned} \tag{3.7}$$

where,

$$a(\theta, \nu, \sigma, c) = \frac{2(\sqrt{2\frac{\sigma^2}{\nu}+\theta^2})^{\frac{1}{2}-\frac{1}{\nu}}}{\sigma\sqrt{2\pi}\nu^{\frac{1}{\nu}}\Gamma(\frac{1}{\nu})}, \tag{3.8}$$

with the domain of the parameters: $\nu > 0$, $\sigma > 0$, $-\infty < \theta < \infty$, $-\infty < c < \infty$.

According to Paolella (2007), the relationship between the parameters of the ghyd and VG is given by: $\mu = c$, $a = \frac{\sqrt{2\frac{\sigma^2}{\nu}+\theta^2}}{\sigma^2}$, $\beta = \frac{\theta}{\sigma^2}$ and $\lambda = \frac{1}{\nu}$.

Another limiting case is the distribution t Student, for which the density function is described by the below function and $\lambda < 0$.

$$f_t(x|\nu, \beta, \delta, \mu) = \frac{\delta^{\nu}|\beta|^{\frac{\nu+2}{2}}K_{\frac{\nu+1}{2}}(\beta\sqrt{\delta^2+(x-\mu)^2})\exp[(x-\mu)]}{2^{\frac{\nu-1}{2}}\Gamma\left(\frac{\mu}{2}\right)\sqrt{\pi}[\sqrt{\delta^2+(x-\mu)^2}]^{(\nu+1)/2}}. \tag{3.9}$$

4. Methodology

In the process of analyzing if ghyd represents a good choice in modeling the stock market data, the first step is to fit the univariate distributions to data. From the family of the ghyd distributions are considered here hyp, VG, NIG, ghyd and t Student.

The parameters of the hyp distribution, VG, NIG, ghyd and t Student distribution are estimated using the maximum likelihood estimation, implemented based on the EM scheme of Dempster et al. (1977) in R ghyp package. EM algorithm is useful in maximum likelihood estimation when data contains missing values.

For a vector of observations $x_1, x_2, \dots, x_n$, the maximum likelihood estimation of the parameters $\lambda, \alpha, \beta, \delta, \mu$ is obtained by maximizing the log-likelihood function:

$$L\left(x_1, x_2, \dots, x_n;\ \lambda, \alpha, \beta, \delta, \mu\right) = loga + \frac{\lambda - \frac{1}{2}}{2} \sum_{i=1}^{n} \log(\delta^2 + (x_i - \mu)^2) +$$

$$+ \sum_{i=1}^{n} logK_{\lambda - \frac{1}{2}}(\alpha \sqrt{\delta^2 + (x_i - \mu)^2} + \sum_{i=1}^{n} \beta\left(x_i - \mu\right), \tag{4.1}$$

where a is defined in equation (3.2).

The aim of this paper is to find the distribution that best approximates the data. The choice between the above mentioned univariate distributions is performed based on several goodness-of-fit measures and plot inspection. The goodness-of-fit measures used are the ones suggested by most of the studies on the distribution of asset or market index returns.

One of the goodness-of-fit measures is the Kolmogorov- Smirnov distance (KS), which is a proper choice in the case of continuous distributions. This test statistic measures the supremum of the distance between the empirical distribution function $F_e(x)$ and the estimated distribution $F(x)$ and tests if the data comes from the given distribution.

$$KS = \sup_{x \in R} |F_e\left(x\right) - F(x)|. \tag{4.2}$$

The KS distances are also computed for the normal distribution.

Another goodness-of-fit measure used is the Akaike Information Criterion (AIC), which is a measure of the relative fit of the model to the data. The best model will be the one with the smallest AIC. If k is the number of parameters in the model, then:

$$AIC = 2k - 2logL, \tag{4.3}$$

where L represents the maximum value of the likelihood function of the estimated model.

Log-likelihood is also a goodness-of-fit measure (LL). The higher the value of LL, the better the fit of the considered distribution to the given data.

In the process of choosing between several distributions are also inspected the density plots and the Q-Q plots of the distributions that offer a good fit. As in Necula (2009) are inspected the first four moments of the estimated distributions and compared to the ones of the empirical one.

5. Data and descriptive analysis

The performance of the ghyd on the Romanian market is inspected on the daily returns of five equities, namely the Investment Funds: SIF1, SIF2, SIF3, SIF4, SIF5 and the index of the market, BET (Ristea et al., 2010, Dumitrana et al., 2010).

These Investment Funds are listed at Bucharest Stock Exchange and they play an important role on the Romanian economy because they hold shares on more than 300 important companies in different and main domains, like: energy, banking, gas. The Investment Funds represent five giants of the economy, with a capitalization 8.8 times greater than in 1999, the moment when they were listed to the stock market. SIF's are managed by well established rules and no investor can hold more than 5% of them.

Data was collected over five years, the time period considered is between the day of the maximum closing price in 2007 and the last trading day of 2012. The returns are computed as the difference between natural logarithm of current day closing price and natural logarithm of previous day closing price:

$$R_t = \ln(\frac{Y_t}{Y_{t-1}}). \tag{5.1}$$

In all further computations were considered returns multiplied by 100.

The choice of the time period is due to the structural break identified in the return series at the middle of 2007. The changes that affected the Romanian stock market after the middle of 2007 can be associated with the economical and political crisis, a moment that brought changes in the portfolio and the main characteristics of each time series were expected to change in the two periods of time, the ante and the crisis periods of time. In this period are expected even more severe departures from normality, with heavier tails that are better highlighted using the five years period of time than a longer one. The purpose of the study is to identify the behavior of the returns on specific and extreme market conditions. Further research should be performed on longer periods of time and verify if the market exhibits the same behavior. Another reason for the choice of the time period is the risk management approach in the specific environment of the crisis, issue that is addressed in Baciu (2014). In the years of the financial crisis, contrary to the expectation, the risk of loss is overestimated when using distributions that approximate well the data. It is of main importance, both for practice and theory, to approximate the behavior of the returns in extreme conditions, like a financial crisis and to be able to create financial models or risk management measures that take into account the effects of the extreme negative returns.

In the below table are presented the descriptive statistics of the daily returns for the five investigated funds and BET index.

Table 1. Descriptive statistics

Equity	Sample size	Mean	Standard deviation	Skewness	Kurtosis	Jarque-Bera
SIF 1	1352	0.0946	3.2077	0.1802	4.1727	997,7*
SIF2	1353	0.0752	3.2264	0.2385	4.5521	1191.9*
SIF 3	1372	0.1053	3.3111	0.6811	7.5740	3460.8*
SIF 4	1362	0.0987	2.9963	0.0860	4.9256	1369.4*
SIF 5	1351	0.0924	3.1551	0.1243	4.1248	965.5*
BET	1368	-0.0295	2.0075	-0.4952	5.8992	2046.9*

Note: * denotes statistical significance at 5%.

From Table 1 it can be noticed that the returns are all skewed and present a higher kurtosis than in the case of the normal distribution. The hypothesis that data is following a normal distribution is rejected for all equities, as suggested by the Jarque- Bera test results. Although the considered period of time is covering a period of economical crisis, except for the BET index, all equities have a positive mean return. The SIFs present an increased risk compared to the risk of the market index.

6. Empirical results

The performance of each distribution is analyzed based on plots and goodness-of-fit measures.

Appendix A presents the estimated values of the parameters using the maximum likelihood method. It can be noticed that with the exception of VG for SIF 4, all estimated distributions are skewed.

In Figure 1 are presented the plots of the densities and log densities of the empirical, normal and ghyd distributions. It is included only the ghyd because among all the distributions from the ghyd family, ghyd approximates the best the behavior of the returns and specially, the weight of the tails. Also, the Q-Q plots of the ghyd and NIG distributions are included in Appendix B, for a better comparison of the fit of these two distributions to the given data, since both exhibited appropriate goodness-of-fit measures. Again, the tails of the empirical distribution are well approximated by both distributions.

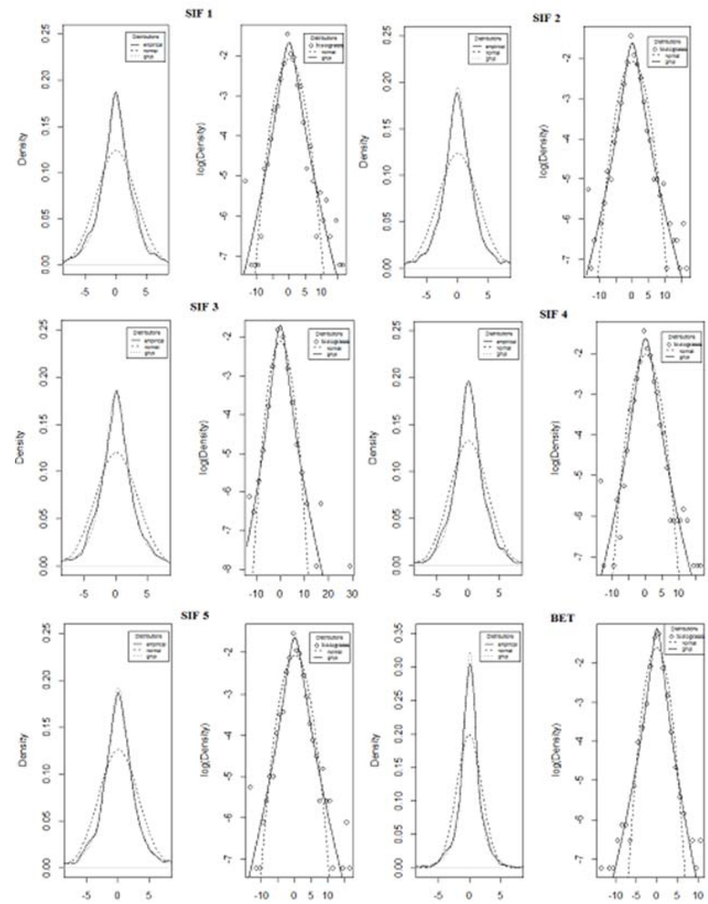

Figure 1. Plots of the densities and log-densities

Table 2 contains the goodness-of-fit measures: KS distance, AIC and LL. For all the equities, the normal distribution is rejected by the KS test and it exhibits the highest KS distances among all the considered distributions. It can be noticed for all the equities that there are very close values among the AIC and LL for the distributions that are not rejected by the KS test.

For SIF 1, although the VG has the minimum AIC and maximum LL values among all the considered distributions, the KS test rejects that data comes from the mentioned distribution. If looking at the KS test, data could follow one of the hyp, t Student or ghyd, with a minimum KS distance for the ghyd distribution.

For SIF 2, KS test suggest that data could follow either hyp, NIG, t Student or ghyd. The minimum KS distance is met in the case of NIG distribution. Again, VG distribution presents the best values for AIC and LL, but is rejected by the KS test.

SIF 3, according to the AIC and LL values, follows a VG distribution, fact rejected again by the KS test. The next distribution with the second best values for AIC and LL is hyp, which is also confirmed by the KS test, but the minimum KS distance is given for ghyd, followed by NIG.

In the case of SIF 4 returns, VG is rejected by the KS test although it exhibits the best values for AIC and LL. KS test suggest that data follows one of the hyp, t Student or ghyd. The minimum KS distance is obtained for ghyd.

For SIF 5 only the normal distribution is rejected by the KS test at a 1% level of confidence. VG distribution presents the best fit according to AIC and LL and t Student has the minimum distance.

The BET index can follow, according to the KS test, any of the considered distributions. The smallest KS distance is in the case of the ghyd distribution, followed by t Student. The minimum AIC value and maximum LL appears for hyp, followed by very close values by the ghyd.

Table 2. Goodness-of-fit results

Equity	Distribution	KS	p-value	AIC	LL
SIF1	Normal	0.2055	0		
	Hyp	0.0472	0.15	6721.53	-3356.77
	NIG	0.1142	0	6728.74	-3360.37
	VG	0.1613	0	5848.36	-2920.18
	t Student	0.0672	0.01	6733.67	-3362.84
	Ghyd	0.0614	0.02	6730.73	-3360.38
SIF2	Normal	0.2052	0		
	Hyp	0.0609	0.02	6701.18	-3346.59
	NIG	0.0399	0.31	6694.27	-3343.13
	VG	0.1793	0	5675.882	-2833.94
	t Student	0.0535	0.07	6700.72	-3346.36
	Ghyd	0.0589	0.03	6696.17	-3343.08
SIF3	Normal	0.1992	0		
	Hyp	0.0507	0.10	6865.74	-3428.74
	NIG	0.0476	0.14	6867.42	-3429.71
	VG	0.1718	0	5677.86	-2834.93
	t Student	0.0737	0	6870.06	-3431.03
	Ghyd	0.0432	0.23	6869.03	-3429.52
SIF4	Normal	0.1866	0		
	Hyp	0.0506	0.10	6577.22	-3284.61
	NIG	0.0829	0	6578.27	-3285.14
	VG	0.1516	0	5361.53	-2677.77
	t Student	0.0559	0.054	6580.67	-3286.34
	Ghyd	0.0496	0.11	6597.91	-3284.95
SIF5	Normal	0.222	0		
	Hyp	0.0618	0.02	6672.1	-3332.05
	NIG	0.0628	0.02	6672.9	-3332.44
	VG	0.0549	0.06	6648.97	-3320.48
	t Student	0.0508	0.10	6679.98	-3335.99
	Ghyd	0.0589	0.03	6674.87	-3332.44
BET	Normal	0.099	0		
	Hyp	0.0312	0.62	5461.43	-2726.72
	NIG	0.0262	0.82	5443.44	-2717.72
	VG	0.0282	0.74	5461.2	-2726.6
	t Student	0.0241	0.89	5452.13	-2722.07
	Ghyd	0.0234	0.91	5445.42	-2717.71

Overall, the distributions that can be considered to fit the data are the ghyd, NIG and for some equities the t Student or hyp. But KS test rejects that SIF 1 or SIF 4 could follow a NIG distribution or SIF 3 a t Student distribution. Ghyd is the only distribution that is not rejected for any equity.

Table 3. First four centered moments

Equity	Distribution	Mean	Variance	Skewness	Kurtosis
SIF1	Empirical	0.0946	10.2970	0.1802	4.1727
	Hyp	0.0951	9.7533	0.0913	6.0055
	NIG	0.0945	10.324	0.1455	8.5254
	VG	-0.3508	8.5052	-1.1274	13.3716
	t Student	0.1011	-	-	-
	Ghyd	0.0955	10.3386	0.1518	8.6664
SIF 2	Empirical	0.0752	10.4173	0.2385	4.5520
	Hyp	0.0764	9.5585	0.0741	6.0036
	NIG	0.0754	10.5006	0.2285	9.7553
	VG	0.9416	20.4404	2.8639	23.4692
	t Student	0.0921	-	-	-
	Ghyd	0.0745	10.5680	0.2404	10.3720
SIF3	Empirical	0.1052	10.9698	0.6810	7.5740
	Hyp	0.1041	10.0656	0.0984	6.0064
	NIG	0.1050	10.7658	0.2192	8.8478
	VG	0.9185	32.8697	2.3934	22.5405
	t Student	0.1157	-	-	-
	Ghyd	0.1050	10.9045	0.2810	10.2090
SIF4	Empirical	0.0987	8.9833	0.0860	4.9255
	Hyp	0.0978	8.4563	0.1009	6.0067
	NIG	0.0980	8.9039	0.1072	8.4537
	VG	0	21.1368	0	18.9063
	t Student	0.1027	-	-	-
	Ghyd	0.0977	9.0440	0.1383	9.9020
SIF5	Empirical	0.0924	9.9603	0.1243	4.1248
	Hyp	0.0929	9.4297	0.0908	6.0054
	NIG	0.0926	10.0527	0.1060	8.8334
	VG	0.1869	10.1727	0.2353	7.0591
	t Student	0.1000	-	-	-
	Ghyd	0.0915	10.0406	0.1023	8.7092
BET	Empirical	-0.0515	4.0245	-0.4951	5.8991
	Hyp	-0.0512	3.6405	-0.2245	5.9809
	NIG	-0.0517	4.0427	-0.4256	10.2783
	VG	-0.0513	3.7288	-0.2588	6.2460
	t Student	-0.0750	-	-	-
	Ghyd	-0.0517	4.0306	-0.4126	10.0534

From Table 3, except the BET index, VG distribution returns values far from the empirical mean. For all equities, NIG exhibits the closest mean value to the empirical mean, while for SIF 3 and BET, ghyd exhibits the same mean value as NIG. NIG has the closest variance to the empirical one with the exception of SIF 3, where the closest value is given by ghyd. The closest skewness value is reached also in the case of the NIG distribution, excepting SIF 1 and SIF 4. All distributions have a higher peak compared to the empirical one, but the closest values are given by the hyp, excepting SIF 3 where is given by NIG.

Among all the distributions, the best fits are in the case of the ghyd and NIG distributions. Based on goodness-of-fit measures ghyd outperforms the NIG distribution. This result is in line with the one of Prause (1997), Rege and Menezes (2012) or Socgnia and Wilcox (2014).

7. Conclusion

The family of ghyd is fitted in the present study on the Romanian stock market index, BET and five of the most important equities on this market. The parameters of the hyp, NIG, VG, ghyd and t Student are estimated by means of the maximum likelihood estimation.

The choice among the several distributions is made based on plots and the goodness-of-fit measures: KS distance, LL and AIC. From the beginning, the KS test ruled out the VG. This result is in contradiction with the one of Cepni et al. (2013), a study that found VG distribution the best fit on a wide data set from emergent and developed markets.

As expected, after inspecting the density plots and based on KS test, the family of the ghyd offers a better and more appropriate fit to stock market data than the normal distribution.

The goodness-of-fit measures provide AIC and LL values that are very close together for all distributions with one exception, VG. KS distances suggested that the most appropriate distributions to model financial data on Romanian stock market are the ghyd and NIG.

Among the five generalized hyperbolic distributions, NIG and ghyd distributions are the ones that represent a good approximation of the data, results that are in line with the ones of Prause (1997) or Socgnia and Wilcox (2014). Based on the goodness-of-fit measures, the ghyd exhibit the best fit of the given data.

The study of the distribution of the returns is of main importance in fields like risk management. These results are further used in Baciu (2014), a study that concludes that over periods of extreme market conditions, like a financial crisis, ghyd overestimates the risk of loss and contrary to the expectation, the normal distribution that does not take into account the extreme values could be more appropriate to predict the loss when the effects of the crisis begin to diminish.

The present research is limited at the investigation of the Romanian market index and five of its main equities and how well the returns follow the distributions of the generalized hyperbolic family. As further work, the study should be extended on a larger portfolio and over a longer period of time. Also, more goodness-of-fit measures should be interrogated. Being able to predict the behavior of the market implies an insight of the past behavior and specially, of the behavior on extreme conditions that could produce the worst damage for investors. For this reason, a research that compares how well different distributions describe the behavior of stock returns would be of main interest.

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Appendix

Appendix A: Table of the estimated parameters

Equity	Distribution				Parameters		
		Lambda	alpha.bar	Nu	Mu	Sigma	gamma
SIF1	hyp	1	0.00000234		0.00000260	3.121591	0.095127
	NIG	-0.5	0.54572713		0.00945616	3.211088	0.085048
	VG	0.315201	0		0.00000005	2.848648	-0.35083
	t Student	-1.54307	0	3.086136	0.00988853	3.481767	0.091249
	Ghyd	-0.55178	0.54177476		0.00890538	3.213182	0.086627
SIF2	Hyp	1	0.00000099		-0.00000042	3.090738	0.076476
	NIG	-0.5	0.44872010		-0.03526060	3.236243	0.110754
	VG	0.202606	0		-0.00000006	4.00796	0.941676
	t Student	-1.37776	0	2.755525	-0.01659794	3.726415	0.108724
	Ghyd	-0.6251	0.43282376		-0.03270474	3.246557	0.107254
SIF3	Hyp	1	0.00000098		0.00000107	3.170933	0.10415
	NIG	-0.5	0.51869660		-0.01929949	3.276591	0.124378
	VG	0.19186	0		0.00000008	5.33593	0.918547
	t Student	-1.52513	0	3.050254	-0.02719967	3.551674	0.142976
	Ghyd	-0.84309	0.46720140		-0.02600721	3.296065	0.131092
SIF4	Hyp	1	0.00000046		0.00000086	2.906329	0.097864
	NIG	-0.5	0.55163570		0.03916995	2.982893	0.058837
	VG	0.188604	0		0.00000001	4.597477	0
	t Student	-1.56962	0	3.139243	0.03708066	3.202651	0.06563
	Ghyd	-0.90372	0.48281620		0.03718524	3.005939	0.060581
SIF5	Hyp	1	0.00000122		-0.00000035	3.06938	0.092972
	NIG	-0.5	0.51560270		0.03484094	3.169586	0.057815
	VG	0.745859	0		0.00000000	3.182118	0.186935
	t Student	-1.47637	0	2.95273	0.03695667	3.514013	0.063116
	Ghyd	-0.45747	0.51819470		0.03463519	3.167722	0.056922
BET	Hyp	1	0.087429370		0.094435810	1.902563	-0.14564
	NIG	-0.5	0.426332000		0.069919490	2.002017	-0.12162
	VG	0.093713	0.000000000		0.105185400	1.924239	-0.15653
	t Student	-1.2557	0.000000000	2.711392	0.062485900	2.329549	-0.13755
	Ghyd	-0.45643	0.430174500		0.069492400	1.999277	-0.12126

Appendix B: Q-Q plots for the ghyd and NIG distributions