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KINEMATIC PROPERTIES AND MESHING CONDITION OF ELLIPTICAL GEAR TRAIN

Summary. This paper presents the concept of eccentric elliptical gear train able to generate a variable gear ratio law. The first step in the noncircular gears virtual design process is the generation of the conjugate pitch curves, starting from a predesigned law of motion for the driven element or a predesigned geometry for the driving gear pitch curve. By designing a pair of non-circular gears, which are able to perform a proper gear ratio function, the output member of a mechanism can be effectively forced to move according to a prescribed law of motion, when operated at a constant input-velocity. This mechanism is designed to obtain a specific motion law. Detailed knowledge of meshing conditions is a prerequisite for studying kinematic conditions in gearings, as well as the strength calculation of gearing.
Keywords: non-circle gear, elliptical gear, kinematic conditions, meshing conditions.

1. INTRODUCTION

The idea of non-circular gears originates from the precursors of the engineering thought. These gears were sketched by Leonardo da Vinci, and found their application in many types of mechanical devices, like locks and toys. In the late 19th century, Franz Reuleaux ordered at

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Gustav Voigt Mechanische Werkstatt in Berlin, a series of non-circular gear models to help study kinematics. The gears made at those times had simplified tooth shapes and, for this reason, the meshing conditions were not always correct [1].

Several mechanical devices can be designed to obtain a prescribed motion law of output element. When a mechanical system is used to obtain a required motion of the output link, non-circular gears are another possible choice. In spite of their poor diffusion, the non-circular gears can be in a variety of mechanical systems. In fact, since the gear ratio function they generate is variable, a purely mechanical control can be performed on the input/output relationship. For this reason, non-circular gears are useful in those mechanisms whose task is to force an output element to move according to a specific law of motion. Automatic equipment in printing presses, textile industry, packaging machines and quick-return mechanisms represent the most diffuse applications.

A common challenge in the design of mechanical systems is the kinematic synthesis of a mechanism in order to satisfy a set of motion characteristics [2, 3]. Frequent requirements are to guide a rigid body through a series of specified positions and orientations (rigid body guidance), to force a coupler point to move along a prescribed trajectory (path generation), or to cause an output member to move according to a specific function of the input motion (function generation) [4].

The application of non-circular gears in function generating mechanisms has been proposed and discussed in [5, 6]. By designing a pair of non-circular gears, which are able to perform a proper gear ratio function, the output member of a mechanism can be effectively forced to move according to a prescribed law of motion, when operated at constant input velocity [7]. In typical arrangements, a pair of variable radius pitch curves are synthesized to drive a slider-crank mechanism according to prescribed motion law. For many applications, non-circular gears provide some benefits over cams, although they are more difficult to design and expensive to manufacture. The main advantages are the lower weight-to-strength ratios and the absence of gross separation or decoupling of moving parts [8].

The gearing with changing transmission gear ratio is used in the practice, even though the "standard" gearing with constant transmission gear ratio is used more often. This article examines the mesh conditions proposed in elliptical gears, as I speed and power ratios in this proposed gearings. The work is devoted to the analysis of these kinematic conditions in the proposed gearings and examines their differences from "standard" ring gear transmission with a constant transference number. The problem is solved for elliptical, eccentric gear with a continuously variable gear ratio.

2. DESIGNED NON-CIRCULAR GEARING

Generation of this noncircular gear was developed starting from the hypothesis such as the law of driven gear motion, variation of gear transmission ratio and design of driving gear pitch curve [9, 10]. This model of non-circular gear was designed for variable gear ratio to a range from 0.5 through 1 to 2. This transfer should be formed by two identical wheels with the number of teeth $z_1 = z_2 = 24$ and gearing module $m_n = 3.75$ mm, the axial distance $a = 90$ mm and for a one direction of rotation.

Given that each gear must satisfy the conditions of proper meshing, it was necessary to determine the geometric shape of the wheels. The pitch curve corresponded to the pitch circles in regular gears. They represent a non-circular gear as two rollers rolling together without slip, provided there is no addendum modification and the nominal axle distance is used.

The gearing is designed such that the pitch curve is composed of an ellipse formed with the basic parameters shown in Figure 1. A geometric centre of the gear is not the centre of wheel's rotation. The centre of gear's rotation is in the focus point of the ellipse.

The pitch ellipse has a large half-axis x = 45 mm, which is half of the axial distance. The second half-axis is determined by the distance from the focus point 45 mm (Fig.1), whose position is determined by considering the desired gear ratio.

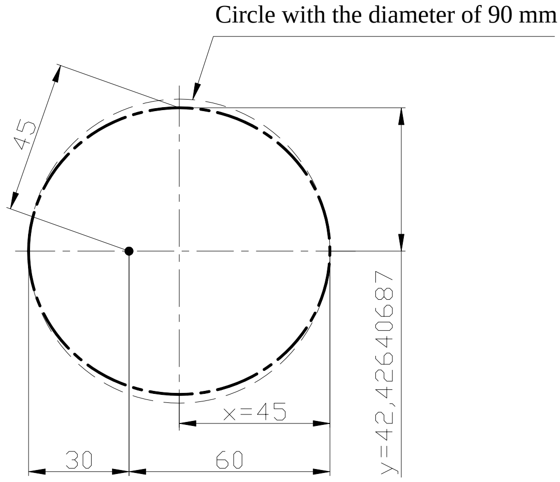

Fig. 1. Pitch ellipse for designed gearing

In pursuit of the kinematical conditions of the proposed gearings with eccentrically selected centres of rotation, we start from one of the conditions of a correct mesh, which states that the circumferential velocities in the pitch point are equal and their projections into the profile normal line are the same.

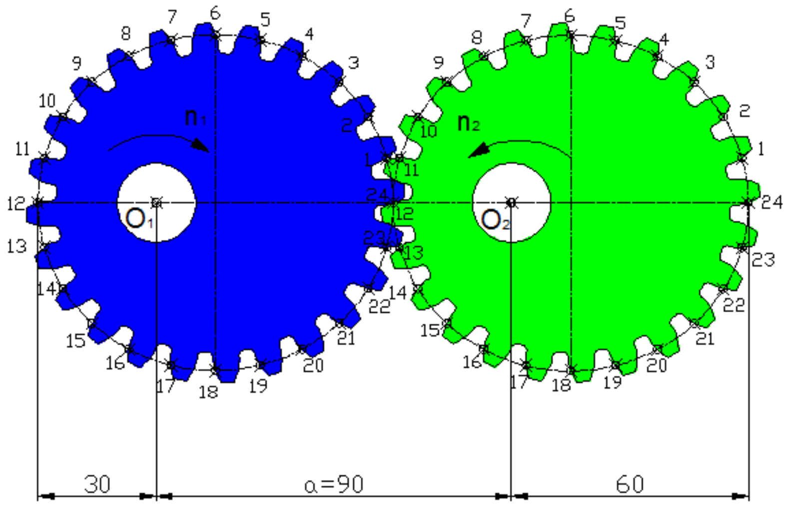

Fig. 2. Designed eccentric elliptical gear train

The conventional gearing involute starts from the base circle, in which case, it is the base of the evolute of the ellipse. The evolute for the left and right side teeth is not the same. Each of the twelve teeth is different; the next twelve teeth of the same wheel are the same. The side curve is the involute, and is different for the active and passive side of the tooth; the teeth are asymmetrical. The gears for a given variable transmission have been proposed as elliptical - eccentrically placed, so that conditions were right shot. The geometric model of the proposed gear is shown in Figure 2. Teeth of gearing are numbered; the picture depicts the mesh of the tooth No. 24 and No. 12.

3. MESHING CONDITION OF DESIGNED ELLIPTICAL GEARING

Detailed knowledge of the meshing conditions is a prerequisite not only for solution of the deformation of the gearing but also for the research of speed, power and energy conditions and for the strength calculation of gearing [11-13]. The characteristic points of meshing in the frontal plane of the spur gear are shown in Figure 3.

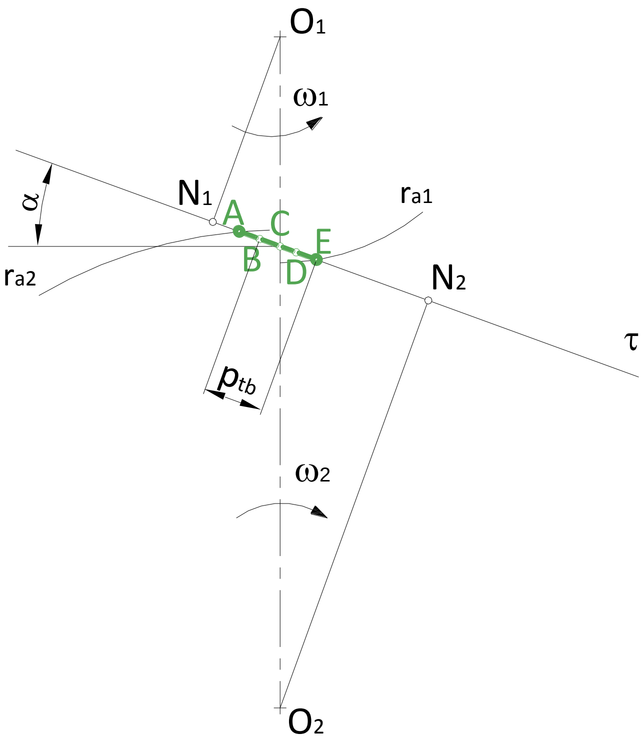

Fig. 3. Characteristic points of meshing

For the sense of rotation shown in Figure 3, the energy is transmitted from the gear pinion 1 to the gear wheel 2 (the gear 1 is the drive and the gear 2 is driven) and the meshing proceeds from point A to point E. Points A, E are the outer or end points of the pressure line AE. The sections AB and DE are sections of two-pair of meshing teeth, that is, when two pairs of teeth meshing the gears are in contact. When there is meshing point in these sections, so this pair of teeth is always considered with the meshing of the adjacent pair of teeth in the second section. The line section BD is a single-pair section, that is, there is only one pair of mating teeth on this section of the pressure line.

Involute gear is characterised by rectilinear pressure line [14]. This is true even for the designed elliptical gearing. The result of the examination for teeth 24 - 12 in meshing is shown in Figure 4.

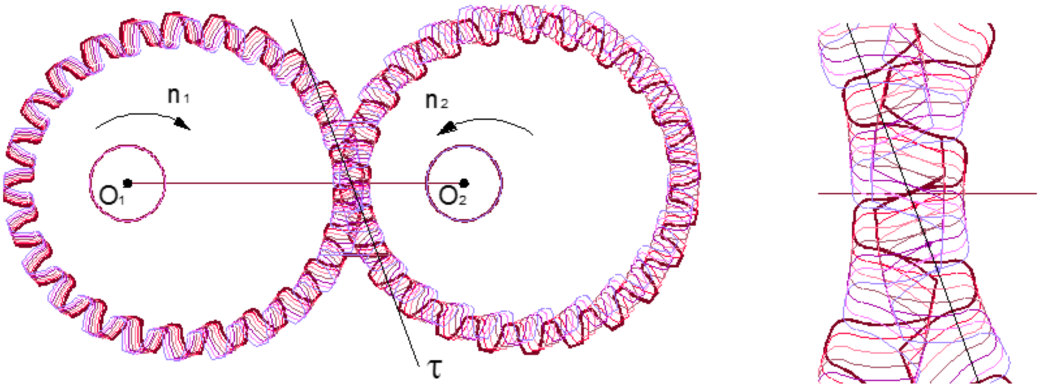

Fig. 4. Face of pressure line for designed gear train

The characteristic points A, E are the end points of the meshing line AE, which is component of pressure line τ, in the meshing line is realise meshing of teeth of gearing.

The length of meshing line for spur gears is calculated by equation:

$$\overline{AE} = \sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a \cdot \sin\alpha \quad (mm) \tag{1}$$

where $r_{a1,2}$ is radius of addendum circle, $r_{b1,2}$ is radius of base circle, a is axial distance and α is pressure angle.

The expression of meshing properties is used coefficient of meshing ε_α, calculate by equation:

$$\varepsilon_\alpha = \frac{\overline{AE}}{p_{tb}} \tag{2}$$

where p_{tb} is base pitch. The value of the coefficient of meshing for spur gears with straight teeth is in the range $1<\varepsilon_\alpha<2$.

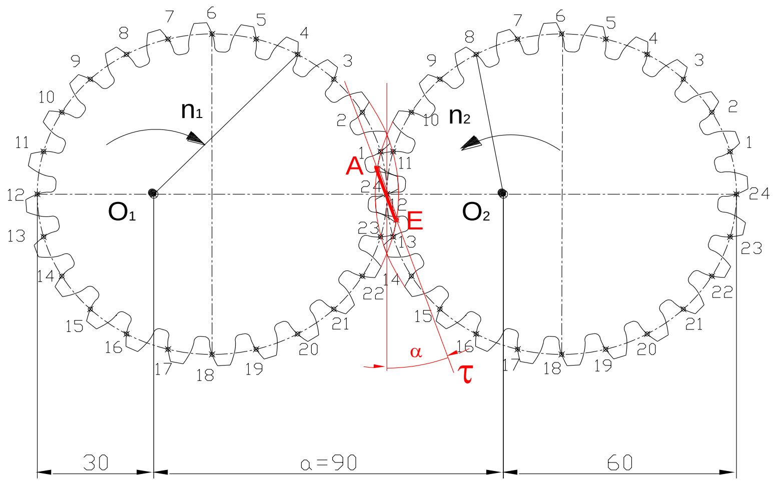

Fig. 5. The meshing line AE for teeth 24 - 12 in meshing

The length of meshing lineaof designed eccentrically elliptical gearing cannot calculate by equation for the circular involute gearing. Therefore, they are intended of graphic for each pair of teeth of meshing (for example, Figure 5 and Figure 6).

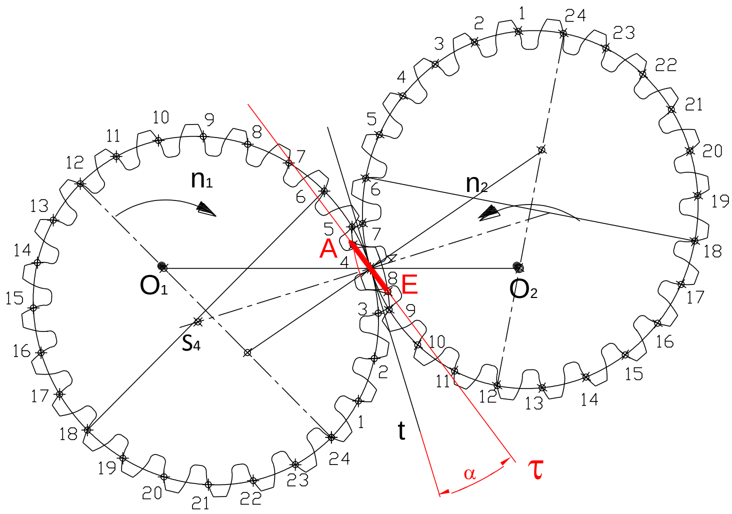

Fig. 6. The meshing line AE for teeth 4 - 8 in meshing

The measured value of length of meshing line AE and calculated value of meshing coefficient ε_α for designer elliptical and spur gearing are shown in Table 1.

Tab. 1

Coefficient of meshing ε_α and length of the meshing line AE

Elliptical gearing			Spur gearing (m_n=3,75mm)				
Meshing teeth input - output	AE (mm)	ε_α	r_{1-i} (mm)	r_{2-j} (mm)	Transmission ratio $u_i=r_2/r_1$	AE (mm)	ε_α
24 - 12	14.406	1.222	60	30	0.5	17.527	1.488
01 - 11	14.412	1.223	59.458	30.541	0.5136	17.542	1.489
02 - 10	14.436	1.225	57.891	32.108	0.5546	17.583	1.492
03 - 9	14.503	1.231	55.449	34.550	0.6230	17.636	1.497
04 - 8	14.52	1.232	52.338	37.663	0.7196	17.686	1.501
05 - 7	14.548	1.235	48.779	41.221	0.8450	17.721	1.504
26 - 6	14.594	1.239	45	45	1	18.235	1.647

There is greatest value of length meshing line AE for a one pair of teeth in designed non-circular gearing, which in meshing have a gear ratio equal value 1.0. The designed eccentric elliptical gearing has different profiles for all teeth, the gearing is comprised of teeth with different profiles. The coefficient of meshing ε_α and the length of the meshing line AE for each pair of teeth in meshing are not constant for this designed gear train. These values are constant for standard circular spur gear.

Necessary detailed knowledge of meshing condition is mandatory for solution deformation and stiffness of gearing as well as for the strength calculation of gearing. There is a rectilinear pressure (meshing) line for involute gear. For a standard circular involute gearing the length of meshing line is the same for all teeth pairs in the meshing. For designed elliptical gearing with variable gear ratio, the length of the meshing line and coefficient of meshing for each pair of teeth in meshing are not constant.

4. KINEMATIC CONDITION OF DESIGNED ELLIPTICAL GEARING

Kinematic conditions of designed elliptical gear train were processed for gear No. 1 (with the rotation centre at point O_1) and gear No. 2 (with the rotation centre at point O_2). The kinematic dependences for both designed gear wheels are shown in one graph together. There are teeth of the first gear wheel on the horizontal axis.

There is a continuously changing gear ratio for designed non-circular gear train in Figure 7, during one rotation of the drive gear wheel. Thus, the gear ratio changes over the time of one rotation of the drive gear wheel, from u=0.5 through u=1.0 until u=2.0 and back. If the value of gear ratio less than 1.0, this is an overdrive. The gear ratio value greater than 1.0 signifies a speed reduction.

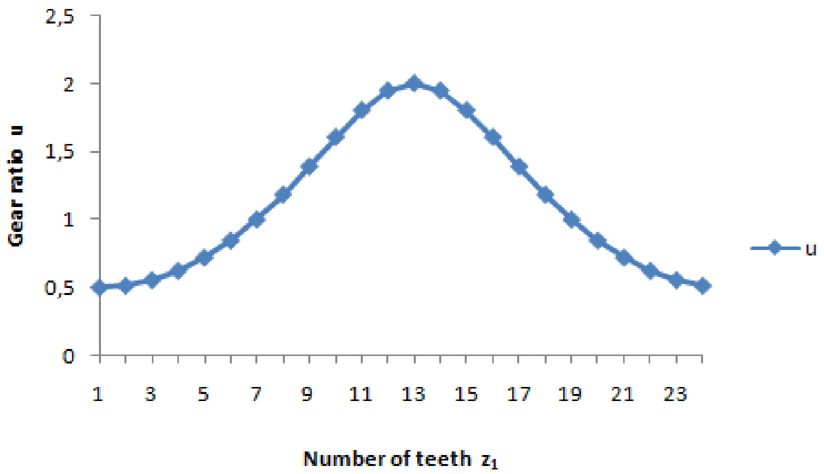

Fig. 7. Gear ratio of designed non-circular gear train

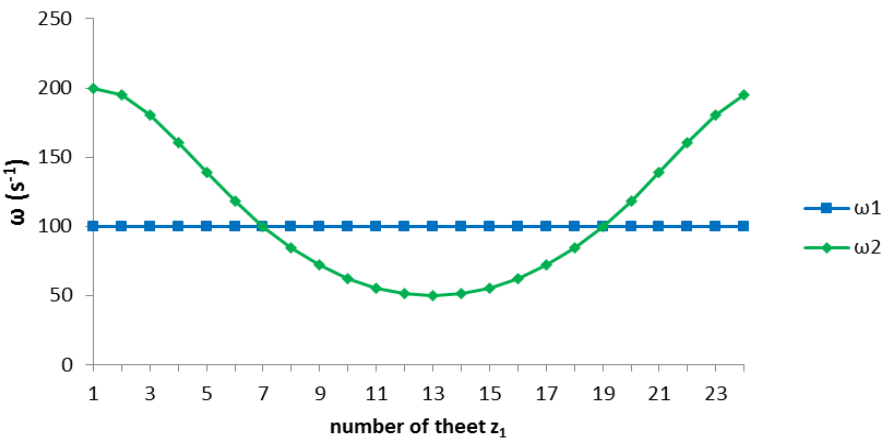

Fig. 8. Rotational speed of designed non-circular gear train

The standard spur gears have a constantly rotational speed on the drive wheel gear and on the driven wheel gear. This non-circular gearing with variable gear ratio has a not constant of rotational speed for the driven gear wheel. Figure 8 shows the change of the rotation speed on the driven gear wheel (ω_2) at a constant rotation speed for the drive gear wheel, for value ω_1 = 100 s^{-1}.

The movement of spur gear is defined in the face plane, therefore, kinematics of this motion is planar [16]. The points of gear are moved along a circular path at a circumferential velocity at the image central point C determined by equation:

$$v = v_1 = v_2 = r_1 \cdot \omega_1 = -r_2 \cdot \omega_2 \tag{3}$$

where $r_{1,2}$ is radius of pitch circle, $\omega_{1,2}$ is a rotational speed.

Figure 9 shows the speed ratios for the meshing of the teeth No. 4 (drive wheel) and teeth No. 8 (driven wheel) at the central point.

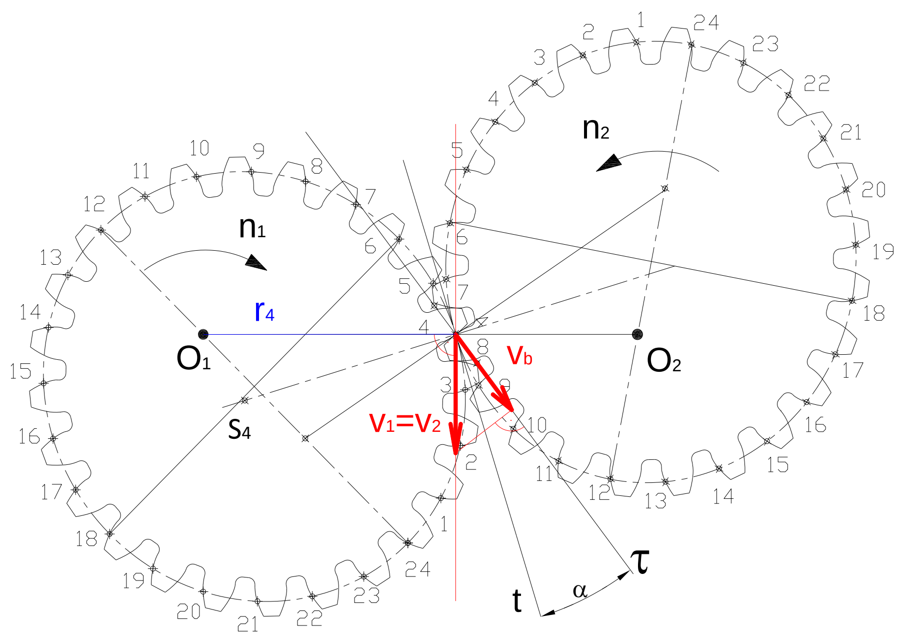

Fig. 9. Circumferential velocity for teeth 4 - 8 in meshing in the central point

In the case of the designed elliptical gearing, the size of the circumferential force in the central point of the shot is different for each pair in the meshing. The direction of the circumferential velocities of the individual pairs of meshing teeth is the same (Fig. 10).

The graph of circumferential velocity in the central points for the pairs of teeth in meshing is presented in Figure 11.

The circumferential velocity for designed non-circular gearing is not constant but changes continuously, depending on the number of gear ratio.

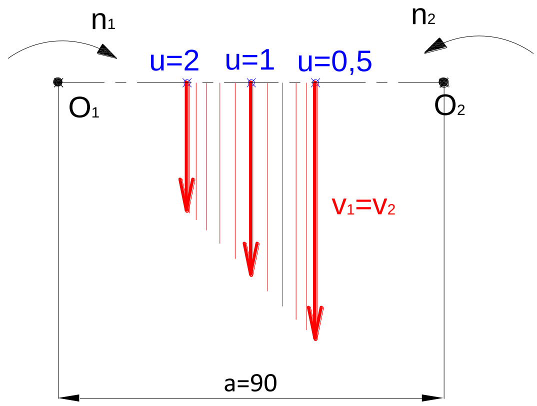

Fig. 10. Changing the size and position of the circumferential velocity at the central points of the pairs of teeth (u - is gear ratio)

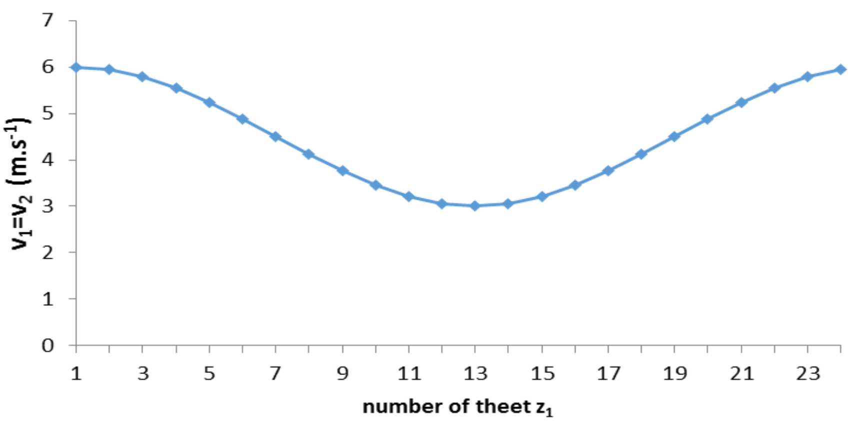

Fig. 11. Circumferential velocity in the central point

5. CONCLUSION

The examined eccentric elliptical gear train with continuously varying gear ratio is based on the requirements, for particular gear transmission parameters. These are gears with asymmetric tooth profile. Properties of this gearing are different from the properties of standard circular gears – spur gear. The gear ratio of this elliptical gear train is not constant but fluidly varies from 0.5 to 2.0 and back. In this way, the gear ratio changes during one revolution of the driven gear. For a standard circular involute gearing, the length of meshing line is the same for all teeth pairs in the meshing. For designed elliptical gearing with variable gear ratio, the length of the meshing line and coefficient of meshing for each pair of teeth in meshing are not the same. There, the size of the circumferential velocity in the central point for the pairs of the teeth is different for each pair in the meshing. The direction of the circumferential velocities of the individual pairs of meshing teeth is the same, the position of these circumferential velocities change with respect to the centres of rotation.

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