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## NATURAL OSCILLATIONS OF VISCOELASTIC CONICAL SHELL


#### Abstract

In this article, the integral-differential equations of natural vibrations of a viscoelastic truncated conical shell are obtained on the basis of the shell equation. Geometrically nonlinear mathematical models of deformation of conical shells are obtained, taking into account the rheological properties of the material. Based on the method of variable separation, a method for solving and an algorithm for equations of natural vibrations of a viscoelastic truncated conical shell with pivotally and freely supported edges is developed. The problem is reduced to solving homogeneous algebraic equations with complex coefficients of large order. For a solution to exist, the main determinant of a system of algebraic equations must be zero. From this condition, we obtain a frequency equation with complex output parameters. The study of natural vibrations of viscoelastic truncated conical shells is carried out and some characteristic features are revealed. The complex roots of the frequency equation are determined by the Muller method. at each iteration of the Muller method, the Gauss method is used with the main element highlighted. As the number of edges increases, the real and imaginary parts of the natural frequencies increase, respectively. Taking into account the rheological properties of the material allows you to increase the frequency values of the shell up to $10 \%$.

Key words: conical shell panel, the non-linear model, the oscillations of visco elasticity, the frequency equation, the frequency.

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## Introduction

Shell calculation is one of the most urgent problems of deformable solid mechanics. This is due to the widespread use of shells in various fields of engineering and construction. The use of shells in shipbuilding, aircraft construction, and rocket technology leads to the need to determine their dynamic characteristics. Of great practical interest is the study and elimination of resonant phenomena in shells. A significant number of theoretical and experimental works have been devoted to the study of natural oscillations of circular cones. However, there are still no reliable solutions that allow us to determine the parameters of resonances in a wide range of changes in physical and geometric parameters. There are also works in which dependences for determining the resonant frequencies [1] and the vibration forms of truncated conical panels are obtained by theoretical and experimental method [2, 3]. The other method is mainly used for the study of shells, which allow us to move from the stability equations of conical shells to the corresponding equations for cylindrical shells with a circular cross-section. Many papers use the momentfree and semi-moment-free shell theory [4, 5]. Approximate methods are also used for solving problems of natural oscillations [6, 7]. The problems of vibrations of reinforced conic shells in a geometrically nonlinear formulation, taking into account the rheological properties of the material, are particularly difficult, and there are practically no solutions for them. Analysis of the literature shows that the existing optimal shell designs for a given geometric and rheological parameters cannot be implemented in practice, and the level of research remains only theoretical. In this regard, despite the long history of the solution, the problem of determining the resonant frequency of natural vibrations, taking into account the rheological properties of shells, remains relevant.

The purpose of this work is to develop a method, algorithm, and program for finding resonant frequencies and waveforms for circular viscoelastic conical shells under various boundary conditions.

## Geometric parameters and strain parameters

Let's direct the $X$ - axis along the generatrix of the cone (see Fig. 1); $\alpha$ denote by the angle of the shell taper; $R_{0}$ and $R_{1}$ - respectively, the radii of the smaller base. Obviously, the radius of an arbitrary ring section will be [8, 9]

$$
\begin{equation*}
r=R_{0}+x \sin \alpha \tag{1}
\end{equation*}
$$

The position on a parallel circle is determined by the angle $\theta$. Adhering to the notation used in shell theory [10], we obtain the following expressions of geometric parameters:
$a=x, A=1, \beta=0, B=r ; \frac{1}{R_{1}}=\frac{1}{R_{12}}=0 ; \frac{1}{R_{2}}=\frac{\cos \alpha}{r}$.
To obtain the parameters of deformations, we denote the movement along the normal to the median surface through $w$, tangent to the circle of radius $r$, though $v$, along the generatrix through $u$. Then for the parameters of the tensile strain we get

$$
\begin{gather*}
\varepsilon_{x}=\frac{\partial u}{\partial x}, \varepsilon_{\theta}=\frac{1}{r}\left(\frac{\partial v}{\partial \theta}+u \sin \alpha-w \cos \alpha\right) .  \tag{3}\\
\omega=\frac{1}{r} \frac{\partial u}{\partial \theta}+r \frac{\partial}{\partial x}\left(\frac{v}{r}\right)
\end{gather*}
$$

Bending strain parameters

$$
\begin{equation*}
\chi_{1}=\frac{\partial^{2} w}{\partial x^{2}}, \tag{4}
\end{equation*}
$$

$$
\begin{aligned}
& \chi_{2}=\frac{1}{r^{2}}\left(\frac{\partial^{2} w}{\partial \theta^{2}}+\frac{\partial v}{\partial \theta} \cos \alpha\right)+\frac{\sin \alpha}{r} \frac{\partial w}{\partial x}, \\
& \tau=\frac{1}{r}\left(\frac{\partial^{2} w}{\partial x \partial \theta}+\frac{\partial v}{\partial x} \cos \alpha\right)-\left(\frac{\partial w}{\partial \theta}+v \cos \alpha\right) \frac{\sin \alpha}{r^{2}} .
\end{aligned}
$$



Figure 1. Calculation scheme
The physical relations for an isotropic viscoelastic body take the form [11]

$$
\begin{aligned}
\sigma_{x} & =\frac{\tilde{\mathrm{E}}}{1-v^{2}}\left(\varepsilon_{x}^{z}+v \varepsilon_{y}^{z}\right) \\
\sigma_{y} & =\frac{\tilde{E}}{1-v^{2}} \cdot\left(\varepsilon_{y}^{z}+v \varepsilon_{x}^{z}\right)
\end{aligned}
$$

$$
\begin{gathered}
\tau_{x y}=\frac{\tilde{E}}{2(1+v)} \gamma_{x y}^{z} ; \tau_{x z}=\frac{\tilde{E}}{2(1+v)} \gamma_{x z} ; \\
\tau_{y z}=\frac{\tilde{E}}{2(1+v)} \gamma_{y z}
\end{gathered}
$$

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Here $\mu$ is the Poisson ratio of the shell material, which is assumed to be constant; $\tilde{\mathrm{E}}_{\mathrm{K}}$ operator elastic modulus of the conical shell,
$\tilde{E}_{k}[f(t)]=E_{0 \kappa}\left[f(t)-\int_{0}^{t} R_{E \kappa}(t-\tau) f(\tau) d \tau\right]$ $E_{0 \kappa}$ - Instantaneous young's modulus of elasticity ( $k=1,2,3 \ldots l$ ); $k=1$-instantaneous modulus of elasticity of the shell, $f(t)$ - continuous function; $R_{E \kappa}(t-\tau)$ - relaxation core.

## Selecting appoximating functions

The Eigen functions of $w, v, u$ the oscillation are chosen as the sum of the products of two functions: one that depends on $X$, and the other that depends on $\theta$, namely:

$$
\begin{aligned}
& w=\sum \sum A_{m n} W_{m}(x) \cos n \theta \\
& v=\sum \sum B_{m n} V_{m}(x) \sin n \theta \\
& u=\sum \sum C_{m n} U_{m}(x) \cos n \theta \\
& A_{m n}, B_{m n}, C_{m n}-\text { Custom parameters. }
\end{aligned}
$$

When choosing approximating functions, we follow the scheme adopted and tested on the calculation of the cylindrical shell, namely, we assume that $W_{m}(x)^{-}$the beam function, also, $X_{m}(x), V_{m}(x), U_{m}(x)$ the parameters $B_{m n}$ and $C_{m n}$ are not independent quantities, but are associated with $W_{m}(x)$ and $A_{m n}$ additional conditions. These terms come down to what we accept $\varepsilon_{0}=\omega=0$.

Substituting (5) into (3), we obtain that for arbitrary $\theta$ and $\varphi$ the following conditions must be met:

$$
\begin{align*}
& B_{m n} V_{m}(x) n+C_{m n} U_{m}(x) \sin \alpha-A_{m n} X_{m} \cos \alpha=0, \\
& B_{m n}\left[V_{m}^{\prime}(x)-\frac{V_{m}(x)}{r} \sin \alpha\right]-n C_{m n} \frac{U_{m}(x)}{r}=0 . \tag{6}
\end{align*}
$$

from the first equation, we obtain up to the longitudinal component

$$
\begin{equation*}
B_{m n} V_{m}(x)=\frac{A_{m n}}{n} X_{m} \cos \alpha \tag{7}
\end{equation*}
$$

Substituting the value $B_{m n} V_{m}(x)$ in the second equation, we find (6)

$$
C_{m n} U_{m}(x)=\frac{A_{m n}}{n^{2}} r \cos \alpha\left(X_{m}^{\prime}(x)-\frac{X_{m}(x)}{r} \sin \alpha\right) . \text { (8) }
$$

So, based on (7), (8) and $W_{m}(x)=X_{m}(x)$ the assumption that, instead of (5), we get the following expression for approximating functions:
$w=\sum \sum A_{m n} X_{m} \cos n \theta$,
$v=\sum \sum \frac{A_{m n}}{\bar{n}} X_{m} \cos \alpha \sin n \theta$,
$u=\sum \sum \frac{A_{m n}}{\bar{n}^{2}}\left(X_{m}^{\prime} r-X_{m} \sin \alpha\right) \cos \alpha \cos n \theta$.

## Determination of strain parameters depending on the selected approximation functions

Based on (5), expressions for the parameters of the tensile strain can be written as follows:

$$
\begin{align*}
& \varepsilon_{1}=\sum \sum \frac{A_{m n}}{n^{2}} X_{m}^{\prime} r \cos \alpha \sin n \theta,  \tag{10}\\
& \varepsilon_{2}=\sum \sum \frac{A_{m n}}{n^{2}}\left(X_{m}^{\prime}-\frac{X_{m}}{r} \sin \alpha\right) \frac{\sin 2 \alpha}{2} \cos n \theta .
\end{align*}
$$

$\varepsilon_{2}$ it follows from expression (10) that with this choice of functions, it vanishes only for shells of small taper or for a large value $n$.

The bending strain parameters will be $\chi_{1}=\sum \sum A_{m n} X_{m}^{\prime \prime} \cos n \theta$,
$\chi_{2}=\sum \sum A_{m n}\left[\frac{X_{m}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\frac{X_{m}^{\prime}}{r} \sin \alpha\right] X_{m} \cos n \theta$,
$\tau=\sum \sum A_{m n}\left(\frac{X_{m}^{\prime}}{r}-\frac{X_{m} \sin \alpha}{r^{2}}\right) \frac{\cos ^{2} \alpha-n^{2}}{n} \sin n \theta$.

## Calculation of coefficients of equations

The natural vibration frequencies of the conical shell are calculated using the Ritz method. To do this, we solve a system of equations of the form (10). We write the diagonal terms of the equation as follows:
$a_{m n}^{m n}=\frac{E h \pi}{1-\mu^{2}} \frac{\cos ^{2} \alpha}{n^{4}}\left\{\int_{0}^{1}\left[X_{m}^{\prime 2} r^{2}+\sin ^{2} \alpha\left(X_{m}^{\prime}-\frac{X_{m}}{r} \sin \alpha\right)^{2}+\right.\right.$
$\left.\left.+2 \mu \sin \alpha X_{m}^{\prime \prime} r\left(X_{m}^{\prime}-\frac{X_{m}}{r} \sin \alpha\right)\right] r d x\right\}+\frac{E h^{3}}{12\left(1-\mu^{2}\right)} \pi\left(\int_{0}^{1}\left\{X_{m}^{\prime 2}+\right.\right.$
$+\left[\frac{X_{m}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\frac{X_{m}^{\prime}}{r} \sin \alpha\right]^{2}+2 \mu X_{m}^{\prime \prime}\left[\frac{X_{m}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\right.$
$\left.\left.\left.+\frac{X_{m}^{\prime}}{r} \sin \alpha\right]+2(1-\mu)\left(\frac{\cos ^{2} \alpha-n^{2}}{n}\right)^{2}\left(\frac{X_{m}^{\prime}}{r}-\frac{X_{m} \sin \alpha}{r^{2}}\right)^{2}\right\} r d x\right)$,

$$
\begin{equation*}
b_{m n}^{m n}=\rho h \frac{\pi}{n^{4}} \int_{0}^{l}\left[\left(n^{4}+n^{2} \cos ^{2} \alpha\right) X_{m}^{2}+\right. \tag{13}
\end{equation*}
$$

$$
\left.+\left(X_{m}^{\prime} r-X_{m} \sin \alpha\right)^{2} \cos ^{2} \alpha\right] r d x
$$

The side coefficients of equations (12) usually do not vanish $X_{m}$ - the functions are chosen in such a way as to satisfy the condition, $\int_{0}^{l} X_{m} X_{p} d x=0(m \neq p)$ and $\int_{0}^{l} X_{m} X_{p} r d x=0^{\text {not }}$.
Therefore, we get

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|  | JIF | $=1.500$ | SJIF (Morocco) $=\mathbf{5 . 6 6 7}$ | OAJI (USA) | $=0.350$ |

$$
\begin{aligned}
& a_{m n}^{p n}=\frac{E h \pi}{1-\mu^{2}} \frac{\cos ^{2} \alpha}{n^{4}}\left\{\int _ { 0 } ^ { l } \left[X_{m}^{\prime 2} X_{p}^{\prime \prime} r^{2}+\sin ^{2} \alpha\left(X_{m}^{\prime}-\frac{X_{m}}{r} \sin \alpha\right)^{2} \times\right.\right. \\
& \times\left(X_{p}^{\prime}-\frac{X_{p}}{r} \sin \alpha\right)+\mu\left[X_{m}^{\prime \prime}\left(X_{p}^{\prime}-\frac{X_{p}}{r} \sin \alpha\right)+\right. \\
& \left.\left.+X_{p}^{\prime \prime}\left(X_{m}^{\prime}-\frac{X_{m}}{r} \sin \alpha\right)\right] r\right\} r d x+\frac{E h^{3} \pi}{12\left(1-\mu^{2}\right)} \times \\
& \times \int_{0}^{\prime}\left\{X_{m}^{\prime \prime} X_{p}^{\prime \prime}+\left[\frac{X_{m}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\frac{X_{m}^{\prime}}{r} \sin \alpha\right] \times\right. \\
& \times\left[\frac{X_{p}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\frac{X_{p}^{\prime}}{r} \sin \alpha\right]+ \\
& +\mu X_{m}^{\prime \prime}\left[\frac{X_{p}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\frac{X_{p}^{\prime}}{r} \sin \alpha\right]+\mu X_{p}^{\prime \prime} \times \\
& \times\left[\frac{X_{m}}{r^{2}}\left(\cos ^{2} \alpha-n^{2}\right)+\frac{X_{m}^{\prime}}{r} \sin \alpha\right]+2(1-\mu)\left(\frac{\cos ^{2} \alpha-n^{2}}{n}\right)^{2} \times \\
& \left.\times\left(\frac{X_{m}^{\prime}}{r}-\frac{X_{m} \sin ^{\prime} \alpha}{r^{2}}\right)\left(\frac{X_{p}^{\prime}}{r^{2}}+\frac{X_{p}^{\prime} \sin \alpha}{r^{2}}\right)\right\} r d x, \\
& b_{m n}^{p n}=\rho h \frac{\pi}{n^{4}} \int_{0}^{l}\left[\left(n^{4}+n^{2} \cos ^{2} \alpha\right) X_{m} X_{p}+\cos ^{2} \alpha \times\right. \\
& \left.\times\left(X_{m}^{\prime} r-X_{m} \sin \alpha\right)\left(X_{p}^{\prime} r-X_{p} \sin \alpha\right)\right] r d x .
\end{aligned}
$$

## Shell with hinged edges

Let's consider the simplest calculated case, namely, the case of hinged fastening of the shell edges.

In this case we assume

$$
\begin{equation*}
X_{m}=\sin \frac{m \pi x}{l}=\sin k_{m} x \tag{15}
\end{equation*}
$$

In this case, the necessary boundary conditions are obviously met at the edges of the shell, namely:

$$
\begin{equation*}
w=0, M=0 . \tag{16}
\end{equation*}
$$

(the Second condition (16) is accurate to ).
Before we start calculating the coefficients (12) - (14), we introduce the following notation:

$$
\begin{equation*}
R_{c p}=\frac{R_{1}+R_{0}}{2}, \lambda=\frac{R_{1}-R_{0}}{R_{0}} . \tag{17}
\end{equation*}
$$

Part of the integrals in closed form is not obtained, so by decomposing the integrand in a series and respectfully integrating, we get the values of the integrals we need.

To simplify writing, we denote

$$
\left.\begin{array}{l}
\int_{0}^{l} \frac{X_{m}^{2}}{r} d x=\frac{l}{2 R_{0}} L_{0},  \tag{18}\\
\int_{0}^{l} \frac{\cos 2 k_{m} x d x}{r} d x=\frac{l}{R_{0}} L_{1}, \\
\int_{0}^{l} \frac{X_{m}^{2}}{r^{3}} d x=\frac{l}{2 R_{0}^{3}} L_{2}
\end{array}\right\}
$$

where

$$
\begin{gather*}
L_{0}=1+\sum_{i}(-\lambda)^{i} q_{i}, \\
L_{1}=-\sum_{i} \lambda^{i}\left(\frac{1}{i+1}-q_{i}\right), \\
L_{2}=1+p_{0} \lambda+\sum_{i=1}^{n} \rho_{i} \lambda^{i+1},  \tag{19}\\
q_{i}=\sum_{n} \frac{(2 m \pi)^{2 n}}{2 n!} \frac{(-1)^{n+i}}{2 n+i+2}, \\
p_{0}=\sum_{n}(-1)^{n+1} \frac{(2 m \pi)^{2 n}}{2 n!}\left[\frac{n-1}{1+\lambda}-\frac{1}{2(1+\lambda)^{2}}-\frac{n(2 n-1)}{2 n+2}\right], \\
p_{i}=\sum_{n}(-1)^{n+1} \frac{(2 m \pi)^{2 n}}{2 n!} \frac{n(2 n-1)}{2 n+i+2},
\end{gather*}
$$

Where $m$ is the number of waves in the shell in the longitudinal direction.

Based on (18)-(19) and the table of elementary integrals, we obtain
$n^{4} \frac{R_{0} R_{c p}\left(1-\mu^{2}\right)}{l R_{c p} E} a_{m n}^{m n}=\cos ^{2} \alpha\left[k_{m}^{4} R_{0}^{3} R_{c p} \frac{(1+\lambda)^{2}+1}{2}+\right.$ $\left.+0,7 k^{2} R_{0} R_{c p} \sin ^{2} \alpha+L_{0} \sin ^{4} \alpha\right]+\beta n^{4}\left\{k^{4} R_{0}^{3} R_{c p}+L_{2}\left(\cos ^{2} \alpha-\right.\right.$ $\left.-n^{2}\right)^{2}\left(1+\frac{2\left(1-\mu^{2}\right)}{n^{2}} \sin ^{2} \alpha\right)+k^{2} R_{0}^{2}\left[\left[\frac{\ln (1+\lambda)}{\beta}\left[\sin ^{2} \alpha+\right.\right.\right.$
$\frac{n^{4}}{l R_{c p}} b_{m n}^{m n}=\left\{n^{4}+\cos ^{2} \alpha\left[n^{2}+\frac{9}{2} \sin ^{2} \alpha+k_{n}^{2} R_{0}^{2} \frac{(1+\lambda)^{2}+1}{2}\right]\right\}$.

We neglect the coefficients $a_{p n}^{m n}$ and $b_{p n}^{m n}$ in the first approximation, the frequency of natural vibrations of the conical shell can be determined by equating the diagonal term to zero, namely:

$$
\begin{equation*}
p_{1 m}^{2}=\frac{a_{m n}^{m n}}{b_{m n}^{m n}} . \tag{21}
\end{equation*}
$$

for large and large tapers, this formula will not be valid, since there is always a connection between the forms of vibrations and in the conical shell.

Therefore, it $p_{1}^{2}$ is necessary to calculate from the determinant of this type:

$$
\left|\begin{array}{ll}
a_{m, n}^{m n}-p_{1}^{2} b_{m n}^{m n} & a_{m, n}^{m+1, n}-p_{1}^{2} b_{m n}^{m+1, n}  \tag{22}\\
a_{m, n}^{m+1, n}-p_{1}^{2} b_{m, n}^{m+1, n} & a_{m+1, n}^{m+1, n}-p_{1}^{2} b_{m+1, n}^{m+1, n}
\end{array}\right|=0,
$$

Where the expression for the side and diagonal coefficients is given by the formulas (12)-(13).

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for one longitudinal half-wave and for 2,4,6,8 waves

## Numeric example

In [12], we tested and calculated two conical shells that are pivotally supported at the ends. Table 1 shows their geometric characteristics. We calculate $m=1, n=2,4,6,8$, the frequencies for the case i.e.
in the circumferential direction.

The calculation is made using the formula (21). Based on the formulas (13), we calculate the coefficients associated with the shell taper: . $L_{0}, L_{1} L_{2}$

Table 1. Geometric characteristics of shells

| № <br> shells | $R_{0}$ | $R_{1}$ | $\Delta R$ | $l$ | $\sin \alpha$ | $\cos \alpha$ | $h$ | $\frac{n^{2}}{12 R_{0}^{2}}=\beta$ |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 10 | 17,5 | 7,5786 | 60 | 0,1254 | 0,9928 | $1 \cdot 10^{-2}$ | $8,35 \cdot 10^{-6}$ |
| 2 | 12,5321 | 15,0 | 2,5067 | 85 | 0,0294 | 0,9997 | $1 \cdot 10^{-2}$ | $5,36 \cdot 10^{-6}$ |


| $\begin{gathered} \text { № } \\ \text { shells } \end{gathered}$ | $R_{c p}$ | $\lambda$ | $\sqrt{R_{0} R_{c p}}$ | $\sqrt{\frac{E}{\rho\left(1-\mu^{2}\right)}}$ | $\frac{(13)}{2 \pi(12)}$ | $\left(\frac{\pi}{l} R_{0}\right)^{2}$ | $\ln (1+\lambda)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 13,75 | 0,75 | 11,7760 | $5,3925 \cdot 10^{5}$ | $0,7221 \cdot 10^{4}$ | 0,275 | 0,5591 |
| 2 | 13,75 | 0,2 | 13,1032 | $5,3925 \cdot 10^{5}$ | $0,6413 \cdot 10^{4}$ | 0,211 | 0,198 |

For the shells under consideration, the angle is small $\alpha$, so in the expressions $a_{m n}^{m n}$ and $b_{m n}^{m n}$ we can neglect the values $\sin ^{2} \alpha, \sin ^{4} \alpha$ and put $\cos ^{2} \alpha=1$, then the approximate formula for $p^{2}$ is
$p_{1}^{2}=\left\{k_{m} \lambda^{4} R_{0}^{3} R_{c p}\left(1+\lambda+\frac{\lambda^{2}}{2}\right)+\beta n^{4}\left[\lambda^{4} R_{0}^{3} R_{c p}+\right.\right.$ $\left.+L_{2}\left(1-n^{2}\right)^{2}+2 k_{m} \lambda^{2} R_{0}^{2}\left(1-n^{2}\right)\right]\left(\frac{\ln (1+\lambda)}{\lambda}\left(\frac{0,7}{n^{2}-1}\right)+\right.$
$\left.\left.+L_{1}\left(2-\frac{0,7}{n^{2}}\right)\right)\right\} \frac{1}{\left[n^{4}+n^{2}+k_{m} \lambda^{2} R_{0}^{2}\left(1+\lambda+\frac{\lambda^{2}}{2}\right)\right]}$,
where

Table 2. Shell№1 $\alpha=7^{\circ} 40^{\prime}$ Shell №2 $\alpha=1^{\circ} 40^{\prime}$

| $n$ | $p$ | theory. |  | $p_{9}$ | theory. |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $p_{\text {b }}$ | cone $p$ |  | $p_{\text {b }}$ | cone $p$ |
| 2 | - | - | - | 276 | 311 | 331,9981 |
| 3 | 310 | 326 | 360,8673 | 200 | 192 | 191,7690 |
| 4 | 285 | 254 | 270,3401 | 244 | 219 | 222,0241 |
| 6 | 456 | 447 | 455,8703 | 410 | 454 | 450,1437 |

$p_{э}-$ Experimental frequencies, $p_{b}-$ job information [12].

$$
\beta=\frac{h^{3}}{12 R_{0}^{2}}
$$

Hz and the frequency of natural vibrations in

$$
p=\frac{p_{1}}{2 \pi \sqrt{R_{0} R_{c p}}} \sqrt{\frac{E}{1-\mu^{2}}}
$$

Table 2 shows the calculated experimental data for comparison.

In addition to our calculated values $p_{1}$ and $p_{2}$, based on the assumption of non-extensibility of the cross section and the absence of shifts in the median surface for the cone, we present calculated data[11] that fully take into account the deformations of the median surface.

From table. 2 it should. That the natural frequency of shell vibrations increases with increasing taper, and as for the accuracy of the theoretical methods of our and, they should be considered the same.

|  | ISRA (India) | $=4.971$ | SIS (USA) | $=\mathbf{0 . 9 1 2}$ | ICV (Poland) | $=\mathbf{6 . 6 3 0}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Impact Factor: | ISI (Dubai, UAE) $=\mathbf{0 . 8 2 9}$ | PИHL (Russia) $=\mathbf{0 . 1 2 6}$ | PIF (India) | $=\mathbf{1 . 9 4 0}$ |  |  |
| GIF (Australia) | $=0.564$ | ESJI (KZ) $=\mathbf{8 . 9 9 7}$ | IBI (India) | $=\mathbf{4 . 2 6 0}$ |  |  |
|  | JIF | $=\mathbf{1 . 5 0 0}$ | SJIF (Morocco) $=\mathbf{5 . 6 6 7}$ | OAJI (USA) | $=\mathbf{0 . 3 5 0}$ |  |

## Conclusions

The advantage of our methods is the ability to obtain simple calculation formulas. With increasing
frequency, it is obviously necessary to determine the eigenfrequencies from equation (22).

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