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TEST CHARGED PARTICLES AROUND REGULAR BLACK HOLES IMMERSED IN MAGNETIC FIELD

Abstract: The article deals with test particles around a charged regular black holes immersed in magnetic field.

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Introduction

The particle collision of two particles around the RBH has also been studied which may cause the production of high energy particles, as goes the permutations between neutral, positively, and negatively charged particles. For the first time Roger Penrose suggested a process towards explaining the origin of UHECR by assuming the decay of a particle in the ergo-region into two particles where one particle falls into the BH with negative energy, and the second one has ejected away from the BH with an energy larger than the initial the energy of the parent particle. Recently, a model called the magnetic Penrose process (MPP) has been developed by to describe the motion of charged particles around a BH which are itself embedded in an external magnetic field. The efficiency of the process in the ordinary Penrose process ranges up to 21% in the case of extreme rotating BHs. It works in three regimes of efficiency: low, moderate, and ultra. It is shown that SMBH can be a source of UHECR where the beta decay of neutrons in the ergo sphere results in the emission of protons that can have energies up 1019-21eV in the ultra-high efficiency regime Frolov, have shown that non-rotating BH can act as particle accelerators when immersed in an external magnetic field considering charged and magnetized particle collisions. In an external magnetic field considering particle collision BH environment. Moreover BSW type collisions .It is always motivate to explain the

origin of UHECR to study testing energy extraction models in different BH models.

II.Literature review

The new urge to considering such processes came from an interesting observation made in Ref. [2]. It was found there that two particles which move towards the horizon of the extremal black holes can produce infinity energy in the centre of mass frame. This effect (called the BSW one after the names of its authors) provoked a large series of works and is under active study currently. The most part of them was restricted to the investigation of the vicinity of the horizon where collision occurs. In terms of energy production, we also consider particle collisions/decay happening inside the accretion disk of the RBH. This discussion is relevant the detection of ultra high energy cosmic rays (UHECR) that could help identify potential black hole candidates and/or further strengthen the argument that BH are indeed sources of these extremely high energy particles. One of the most interesting properties of BHs is energy extraction from them, there are several models whose purpose is to explain particle acceleration around a rotating BH.

III.Analysis

Throughout this work we use signature (-; +; +; +) for the space-time and geometrized unit system $G = c = 1$ (However, for an astrophysical application we have written the speed of light explicitly in our

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expressions). Latin indices run from 1 to 3 and Greek ones from 0 to 3.

The space-time around a RBH can be obtained using GR coupled to nonlinear electrodynamics (NED) and the corresponding action for these coupled fields is written

$$S = \frac{1}{16\pi} \int dx^4 \sqrt{-g} (R - L(F)) \quad (1)$$

where $F = F^{\mu\nu}F_{\mu\nu}$ is the electromagnetic field invariant and $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$ is the electromagnetic field tensor and A_μ is the electromagnetic field four potential. The spacetime around the RBH has been found by coupling Einstein's theory of gravity to NED where the Lagrangian is found as a function of the electromagnetic field invariant.

$$L(F) = \frac{4n}{\alpha} \frac{(\alpha F)^{\frac{k+3}{4}}}{\left[1 + (\alpha F)^{\frac{k}{4}}\right]^{1+\frac{n}{k}}} \quad (2)$$

For the case $k = 1$ and $n \geq 3$, where n is assumed to be an integer [1], the metric tensor is,

$$ds^2 = -f dt^2 + f^{-1} dr^2 + r^2 d\Omega^2 \quad (3)$$

The effective potential for a charged particle a constant plane ($\theta = \text{const}$ and $\dot{\theta} = 0$) can be found by solving equation $E = V_{eff}$ (taking $\dot{r} = 0$) and we have and four-velocities of the charged particle

$$\dot{t} = \frac{1}{f} (\mathcal{E} - qA_t)$$

$$\dot{r}^2 = (\mathcal{E} - qA_t)^2 - f \left[1 + \left(\frac{l}{r \sin \theta} - \frac{qB}{2} r \sin \theta \right)^2 \right]$$

$$\begin{aligned} \varepsilon_{cm}^2 = & 1 + N^2 - \frac{2N}{f} \left(f r^2 \left(\frac{l_1}{r^2} - \frac{q_1 B}{2} \right) \left(\frac{l_2}{r^2} - \frac{q_2 B}{2} \right) - (\varepsilon_1 - q_1 A_t) (\varepsilon_2 - q_2 A_t) \right) + \\ & \sqrt{\left\{ (\varepsilon_1 - q_1 A_t)^2 - f \left[1 + \left(\frac{l_1}{r} - \frac{q_1 B}{2} r \right)^2 \right] \right\} \left\{ (\varepsilon_2 - q_2 A_t)^2 - f \left[1 + \left(\frac{l_2}{r} - \frac{q_2 B}{2} r \right)^2 \right] \right\}} \end{aligned} \quad (9)$$

Here we will consider the collision of the charged particles with the same mass $m_1 = m_2 = m$ (charge might be different, for example, electron and positron) and initial energy $\varepsilon_1 = \varepsilon_2 = 1$, then the expression for the center-of-mass energy takes the following form. Now we will study in detail, center-of-mass energy of two colliding (neutral/charged) particles with different cases, i.e. particles with the same mass ($m_1 = m_2 = m$) and different mass $m_2 \neq m_1$ (assuming $m_1 = N m_2$, here N is some non-zero number) and the angular momentum ($l_1 = -l_2 = l$ and $l_1 \neq l_2$), and initial energies $\varepsilon_1 = \varepsilon_2 =$

$$\dot{\phi} = \frac{l}{r^2 \sin^2 \theta} - \frac{qB}{2} \quad (4)$$

In this section, we will study the centre-of-mass energy of two particles in the case of charged-charged, charged neutral particles collisions. The expression for the centre of-mass energy for two particle system with mass m_1 and m_2 , in a given gravitational field is as a sum of two-momenta

$$\{E_{cm}, 0, 0, 0\} = m_1 u_1^\mu + m_2 u_2^\mu \quad (5)$$

where, u_1^α and u_2^β are four-velocity of the two colliding particles and the velocities satisfy the condition $u_\mu u^\mu = -1$. Keeping the condition one can square (5) and we have.

$$E_{cm}^2 = m_1^2 + m_2^2 - 2m_1 m_2 g_{\mu\nu} u_1^\mu u_2^\nu \quad (6)$$

and after simplifying

$$\frac{E_{cm}^2}{m_1 m_2} = \frac{m_1}{m_2} + \frac{m_2}{m_1} - 2g_{\mu\nu} u_1^\mu u_2^\nu \quad (7)$$

Let us consider simple estimation, assuming that the mass of the particles is different from each other N times, i.e. $m_1 = N m_2$, N cannot be zero, obviously that $N > 1$ corresponds to $m_1 > m_2$, and vice versa $N < 1$ to $m_1 < m_2$. Thus, the expression for center-of-mass energy (7) takes the following form.

$$\varepsilon_{cm}^2 = \frac{E_{cm}^2}{m^2} = 1 + N^2 - 2N g_{\mu\nu} u_1^\mu u_2^\nu \quad (8)$$

taking into consideration equations

1 in the equatorial plane using equations of motion charged particles. In figure 1 radial dependence of center-of-mass is plotted in different values of Q and n .

One can see from left panel of this figure that the center-of-mass energy increases as the charge of RBH increase, but the increase of n cause to decrease the energy (middle panel), left panel is plotted for extreme charged RBH case for given values of n obviously that the maximum of the energy is at horizon when $r \rightarrow r_h$

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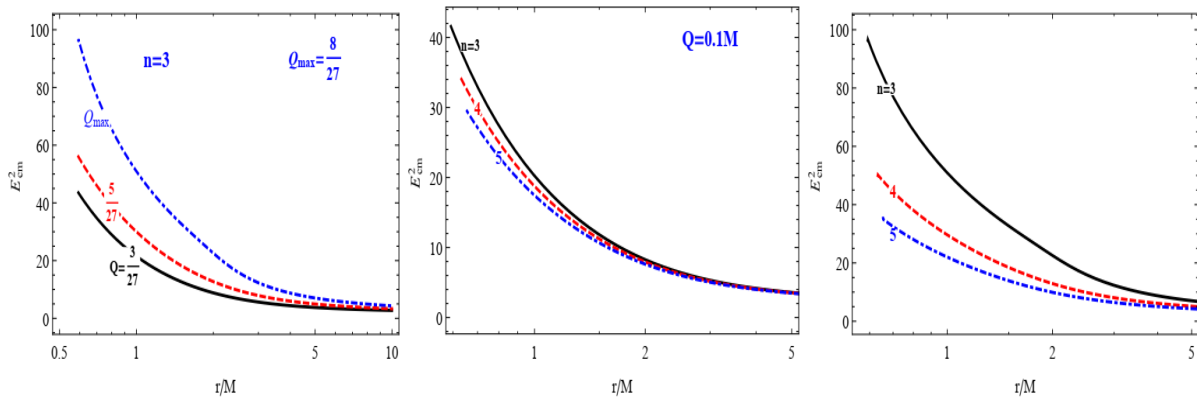


FIG. 1: Radial dependence of center-of-mass energy for charges with value $q = -100$ and $q = 2$

Let us consider that two charged particle having the same charge and the same angular momentum collision with opposite direction. The question that what is the minimum values of charge q and angular

momentum l that the center-of-mass energy $\epsilon > 100$ can be greater than 100.

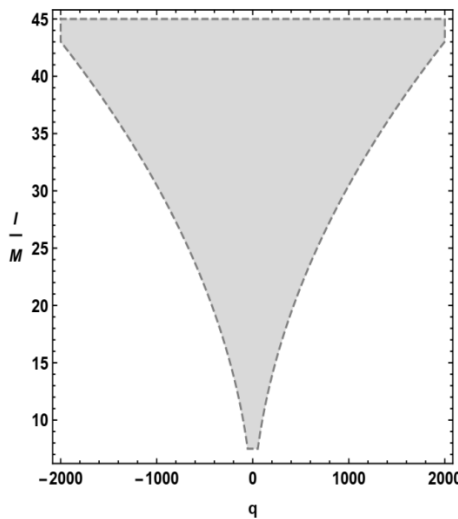


FIG. 2: Figure shows they are where $\epsilon > 10$ for different cases in the case $B = 0$

”The colored area” in figure 2, where a field which set of points consisting of the values of l and q , correspond ”the area” where $\epsilon > 10$. In the border of the colored area $\epsilon = 10$ and in the white-

uncolored area $\epsilon < 10$ the figure in below we try to find minimum values of charged particles in the collision with angular momentum $l_1 = -l_2=10$ the energy to be $\epsilon > 100$

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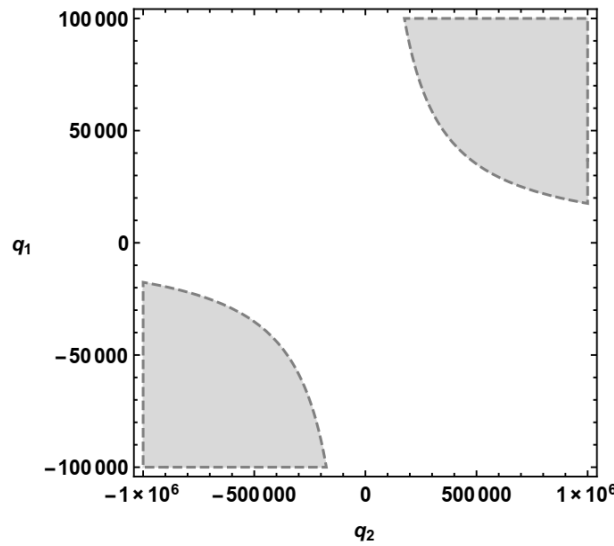


FIG. 3: the same figure with Fig.2 but for $\varepsilon > 100$

In figure 3 we set up the shaded region consisting of the values of q_1 and q_2 for $\varepsilon_{cm} > 100$. In both figures 2 and 3 we have considered the particle with the same mass. This can be attributed to the values for q_1 vs q_2 for both figures interchanging places, so the plot just had a change of axis. Now the momentum value change had no effect because q_1 vs q_2 have very high values compared to the angular momentum. In figure 2 we made $q_1 = q_2 = q$ and $l_1 = l_2 = l$ and then plotted q with respect to l . For this plot $\varepsilon_{cm} > 10$. It can be seen that as q increases at a greater rate l will still have a smaller increasing rate. In figure 3 we again made $q_1 = q_2 = q$, and $l_1 = l_2 = l$. In this case $\varepsilon_{cm} > 100$ and l had a much higher range compared to the figure 1 and 2. This illustrated how when the angular momentum. It can be seen that when q is of a small range then the value of l will have an effect on the plot.

IV. Discussion

A. Harmonic oscillations. The effective potential of a charged particle becomes minimum at a distance r_0 in the equatorial plane ($\theta = \pi/2$). Let us consider the case when the charged particle slightly shifts radially its position on a given circular orbit, the charge tries to back its equilibrium orbits where the effective potential is minimum corresponding charges parameters, then the charge starts oscillate around the circular orbits where the charge initially has been. If the derivation of the effective potential by radial coordinate is small enough, the condition of linear harmonic oscillation can be satisfied. Variation of the charged particle around the distance r_0 is $\delta r = r - r_0$ and the equation of the linear harmonic oscillations can be described by using the Taylor expansion around the distance r_0

$$V_{eff}(r) = V_{eff}(r_0) + \frac{\partial V_{eff}(r)}{\partial r} \Big|_{r=r_0} (r-r_0) + \frac{1}{2} \frac{\partial^2 V_{eff}(r)}{\partial r^2} \Big|_{r=r_0} (r-r_0)^2 + (\delta r)^3 \quad (10)$$

In cases small displacement of charged particle around the stable circular orbits the higher orders (more than three) of δr tends zero. In fact that the first derivative of the effective potential is zero in circular orbits, then the equation (10) can be rewritten as

$$V_{eff}(r) = V_{eff}(r_0) + \frac{1}{2} \frac{\partial^2 V_{eff}(r)}{\partial r^2} \Big|_{r=r_0} (r-r_0)^2 \quad (11)$$

III. PSEUDO - NEWTONIAN POTENTIAL.

Here we derive the pseudo-Newtonian potential (or the Paczyński-Wiita (PW) potential for the RBH (a Maxwellian solution), which is an interesting astrophysical object. First, we calculate the Keplerian angular momentum to derive the PW potential

$$\Omega_k = \frac{L_{crit}^2}{\varepsilon^2} = \frac{Mr^3(1+\frac{Q}{r})^{n-1} [1+(1-n)\frac{Q}{r}]}{[r(1+\frac{Q}{r})^{n-2M}]^2} \quad (12)$$

The general form of the pseudo-Newtonian potential

$$V_{PW} = \int F_{CF} dr, \quad F_{CF} = \frac{\Omega_k}{r^3} \quad (13)$$

Here F_{CF} is centrifugal force. Taking into consideration (13) one can easily calculate the PW potential as

$$V_{PW} = -\frac{M}{r(1+\frac{Q}{r})^{n-2M}} \quad (14)$$

Equation (14) at $Q = 0$ reduces to the potential for a Schwarzschild BH

$$V_{PW} = -\frac{M}{r-2M} \quad (15)$$

FIG. 4: The topmost panel illustrates how the PW potential for marginally bounded neutral particles increases around a RBH for a fixed charge Q while varying n . The bottom panel illustrates the same

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pattern for a fixed degree of nonlinearity n while varying Q . In figure 4 we plot the radial profile of the PW potential to illustrate the effects of the RBH charge Q and the degree n . The presence of both

parameters Q and n causes the PW potential to increase. This corresponds to a shorter radial distance to the RBH for larger values of n

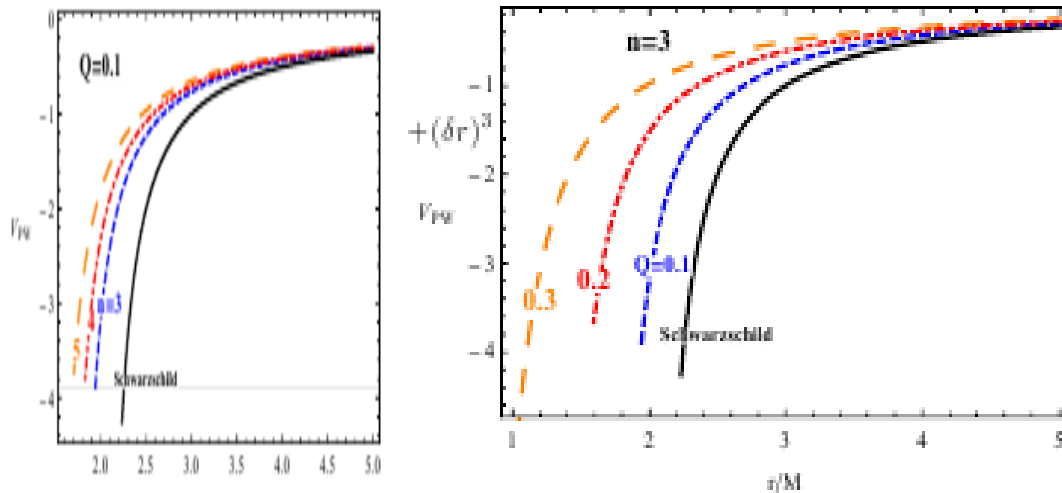


FIG. 4

V. Conclusion

We studied in detail properties of the RBH space-time obtained by coupling general relativity to nonlinear electrodynamics [1], focusing on the solutions having the proper Maxwell weak-field limit of the nonlinear model of electrodynamics. We concentrated on the properties of the space-time curvature, the electric field, and the motion of neutral particles and electrically charged particles. An analytical expression for the radius of the outer event horizon for the cases of $n = 1/3$ and $n = 1/4$ was obtained, and it was shown that the radius of the event horizon decreases as the RBH charge Q , and degree of nonlinearity n , increase.

The rate at which the event horizon decreases as these parameters increase speeds up much more than in the Reissner Nordstrom case. For the electric field around the RBH we can state that an exact expression for the radial component of the electric field strength was derived. It was understood that the strength of the electric field E increases at large distances when $n = 1/4$

and as Q is increased. In extreme charged RBH case the value of E-field becomes negative, near the event horizon.

The motion of neutral particles was considered and it was shown that (a) the ISCO and marginally bounded radius decreases as Q and n are both increased. The rate of this decrease is larger for RBHs than in the RNBH case. (b) The value of the effective potential for neutral particles increases with increasing the values of the parameters Q and n . This research is supported by Grants No. VA-FA-F-2-008 and No. YFA-Ftech-2018-8 of the Uzbekistan Ministry for Innovation Development, F.4-18 of the Uzbekistan Academy of Sciences, and by the Abdus Salam International Centre for Theoretical Physics through Grant No. OEA-NT-01. This research is partially supported by an Erasmus + exchange grant between SU and NUUZ.BA thanks the Institut für Theoretische Physik and the Silesian University for the warm hospitality during his stay in Frankfurt and Opava.

References:

1. Toshmatov, B., Stuchlík, Z., & Ahmedov, B. (2018). *Phys. Rev. D* **98**, 028501, arXiv:1807.09502 [gr-qc].
2. Wald, R. M. (1974). *Phys. Rev. D* **10**, 1680.

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IBI (India) = 4.260
OAJI (USA) = 0.350

3. Aliev, A. N., & G`umr`uk,c`uo`glu, A. E. (2005). *Phys. Rev. D* **71**,104027, hep-th/0502223.
4. Abdujabbarov, A. A., Ahmedov, B. J., & Kagramanova, V. G. (2008). *General Relativity and Gravitation* **40**, 2515, arXiv:0802.4349 [gr-qc].
5. Abdujabbarov, A., & Ahmedov, B. (2010). *Phys. Rev. D* **81**,044022, arXiv:0905.2730 [gr-qc].
6. Grib, A. A., & Pavlov, Y. V. (2011). *Gravitation and Cosmology* **17**, 42, arXiv:1010.2052 [gr-qc].
7. Grib, A.A., & Pavlov, Y. V. (2011). *Astroparticle Physics* **34**,581, arXiv:1001.0756 [gr-qc]
8. Mirzaev, M.S., Samiev, K.A., & Mirzaev, S.M. (2019). Experimental Study of Distance between Evaporator and Condensate of Inclined Multistage Desalination Plant. *Applied Solar Energy*, 55:1, pp. 36-40.
9. Djuraev, D.R., Niyazov, L.N., Saidov, K.S., & Sokolov, B.Yu. (2011). Changing the cubic ferrimagnetic domain structure in temperature region of spin flip transition. *Uzbekiston Fizika Zhurnali*, 13:5, pp. 359-366.
10. Valiev, U.V., Dzhuraev, D.R., Malyshev, E.E., & Saidov, K.S. (1999). Electronic structure of the ground multiplet of the Dy³⁺ ion in the DyAlO₃ orthoaluminate. *Optics and Spectroscopy*, 86:5, pp. 703-706.