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2 LOCAL TWO-SIDED SIMMETRIC MULTIPLICATIONS IN THE BANACH ALGGEBRA OF MATRIX

Abstract: *This article is about learning the notion of 2 local two-sided symmetric multiplications in the Banach algebra of matrixes. The lemma and the theorem concerning the above mentioned matter are proven.*

Key words: *matrix, unit matrix, the Banach algebra, 2 local two-sided multiplication, equality.*

Language: *English*

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Introduction

Description: Let us mark the two-dimensional algebra as $M_2(R)$. Let us assume that if such $A \in M_2(R)$ exists in case of taking a random $x, y \in M_2(R)$ in purpose of reflection of $\Delta: M_2(R) \to M_2(R)$, and if to fulfil the $\Delta(x) = AXA$ $\Delta(y) = AYA$ equality in it, then Δ is defined as 2 local two-sided multiplication According to this notion, the following lemma is relevant:

Lemma: There is such a matrix as $A \in M_2(R)$ in the algebra of two-dimensional matrixes, and in case of two-sided multiplication for all $e_{ij} \in M_2(R)$, $i, j = 1{,}2$ unit matrixes, the $\Delta(e_{ij}) = Ae_{ij}A$ equality is fulfilled. Which means:

$$\Delta(e_{3.1}) = Ae_{3.1}A, \Delta(e_{12}) = Ae_{12}A$$

$$\Delta(e_{21}) = Ae_{21}A, \Delta(e_{22}) = Ae_{22}A.$$

Proof: Let us mark the product of the matrix of 2 local two-sided multiplication of all four unit matrixes.

Then the following equalities are relevant in this case.

$$\Delta(e_{3.1}) = Be_{3.1}B = Ce_{3.1}C = Ne_{3.1}N,$$

$$\Delta(e_{12}) = Be_{12}B = De_{12}D = Me_{12}M,$$

$$\Delta(e_{21}) = Ce_{21}C = Ge_{21}G = Me_{21}M,$$
$$\Delta(e_{22}) = Ge_{22}G = De_{22}D = Ne_{22}N.$$

Let us calculate the 2 local two-sided multiplication for each matrix.

$$\Delta(e_{3.1}) = Be_{3.1}B = \begin{pmatrix} b_{3.1} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} b_{3.1} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} b_{3.1}^2 & b_{3.1}b_{12} \\ b_{21}b_{3.1} & b_{21}b_{3.1} \end{pmatrix}$$

$$\Delta(e_{12}) = Be_{12}B = \begin{pmatrix} b_{3.1} & b_{12} \\ b_{21} & b_{22} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} b_{3.1} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} b_{3.1}b_{21} & b_{3.1}b_{22} \\ {b_{21}}^2 & b_{21}b_{22} \end{pmatrix}$$

$$\Delta(e_{21}) = Ce_{12}C = \begin{pmatrix} c_{3.1} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} c_{3.1} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} c_{3.1}c_{12} & {c_{12}}^2 \\ c_{3.1}c_{22} & c_{22}c_{12} \end{pmatrix}$$

$$\Delta(e_{22}) = Ge_{22}G = \begin{pmatrix} g_{3.1} & g_{12} \\ g_{21} & g_{22} \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} g_{3.1} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} = \begin{pmatrix} g_{12}g_{21} & g_{12}g_{22} \\ g_{21}g_{22} & {g_{22}}^2 \end{pmatrix}$$

The indexes of the elements of the product matrix are the same even if we calculate the multiplication of the two other matrixes while fulfilling the 2 local two-sided multiplication. Therefore,

$$Be_{3.1}B = Ce_{3.1}C = Ne_{3.1}N,$$

$$Be_{12}B = De_{12}D = Me_{12}M,$$

$$Ce_{21}C = Ge_{21}G = Me_{21}M,$$

$$Ge_{22}G = De_{22}D = Ne_{22}N$$

let us equalize all the their elements from the equality

$$b_{3.1}^2 = c_{3.1}^2 = n_{3.1}^2, \quad b_{3.1}b_{12} = c_{3.1}c_{12} = n_{3.1}n_{12}, \quad b_{3.1}b_{21} = c_{3.1}c_{21} = n_{3.1}n_{21}$$

$$b_{12}b_{21} = c_{12}c_{21} = n_{12}n_{21}, \quad b_{3.1}b_{21} = d_{3.1}d_{21} = m_{3.1}m_{21}, \quad b_{21}^2 = d_{21}^2 = m_{21}^2$$

$$b_{21}b_{22} = d_{21}d_{22} = m_{21}m_{22}, \quad c_{3.1}c_{12} = g_{3.1}g_{12} = m_{3.1}m_{12}, \quad c_{12}^2 = g_{12}^2 = m_{12}^2$$

$$c_{3.1}c_{22} = g_{3.1}g_{22} = m_{3.1}m_{22}, \quad c_{12}c_{22} = g_{12}g_{22} = m_{12}m_{22}, \quad g_{12}g_{21} = d_{12}d_{21} = n_{12}n_{21}$$

$$g_{12}g_{22} = d_{12}d_{22} = n_{12}n_{22}, \quad g_{22}g_{21} = d_{22}d_{21} = n_{22}n_{21}, \quad g_{22}^2 = d_{12}^2 = n_{12}^2$$

Let us assume that the matrixes which we are looking through consist of positive elements different only from zero. Then, the elements of quadratic equality are equal among each-other. Which means:

$$b_{3.1} = c_{3.1} = n_{3.1}, \quad b_{21} = d_{21} = m_{21}$$
$$c_{12} = g_{12} = m_{12}, \quad g_{22} = d_{22} = n_{22}$$

We get the following result if we apply these equalities to the above mentioned ones.

$$b_{3.1} = c_{3.1} = d_{3.1} = g_{3.1} = n_{3.1} = m_{3.1},$$

$$b_{12} = c_{12} = d_{12} = g_{12} = n_{12} = m_{12},$$

$$b_{21} = c_{21} = d_{21} = g_{21} = n_{21} = m_{21},$$

$$b_{22} = c_{22} = d_{22} = g_{22} = n_{22} = m_{22}$$

The result is B=C=D=G=N=M. It means that we have achieved the equality among all the matrixes. The proof is completed.

Theorem: $\in M_2(R)$ is being considered, in this case, if to take a random $x \in M_2(R)$ for the reflection of $\Delta: M_2(R) \to M_2(R)$ in the matrixes algebra as $5A = \{a_{ij}, a_{ij} > 0\}$, there is such $A \in M_2(R)$ in which the $\Delta(x) = AXA$ equality is fulfilled.

Proof: For a random $x \epsilon M_2(R)$

$$\Delta(x) = BxB, \quad \Delta(e_{3.1}) = Be_{3.1}B,$$

$$\Delta(x) = CxC, \quad \Delta(e_{12}) = Ce_{12}C$$

$$\Delta(x) = DxD, \quad \Delta(e_{21}) = De_{21}D,$$

$$\Delta(x) = FxF, \quad \Delta(e_{22}) = Fe_{22}F.$$

According to the lemma, there is such an A

$$Be_{3.1}B = Ae_{3.1}A, \quad Ce_{12}C = Ae_{12}A,$$

$$De_{21}D = Ae_{21}A, \quad Fe_{22}F = Ae_{22}A$$

that the equalities given are relevant. And this gives the following results:

$$\begin{cases} b_{3.1}^2 = a_{3.1}^2 \\ b_{3.1}b_{12} = a_{3.1}a_{12} \\ b_{3.1}b_{21} = a_{3.1}a_{21} \\ b_{21}b_{12} = a_{21}a_{12} \end{cases} \qquad \begin{cases} c_{3.1}c_{21} = a_{3.1}a_{21} \\ c_{3.1}c_{22} = a_{3.1}a_{22} \\ c_{21}^2 = a_{21}^2 \\ c_{21}c_{22} = a_{21}a_{22} \end{cases}$$

$$\begin{cases} d_{3.1}d_{12} = a_{3.1}a_{12} \\ d_{12}^2 = a_{12}^2 \\ d_{3.1}d_{22} = a_{3.1}a_{22} \\ d_{22}d_{12} = a_{22}a_{12} \end{cases} \qquad \begin{cases} f_{21}f_{12} = a_{21}a_{12} \\ f_{12}f_{22} = a_{12}a_{22} \\ f_{21}f_{22} = a_{21}a_{22} \\ f_{22}^2 = a_{22}^2 \end{cases}$$

we get these ones. It means:

$$a_{3.1} = b_{3.1} = c_{3.1} = d_{3.1}, \qquad a_{12} = b_{12} = f_{12} = d_{12},$$

$$a_{21} = b_{21} = c_{21} = f_{21}, \qquad a_{22} = c_{22} = d_{22} = f_{22}$$

For the comfort, let us enter the following markings.

$$\alpha_{3.1} = a_{3.1}^2 x_{3.1} + a_{3.1}a_{12}x_{21} + a_{3.1}a_{21}x_{12} + a_{12}a_{21}x_{22},$$

$$\alpha_{12} = a_{3.1}a_{12}x_{3.1} + a_{12}^2 x_{21} + a_{3.1}a_{22}x_{12} + a_{12}a_{22}x_{22},$$

$$\alpha_{21} = a_{21}a_{3.1}x_{3.1} + a_{3.1}a_{22}x_{21} + a_{21}^2 x_{12} + a_{21}a_{22}x_{22},$$

$$\alpha_{22} = a_{21}a_{12}x_{3.1} + a_{12}a_{22}x_{21} + a_{3.1}a_{22}x_{21} + a_{22}^2 x_{22},$$

$$\beta_{3.1} = b_{3.1}^2 x_{3.1} + b_{3.1}b_{12}x_{21} + b_{3.1}b_{21}x_{12} + b_{12}b_{21}x_{22},$$

$$\gamma_{21} = c_{21}c_{3.1}x_{3.1} + c_{3.1}c_{22}x_{21} + c_{21}^2 x_{12} + c_{21}c_{22}x_{22},$$

$$\delta_{12} = d_{3.1}d_{12}x_{3.1} + d_{12}^2 x_{21} + d_{3.1}d_{22}x_{12} + d_{12}d_{22}x_{22},$$

$$\varepsilon_{22} = f_{21}f_{12}x_{3.1} + f_{12}f_{22}x_{21} + f_{3.1}f_{22}x_{21} + f_{22}^2 x_{22}.$$

Then in this case,

$$\Delta(x) = BXB = \begin{pmatrix} \beta_{3.1} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}, \qquad \Delta(x) = CXC = \begin{pmatrix} \alpha_{3.1} & \alpha_{12} \\ \gamma_{21} & \alpha_{22} \end{pmatrix},$$

$$\Delta(x) = DXD = \begin{pmatrix} \alpha_{3.1} & \delta_{12} \\ \alpha_{21} & \alpha_{22} \end{pmatrix}, \qquad \Delta(x) = \mathrm{FX}F = \begin{pmatrix} \alpha_{3.1} & \alpha_{12} \\ \alpha_{21} & \varepsilon_{22} \end{pmatrix}.$$

Then,

$$\Delta(x) = BXB = CXC = DXD = FXF = \begin{pmatrix} \alpha_{3.1} & \alpha_{12} \\ \alpha_{21} & \varepsilon_{22} \end{pmatrix}.$$

Then, if to take a random $x \in M_2(R)$, there is such a $A \in M_2(R)$, here, $\Delta(x) = AXA$ can be seen. The theorem is proven.

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