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METHODS OF PRECISE AIRCRAFT POSITIONING IN THE GPS SYSTEM WITH AN APPLICATION OF THE TROPOSPHERE CORRECTION

Summary. This article presents the results of studies concerning the designation of accuracy in aircraft navigation positioning by means of the SPP and the SBAS code methods. The examination of the aircraft positioning accuracy was made in the aspect of the use of tropospheric correction in observation equations of the SPP and the SBAS positioning methods. The accuracy of the coordinates of the aircraft in the SPP and the SBAS solutions was referenced to the DGPS reference solution. The investigations were conducted on raw observation data and GPS navigation data in an air test in Dęblin. Based on the conducted calculations, it was proved that the lack of use of tropospheric correction in the SPP method causes an error in an aircraft position up to 18.5 m, and in the SBAS method up to 23.2 m. In addition, the statistical measure of RMS accuracy in the absence of applying the tropospheric correction in the SPP method results in an accuracy decrease to 8.6 m, and in the SBAS method to 12.2 m, accordingly.

Keywords: GPS system, troposphere correction, SPP method, SBAS method, DGPS method, accuracy

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1. INTRODUCTION

Along with the use of GPS satellite technology in aviation, there has been a rapid increase in the methods of aircraft precise positioning in the area of air navigation. The methods of aircraft positioning for GPS satellite technology can be divided into absolute methods (non-differential) and differential methods (relative) [16]. Moreover, among the absolute and differential methods, both code and phase observations of GPS measurements are exploited [3]. In addition, the absolute and differential positioning methods may be single- frequency, dual-frequency or multi-frequency [8].

The basic method of GPS positioning in aviation is a construction of observation equations, elimination or modelling of systematic errors as well as determining unknown parameters. The most common methods of GPS positioning in aviation are as follows: SPP code method [7], SBAS positioning method [4] and the DGPS differential method [5]. In the SPP code method, the determined parameters are the aircraft position and the GPS receiver on-board clock. The modelled parameters are satellite-receiver geometric distance, satellite clock error, ionospheric and tropospheric correction, relativistic correction, TGD hardware delay and a multipath effect [15]. The method of positioning designated SBAS parameters are also the position of the aircraft and the GPS receiver clock error. Furthermore, the modelled parameters are as well: the geometric distance satellite-receiver, satellite clock error, tropospheric and ionospheric correction, relativistic correction, TGD hardware delay and a multipath effect. It should be emphasised that the coordinates of the GPS satellite, the GPS satellite clock error, tropospheric and ionospheric correction are modelled by means of the SBAS correction algorithms [4]. In the DGPS positioning method, the determined parameters are finally the aircraft position and the clock difference of a GPS on-board receiver as well as a GPS receiver mounted on the reference station. Then, the modelled parameters are similarly: the satellite-receiver geometric distance, tropospheric and ionospheric correction and a multipath effect. However, the satellite clock error, relativistic correction and TGD hardware delay are eliminated from the observation equations by applying the differentiation operator [1].

Within this work, the authors intend to present the research results of aircraft positioning accuracy in the aspect of using the tropospheric correction in the navigation calculations. The position of the aircraft will be determined based on the SPP code method in the GPS system, the SBAS method for EGNOS corrections, and the DGPS differential method. This work explains how a systematic error in the form of the tropospheric correction affects the accuracy of aircraft positioning for the abovementioned three methods of satellite positioning in air navigation. The examination exploits real GPS data from an on-board receiver and a ground reference station. The recorded data were used for numerical calculations in the RTKLIB v.2.4.2. programme and to develop the results in the Scilab v.6.0.0. programme.

2. RESEARCH METHODOLOGY

A mathematical model to determine the position of the aircraft in the SPP code method, in the GPS system, can be described as below [12]:

$$l = \rho + c \cdot (dtr - dts) + Ion + Trop + \mathrm{Re}l + TGD + Mp \qquad (1)$$

where:

l - code measurement (pseudorange) registered by the airborne receiver in the GPS system,

ρ - the geometric distance satellite and the airborne receiver in the GPS system,

$\rho = \sqrt{\left(x - X_{GPS}\right)^2 + \left(y - Y_{GPS}\right)^2 + \left(z - Z_{GPS}\right)^2}$,

$\left(X_{GPS},\ Y_{GPS},\ Z_{GPS}\right)$ - satellites coordinates in the GPS system,

$\left(x,\ y,\ z\right)$ - aircraft coordinates in the geocentric XYZ frame,

c - speed of light,

dtr - receiver clock bias,

dts - satellite clock bias,

Ion - ionosphere correction,

$Trop$ - troposphere correction,

$\mathrm{Re}\,l$ - relativistic correction,

TGD - timing group delay,

Mp - multipath effect.

The mathematical model of the aircraft position determination in the SBAS method for EGNOS corrections can be described as below [10]:

$$l = \rho^* + c\cdot\left(dtr - dts^*\right) + Ion^* + Trop^* + \mathrm{Re}\,l + TGD + Mp + PRC \tag{2}$$

where:

l - code measurement (pseudorange) registered by the airborne receiver in the GPS system,

ρ^* - the geometric distance satellite and the airborne receiver in the GPS system, after long-term EGNOS correction,

$\rho = \sqrt{\left(x - X^*_{GPS}\right)^2 + \left(y - Y^*_{GPS}\right)^2 + \left(z - Z^*_{GPS}\right)^2}$,

$\left(X^*_{GPS},\ Y^*_{GPS},\ Z^*_{GPS}\right)$ - satellites coordinates in the GPS system, after long-term EGNOS correction,

$\left(x,\ y,\ z\right)$ - aircraft coordinates in the geocentric XYZ frame,

c - speed of light,

dtr - receiver clock bias,

dts^* - satellite clock bias, after long-term EGNOS correction,

Ion^* - ionosphere correction, based on GRID SBAS model,

$Trop^*$ - troposphere correction, based on RTCA-MOPS SBAS troposphere model,

$\mathrm{Re}\,l$ - relativistic correction,

TGD - timing group delay,

Mp - multipath effect,

PRC - fast EGNOS correction.

The mathematical model of determining the aircraft position in the DGPS differential method in the post-processing mode can be described as below [2]:

$$\Delta l = \Delta\rho + c\cdot\Delta dtr + \Delta Ion + \Delta Trop + \Delta Mp \tag{3}$$

where:

Δl - difference between pseudorange registered by the airborne receiver and reference station in the GPS system,

$\Delta\rho$ - difference between geometric distance: satellite-airborne receiver, and satellite-reference station in the GPS system,

c - speed of light,
Δdtr - difference between airborne receiver clock bias and reference station receiver clock bias,
ΔIon - difference between ionosphere correction for airborne receiver and reference station,
$\Delta Trop$ - difference between troposphere correction for airborne receiver and reference station,
ΔMp - difference between multipath effect for airborne receiver and reference station.

3. THE RESEARCH EXPERIMENT

In the examination test, scientific examinations were conducted to determine the accuracy of aircraft positioning in the aspect of exploiting the tropospheric correction in navigation computations. The aircraft position was determined given the SPP code method, the SBAS positioning method, and the DGPS differential method in post-processing. In the calculations, to determine the accuracy of aircraft positioning, a comparison between the designated coordinates in the SPP vs. DGPS and SBAS vs. DGPS solutions was made. Within the SPP and the SBAS method, the authors obtained two SBAS positioning solutions: the former included the tropospheric correction, whereas the latter disregarded the tropospheric correction. In the DGPS solution, the authors considered the tropospheric correction. Furthermore, the designated aircraft position in the DGPS solution is a reference position for the performed calculations. The SPP method model had the tropospheric correction model used as Saastamoinen model [9]. In the SBAS solution, the authors used a model of the tropospheric correction as the RTCA MOPS-SBAS model [11]. Then, in the DGPS solution, the authors used a model of the tropospheric correction as the Saastamoinen model. The calculations were made in the RTKLIB v.2.4.2 programme [17]. The calculations are based on the GPS data derived from the on-board receiver mounted in a Cessna 172. The data comes from a test flight over Dęblin. Moreover, in the DGPS differential method, the authors used the data derived from the GPS receiver mounted on the reference station in Dęblin. The SBAS method used corrections from the EGNOS S120 satellite. The calculations performed with an interval and time synchronisation were equal to 1 s. In addition, the remaining comparative analyses were performed in the Scilab v.6.0.0 programme [18].

In the first stage of the research, the authors determined the position of the Cessna 172 for the SPP method, twice: initially with the tropospheric correction, and later without its use. In the second stage of the research, it was possible to designate the position of the Cessna 172, first, for the SBAS method, and second, without it. In the next step, the authors designated the reference position of the Cessna 172 for the DGPS differential method, using the tropospheric correction. The accuracy analysis is presented in section 4.

4. RESULTS

The analysis of accuracy was made to compare the designated aircraft coordinates in the SPP, SBAS and DGPS solutions. The comparative analysis was performed for geocentric XYZ aircraft coordinates. In the second stage, the authors specified the accuracy of coordinates of the aircraft in the SPP code solution, as below [6]:

$$\begin{aligned} dx &= x_{SPP} - x_{DGPS} \\ dy &= y_{SPP} - y_{DGPS} \\ dz &= z_{SPP} - z_{DGPS} \end{aligned} \tag{4}$$

where:
$(x_{SPP}, y_{SPP}, z_{SPP})$ - obtained aircraft coordinates from Equation 1,
$(x_{DGPS}, y_{DGPS}, z_{DGPS})$- obtained aircraft coordinates from Equation 3.

Furthermore, for the results of Equation 4, the authors determined a statistic quantity, which determines the positioning accuracy in the form of RMS parameter, as below [13]:

$$\begin{aligned} RMS_{dx} &= \sqrt{\frac{\left[dx^2\right]}{N}} \\ RMS_{dy} &= \sqrt{\frac{\left[dy^2\right]}{N}} \\ RMS_{dz} &= \sqrt{\frac{\left[dz^2\right]}{N}} \end{aligned} \tag{5}$$

where:
N - number measurement epochs.

In the second stage, the authors specified the accuracy of aircraft coordinates in the SBAS code solution, as below [6]:

$$\begin{aligned} rx &= x_{SBAS} - x_{DGPS} \\ ry &= y_{SBAS} - y_{DGPS} \\ rz &= z_{SBAS} - z_{DGPS} \end{aligned} \tag{6}$$

where:
$(x_{SBAS}, y_{SBAS}, z_{SBAS})$- obtained aircraft coordinates from Equation 2,
$(x_{DGPS}, y_{DGPS}, z_{DGPS})$- obtained aircraft coordinates from Equation 3.

Furthermore, for the results of Equation 6, the authors determined a statistic quantity, which determines the positioning accuracy in the form of RMS parameter, as below [13]:

$$\begin{aligned} RMS_{rx} &= \sqrt{\frac{\left[rx^2\right]}{n}} \\ RMS_{ry} &= \sqrt{\frac{\left[ry^2\right]}{n}} \\ RMS_{rz} &= \sqrt{\frac{\left[rz^2\right]}{n}} \end{aligned} \tag{7}$$

where:
n - number measurement epochs.

Fig. 1 shows the results of the aircraft positioning accuracy using the SPP code method. The results in Fig. 1 considers the impact of the tropospheric correction in the process of computing the position of the aircraft in the SPP method. The aircraft positioning accuracy along the X-axis ranged from -4.2 to +3.4 m. Next, the aircraft positioning accuracy along the Y-axis ranged from -1.5 to +1.2 m. In addition, the accuracy of aircraft positioning along the Z-axis ranged from -1.7 to +2.2 m. It is worth emphasising that the average positioning accuracy is equal to +0.4 m along the X-axis, +0.2 m along the Y-axis, and +0.1 m along the Z-axis.

Fig. 2 shows the results of the aircraft positioning accuracy using the SPP code method. The results in Fig. 2 do not consider the effect of the tropospheric correction in the computational process of the aircraft position in the SPP method. The aircraft positioning accuracy along the X-axis ranged from -1.1 to +15.6 m. Next, the aircraft positioning accuracy along the Y-axis ranged from -0.1 to +6.9 m. In addition, the accuracy of aircraft positioning along the Z-axis ranged from +4.5 to +18.5 m. It is worth to note that the average positioning accuracy is equal to +7.7 m along the X-axis, +2.3 m along the Y-axis, and +8.4 m along the Z-axis.

Fig. 3 shows the results of the aircraft positioning accuracy using the SBAS code method. The results in Fig. 1 do not include the effects of the tropospheric correction in the computational process of the aircraft in the SBAS positioning method. The aircraft positioning accuracy along the X-axis ranged from +2.0 to +6.9 m. Next, the aircraft positioning accuracy along the Y-axis ranged from -1.0 to +0.5 m. In addition, the accuracy of aircraft positioning along the Z-axis ranged from +1.1 to +5.7 m. It is imperative to note that the average positioning accuracy is equal to +3.9 m along the X-axis, -0.3 m along the Y-axis, and +3.2 m along the Z-axis.

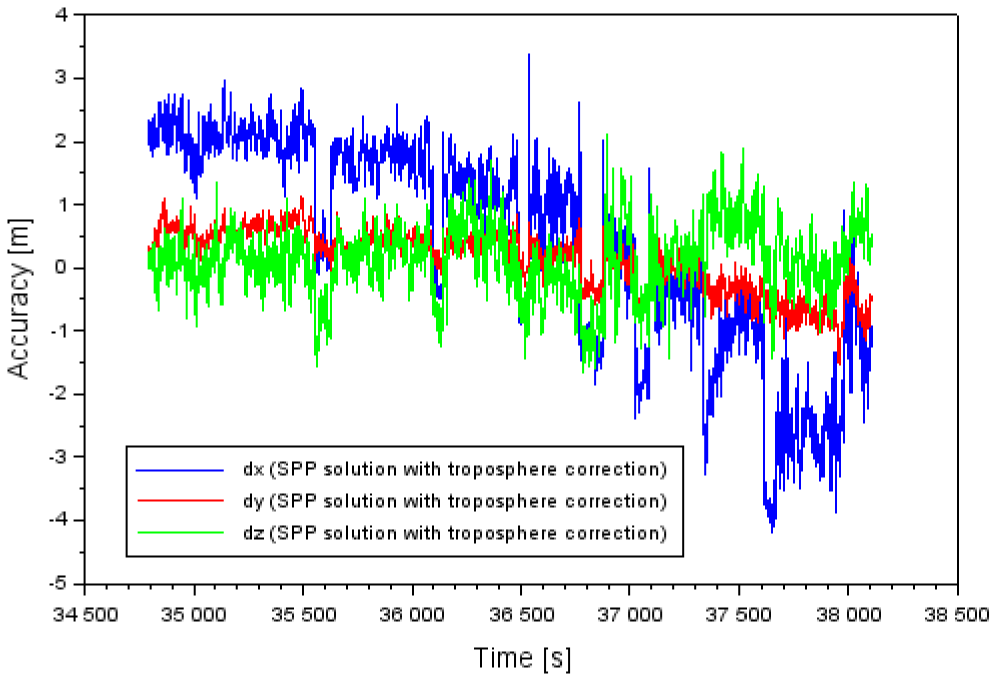

Fig. 1. The accuracy of aircraft position based on the SPP solution with troposphere correction [Source: Based on Scilab software]

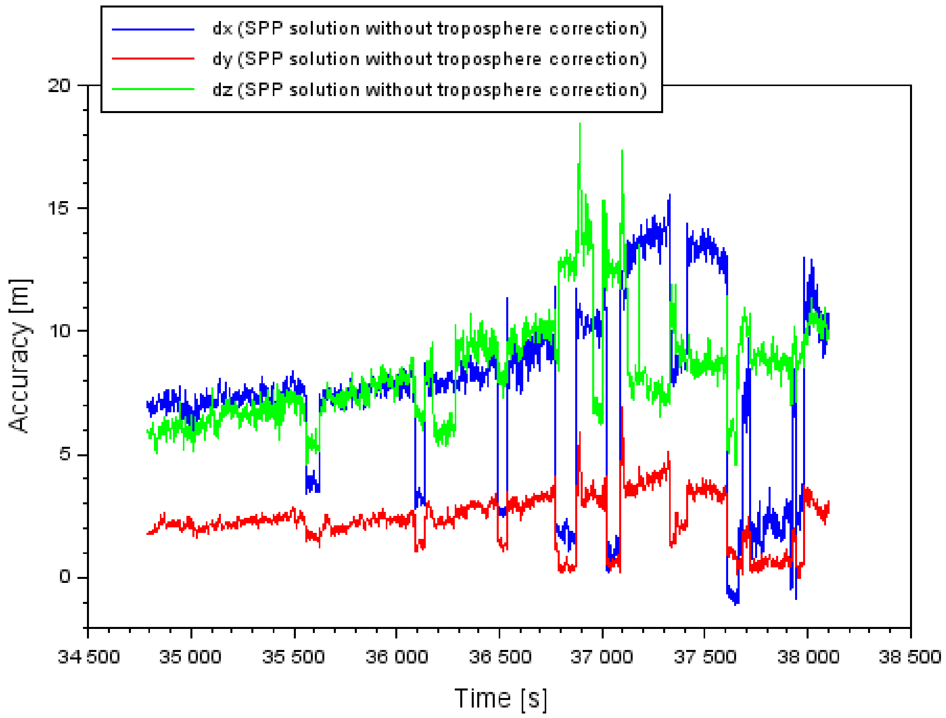

Fig. 2. The accuracy of aircraft position based on the SPP solution without troposphere correction [Source: Based on Scilab software]

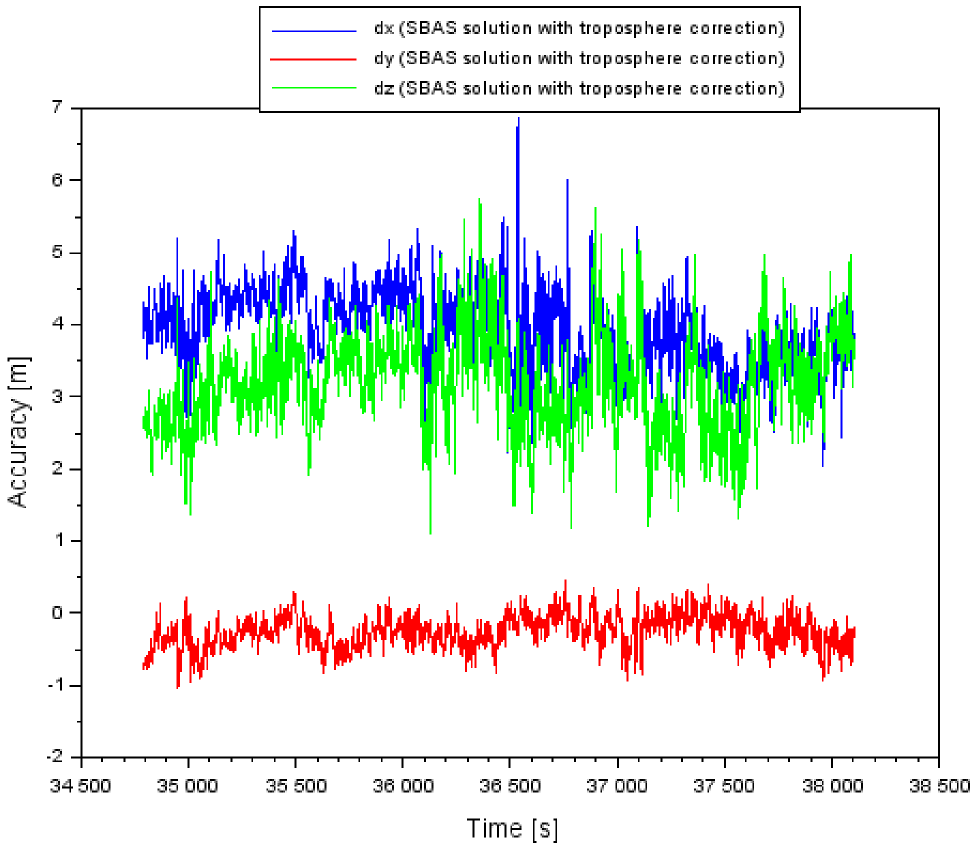

Fig. 3. The accuracy of aircraft position based on the SBAS solution with troposphere correction [Source: Based on Scilab software]

Fig. 4 shows the results of the aircraft positioning accuracy using the SBAS positioning method. The results in Fig. 1 do not include the impact of the tropospheric correction in the process of computing the position of the aircraft in the SBAS positioning method. The aircraft positioning accuracy along the X-axis ranged from +4.8 to +18.5 m. Next, the aircraft positioning accuracy along the Y-axis ranged from -0.6 to +5.2 m. In addition, the accuracy of aircraft positioning along the Z-axis ranged from +7.2 to +23.2 m. It is of considerable note that the average positioning accuracy is equal to +9.9 m along the X-axis, +1.4 m along the Y-axis, and +11.8 m along the Z-axis.

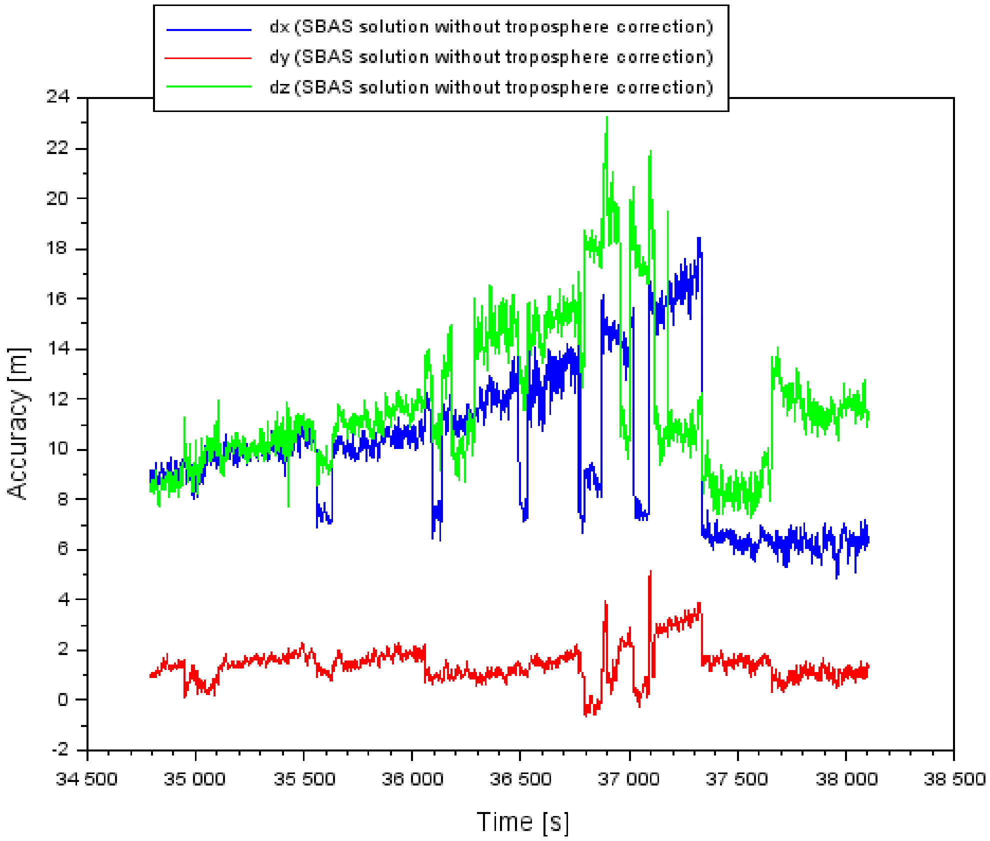

Fig. 4. The accuracy of aircraft position based on the SBAS solution without troposphere correction [Source: Based on Scilab software]

Figs. 5 and 6 illustrate the results of 3D-error aircraft position in a 3D plane. The shift value of the designated aircraft position in the SPP and the SBAS solutions against the reference position determined by the DGPS technique is defined as follows [14]:

$$3D-error = \begin{cases} \sqrt{dx^2 + dy^2 + dz^2} \\ \sqrt{rx^2 + ry^2 + rz^2} \end{cases} \tag{8}$$

The values of the 3D-error parameter for the SPP method range from 0.1 to 4.4 m, using the troposphere correction in navigation computations for the aircraft position. The 3D-error parameter values for the SPP method range from 4.7 to 22.4 m, when the tropospheric correction is not included in the navigation computations of the aircraft position. The 3D-error parameter value for the SBAS method ranges from 2.6 to 8.0 m, using the tropospheric correction in the navigation calculations of the aircraft position. Moreover, the values of the 3D-error parameter for the SBAS method range from 9.2 to 28.1 m, when the tropospheric correction is not included in navigation computations of the aircraft position. Based on the 3D-error parameter findings, it can be observed that disregarding the tropospheric correction in calculations results in massive degradation of an aircraft position against the reference trajectory, for example, even up to 22.4 m in the SPP method and 28.1 m in the SBAS method, respectively. Therefore, it can be concluded that the tropospheric correction is of huge importance in determining an aircraft position during flight operations.

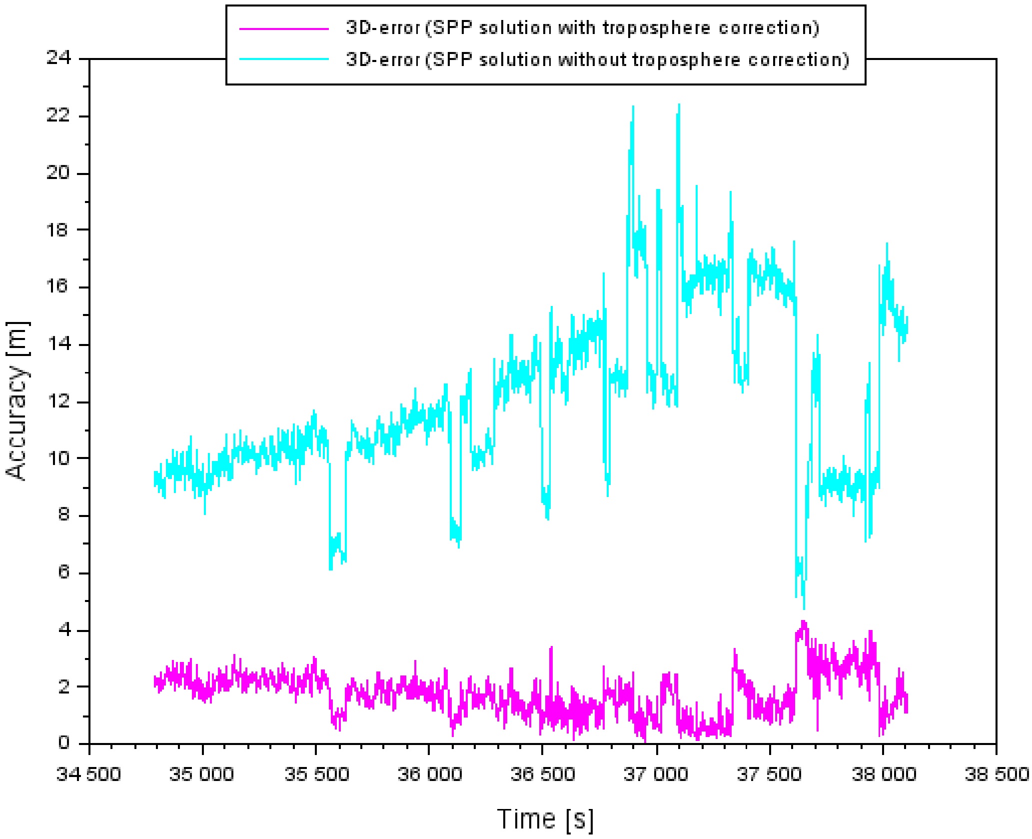

Fig. 5. The values of 3D-error of aircraft position accuracy based on the SPP solution [Source: Based on Scilab software]

Tab. 1

Comparison of obtained RMS parameter [Authors' study]

Positioning method	RMS along to X-axis [m]	RMS along to Y-axis [m]	RMS along to Z-axis [m]
SPP (with troposphere correction)	1.7	0.5	0.6
SPP (without troposphere correction)	8.4	2.6	8.6
SBAS (with troposphere correction)	4.0	0.3	3.2
SBAS (without troposphere correction)	10.4	1.6	12.2

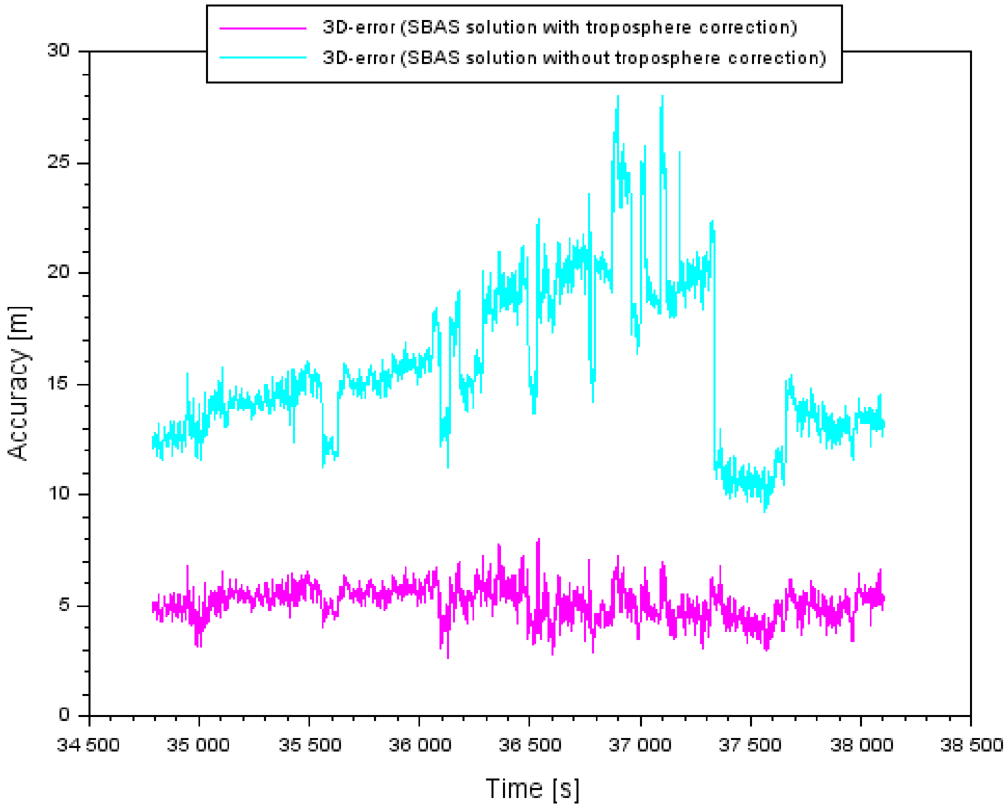

Fig. 6. The values of 3D-error of aircraft position accuracy based on the SBAS solution [Source: Based on Scilab software]

Tab. 1 shows the results of accuracy in the form of the statistical parameter RMS, in accordance with Equations 5 and 7. It can be observed that disregarding the tropospheric correction in the SPP method and the SBAS method causes significant degradation of the aircraft position. In the SPP method, the lowest RMS accuracy equals 1.7 m, using the tropospheric correction in the observation model. Next, the lack of troposphere correction in the SPP method causes degradation in aircraft position accuracy, even to the level of 8.6 m. In the SBAS method, the lowest RMS accuracy equals 4.0 m when using the tropospheric correction in the observation model. Besides, the lack of tropospheric correction in the SBAS method leads to the degradation of aircraft position accuracy, even to the level of 12.2 m.

5. CONCLUSION

This paper demonstrates the results of research into the aircraft positioning accuracy by means of the SPP, SBAS and DGPS methods. In particular, the study focuses on examining the impact of the tropospheric correction on aircraft positioning accuracy in air navigation. In practice, this work presents research tests showing how the use of the tropospheric correction in the SPP and the SBAS methods affect the accuracy of determining aircraft coordinates. The study uses real observation and GPS navigation data derived from an experimental air test made by the Cessna 172. Based on the conducted calculations, it was found that:

- the lack of use of the tropospheric correction in the SPP method results in a drop in accuracy of determining the XYZ aircraft coordinates even to the level of 18.5 m;

- the lack of use of the tropospheric correction in the SBAS method results in a drop in accuracy of determining the XYZ aircraft coordinates even to the level of 23.2 m;
- the lack of use of the tropospheric correction in the SPP method results in a rise in the RMS error to the level of 8.6 m;
- the lack of use of the tropospheric correction in the SBAS method results in a rise in the RMS error to the level of 12.2 m.

The obtained research findings, in the work, indicate that the systematic error in the form of the tropospheric correction exerts a tremendous influence on the designation of an aircraft position. Furthermore, ignoring the systematic error in an observational equation of determining the aircraft position may cause large degradation of accuracy in the designated coordinates of a moving object in air navigation. Hence, it is important to make a correct interpretation of observational equations in a given method for determining the position of a moving object in the aspect of modelling systematic errors.

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References

1. Bakuła M. 2007. “Static Network Code DGPS Positioning vs. Carrier Phase Single Baseline Solutions For Short Observation Time and Medium-Long Distances”. *Artificial Satellites* 42(3): 167-183. DOI: 10.2478/v10018-008-0009-9.
2. Bakuła M. 2010. “Network code DGPS positioning and reliable estimation of position accuracy”. *Survey Review* 42(315): 82-91. DOI: 10.1179/003962610X12572516251448.
3. Bosy J. 2005. “Precise processing of satellite GPS observations in local networks located in mountain areas”. *Zeszyty Naukowe Akademii Rolniczej we Wrocławiu* 522. ISSN: 0867-7964.
4. Ciećko A. 2019. “Analysis of the EGNOS quality parameters during high ionosphere activity”. *IET Radar, Sonar & Navigation* 13(7): 1131-1139. DOI: 10.1049/iet-rsn.2018.5571.
5. Grzegorzewski M., W. Jaruszewski, A. Fellner, S. Oszczak, A. Wasilewski, Z. Rzepecka, J. Kapcia, T. Popławski. 1999. “Preliminary results of DGPS/DGLONASS aircraft positioning in flight approaches and landings”. *Annual of Navigation* 1: 41-53.
6. Grzegorzewski M. 2005. “Navigating an aircraft by means of a position potential in three dimensional space”. *Annual of Navigation* 9: 1-11.
7. Grzegorzewski M., A. Ciećko, S. Oszczak, D. Popielarczyk. 2008. “Autonomous and EGNOS positioning accuracy determination of cessna aircraft on the edge of EGNOS coverage”. Proceedings of the 2008 National Technical Meeting of the Institute of Navigation. San Diego, CA. January 2008. P. 407-410.
8. Hofmann-Wellenhof B., H. Lichtenegger, E. Wasle. 2008. *GNSS – Global Navigation Satellite Systems: GPS, GLONASS, Galileo and More.* Springer Wien NewYork, Wien, Austria. ISBN: 978-3-211-73012-6.

9. Kleijer F. 2004. *Troposphere Modeling and Filtering for Precise GPS Leveling.* PhD thesis. Department of Mathematical Geodesy and Positioning, Delft University of Technology. Kluyverweg 1, P.O. Box 5058, 2600 GB DELFT, the Netherlands. 260 p.
10. Li L., Jia C., Zhao L., Cheng J., Liu J., Ding J. 2016. "Real-time single frequency precise point positioning using SBAS corrections". *Sensors* 16(8). DOI: 10.3390/s16081261.
11. MOPS. 1998. *Minimum operational performance standards for global positioning system/wide area augmentation system airborne equipment.* Document No. RTCA/DO-229A, June 8, 1998, prepared by SC-159.
12. Pan L., C. Cai, J. Zhu, X. Cui. 2019. „Kinematic absolute positioning with quad-constellation GNSS". *Kinematics - Analysis and Applications.* Joseph Mizrahi, IntechOpen. DOI: 10.5772/intechopen.86368. Available in: https://www.intechopen.com/books/kinematics-analysis-and-applications/kinematic-absolute-positioning-with-quad-constellation-gnss.
13. Przestrzelski P., M. Bakuła. 2014. "Performance of real time network code DGPS services of ASG-EUPOS in North-Eastern Poland". *Technical Sciences* 17(3): 191-207.
14. Rodríguez-Bilbao I., S. M. Radicella, G. Rodríguez-Caderot, M. Herraiz. 2015. "Precise point positioning performance in the presence of the 28 October 2003 sudden increase in total electron content". *Space Weather* 13: 698-708. DOI: 10.1002/2015SW001201.
15. Sanz Subirana J., Juan Zornoza J.M., Hernández-Pajares M. 2013. *GNSS Data Processing. Vol. I: Fundamentals and Algorithms.* Publisher: ESA Communications, ESTEC, Noordwijk, Netherlands. ISBN: 978-92-9221-886-7. ESA TM-23/1.
16. Seeber G. 2003. *Satellite Geodesy – 2nd completely revised and extended edition.* Walter de Gruyter GmbH & Co. KG, 10785 Berlin, Germany. ISBN: 3-11-017549-5.
17. Takasu T. 2013. *RTKLIB ver. 2.4.2 Manual, RTKLIB: an open source program package for GNSS positioning*. Available at: http://www.rtklib.com/prog/manual_2.4.2.pdf.
18. Scilab. Available at: https://www.scilab.org/.

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