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On Fuzzy Distance in Fuzzy Graphs

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Abstract: Distance and distance related concepts are well studied and used in many applications of graph theory. In this we paper, we introduce and study a new idea of fuzzy distance in fuzzy graphs and investigate the concepts like fuzzy eccentricity, fuzzy centre, etc. and study the fuzzy distance matrix and centre of fuzzy graphs.

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1. Introduction

Graph theory had emerged as a major branch of applied mathematics in the last three decades and it is generally regarded as a branch of Combinatorics. Graph is a widely used tool for solving combinatorial optimization problems in different areas such as Geometry, Algebra, Number Theory, Topology, Optimization, Computer Science and Engineering. Most important thing which is to be noted is that, when we have uncertainty regarding either the set of vertices or edges or both, the model becomes a fuzzy graph. Distance and Central concepts play important roles in applications related with graphs and fuzzy graphs. In this paper, the authors introduced the concept of a new distance called fuzzy distance in fuzzy graphs. In the already existing distances, we simply sum the weights of edges in a geodesic between the given pair of nodes. The fuzzy distance between a given pair of nodes represents the amount of flow between these nodes so that this distance concept is more relevant, where the network modelled as a fuzzy network. In order to minimize the amount of money, time, energy, the location of the centre of a fuzzy graph is always needy and useful in real life problems.

A fuzzy graph [3] is a pair $G:(\sigma,\mu)$ where σ is a fuzzy subset of a set S and μ is a fuzzy relation on σ. We assume that S is finite and nonempty, μ is reflexive and symmetric. In all the examples σ is chosen suitably. Also, we denote the underlying crisp graph by $G^*:(\sigma^*,\mu^*)$ where $\sigma^*=\{u\in S:\ \sigma(u)>0\}$ and $\mu^*=\{(u,v)\in S\times S:\ \mu(u,v)>0\}$. A fuzzy graph $H:(\tau,\pi)$ is called a partial fuzzy subgraph of $G:(\sigma,\mu)$ if $\tau(u)\leq\sigma(u)$ for every vertex $u\in\sigma$ and $\tau(u,v)\leq\mu(u,v)$ for every pair of nodes $(u,v)\in\mu$. In particular a fuzzy graph $H:(\tau,\pi)$ is called a fuzzy subgraph of $G:(\sigma,\mu)$ if $\tau(u)=\sigma(u)$ for every vertex $u\in\sigma$ and $\tau(u,v)=\mu(u,v)$ for every pair of nodes $(u,v)\in\mu$. A path P of length n is a sequence of distinct nodes $(u_1,u_2,\dots u_n)$ such that $\mu(u_i,u_{i-1})>0$, $i=1,2,\dots,n$ and the membership of a weakest edge is defined as its strength. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths

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of all paths between x and y and is denoted by $CONNG(x, y)$. An $x-y$ path P is called a strongest $x-y$ path if its strength equals $CONNG(x; y)$ [4, 5]. A fuzzy graph $G : (\sigma, \mu)$ is connected if for every x, y in σ, $CONNG(x; y) > 0$. Unless otherwise specified, as assume that G is connected. For the graph theoretical terms and definitions not explicitly defined here, reader may refer Harary [1]. For the fuzzy set theoretical terms and definitions not explicitly defined here, reader may refer [6].

2. Main Results

In this section, a new type of distance in fuzzy graphs is introduced and studied.

Definition 2.1. *Let $G : (\sigma, \mu)$ be a fuzzy graph. Then the fuzzy distance between two nodes u and v is defined as $d_{vf}(u, v) = \wedge \sum \{\wedge(\sigma(u), \sigma(v)) * \mu(u, v)\}$ where $\wedge$ represents the minimum and $*$ represents the ordinary product.*

Remark 2.2. *The distance d_{vf} satisfies the properties of a metric so that (σ^*, d_{vf}) is a Metric Space.*

1. *$d_{vf}(u, v) \geq 0$ for all $u \neq v$ and $d_{vf}(u, v) = 0$ iff $u = v$ (Positivity).*

2. *$d_{vf}(u, v) = d_{vf}(v, u)$ for all $u, v \in \sigma^*$ (Symmetry).*

3. *$d_{vf}(u, v) \leq d_{vf}(u, w) + d_{vf}(w, v)$ for all $u, v, w \in \sigma^*$ (Triangle Inequality).*

Definition 2.3. *The fuzzy eccentricity of a node u in the fuzzy graph $G : (\sigma, \mu)$ is defined as $e_{vf}(u) = \bigvee_{v \in V} (d_{vf}(u, v))$ where $\vee$ represents the maximum.*

Definition 2.4. *The fuzzy radius of the graph $G : (\sigma, \mu)$ is the minimum of the fuzzy eccentricities of all nodes and denoted as $r_{vf}(G)$. i.e., $r_{vf}(G) = \bigwedge_{u \in V} (e_{vf}(u))$.*

Definition 2.5. *Fuzzy diameter of the graph G is the maximum of the fuzzy eccentricities of all nodes in G and denoted as $d_{vf}(G)$. i.e,. $d_{vf}(G) = \bigvee_{u \in V} (e_{vf}(u))$.*

Definition 2.6. *A node v in the fuzzy graph G is called fuzzy eccentric node of another node u if $e_{vf}(u) = d_{vf}(u, v)$. The set of all fuzzy eccentric nodes u is denoted by u^*_{vf}.*

Definition 2.7. *Nodes with minimum fuzzy eccentricity are called fuzzy central nodes or fuzzy radial nodes and nodes with maximum eccentricity are called fuzzy diametrical nodes.*

Theorem 2.8. *In any connected fuzzy graph G, $r_{vf}(G) \leq d_{vf}(G) \leq 2r_{vf}(G)$.*

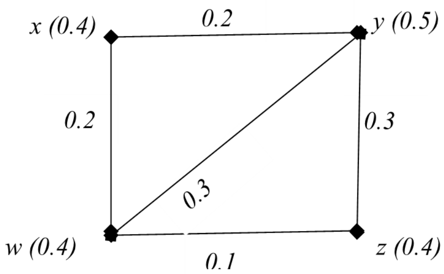
Proof. The inequality $r_{vf}(G) \leq d_{vf}(G)$ follows from definition. Let u, v be two nodes such that $d_{vf}(u, v) = d_{vf}(G)$. Let w be a fuzzy central node of G. By triangle inequality $d_{vf}(u, v) \leq d_{vf}(u, w) + d_{vf}(w, v) \leq r_{vf}(G) + r_{vf}(G) = 2r_{vf}(G)$. □

Definition 2.9. *Centre of a fuzzy graph G is the subgraph induced by the fuzzy central nodes and Periphery of the fuzzy graph G is the subgraph induced by the set of all fuzzy diametrical nodes.*

3. Fuzzy Self-centred Graphs

Now, we discuss the properties of self-centred graphs with respect to the new distance.

Definition 3.1. *A fuzzy graph G is fuzzy self-centred if it is isomorphic with its fuzzy centre.*



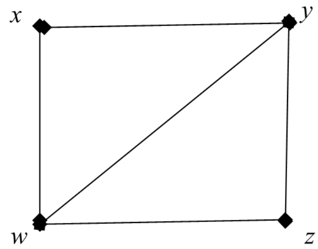


Figure 1.

In fuzzy graph, $d_{vf}\,(x,y) = 0.08$, $d_{vf}\,(x,z) = 0.12$, $d_{vf}\,(x,w) = 0.08$, $d_{vf}\,(y,w) = 0.12$, $d_{vf}\,(y,z) = 0.12$, $d_{vf}\,(z,w) = 0.04$. From the underlying graph, $d\,(x,y) = 1$, $d\,(x,z) = 2$, $d\,(x,w) = 1$, $d\,(y,z) = 1$, $d\,(y,w) = 1$, $d\,(z,w) = 1$. Hence, $e_{vf}\,(x) = e_{vf}\,(y) = e_{vf}\,(z) = e_{vf}\,(w) = 0.12$ so that $e\,(x) = e\,(z) = 2$, $e\,(y) = e\,(w) = 1$. Thus the fuzzy graph in Figure 1 is fuzzy self-centred, but the underlying graph (Figure 2) is not self-centred,

Theorem 3.2. *If a connected fuzzy graph G is fuzzy self-centred, then each node of G is fuzzy eccentric.*

Proof. Assume that G is fuzzy self-centred. Let u be an arbitrary node of G and let $u \in u^*_{vf}$. From the definition of fuzzy eccentric node, $e_{vf}(u) = d_{vf}(u,v)$. Since G is fuzzy self-centred, $e_{vf}\,(u) = e_{vf}\,(v)$ and hence $e_{vf}(u) = d_{vf}(u,v) = d_{vf}(v,u)$. Thus u is a fuzzy eccentric node of v. Hence every node of G is fuzzy eccentric. □

Theorem 3.3. *If a connected fuzzy graph G is fuzzy self-centred, then for every pair of nodes u, v such that whenever u is a fuzzy eccentric node of v, then v should be one of the fuzzy eccentric node of u.*

Proof. We assume that the fuzzy graph G is fuzzy self-centred. Let u be a fuzzy eccentric node of v. Then, $e_{vf}(v) = d_{vf}(v,u)$. Since G is a fuzzy self-centred graph, all nodes will have the same fuzzy eccentricity. $e_{vf}(v) = e_{vf}(u)$. Hence, $e_{vf}(u) = d_{vf}(v,u) = d_{vf}(u,v)$ so that $e_{vf}(u) = d_{vf}(u,v)$. Thus, v is a fuzzy eccentric node of u. □

Theorem 3.4 (Embedding Theorem)**.** *Every fuzzy graph G is the fuzzy centre of some fuzzy connected graph G.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph with n nodes. Construct a new graph H by adding four nodes $\{u, v, w, x\}$. Construct H as the sequential join $u + v + G + w + x$. Let k be the minimum weight among all the edges of G. Assign the value k to all the new edges of H. Then $e_{vf}(u) = e_{vf}(x) = 4k$, $e_{vf}(v) = e_{vf}(w) = 3k$. $e_{vf}(y) = 2k$, $\forall\ y \in G$. Thus every node of G is a fuzzy central node of H and hence G is a fuzzy centre of H.

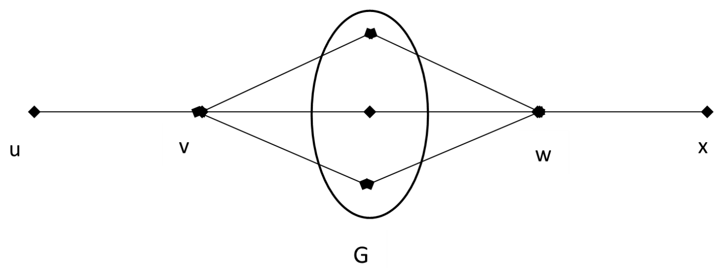


Figure 2.

□

4. Fuzzy Distance Matrix

Definition 4.1. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph with n nodes. The fuzzy distance matrix $D_{vf} = (d_{i,j})$ is a square matrix of order n and is defined by $(d_{i,j}) = d_{vf}(v_i, v_j)$.*

Definition 4.2. *The max-max composition of a square matrix with itself is a square matrix of same order whose $(i,j)^{th}$ entry is given by $d_{i,j} = \max\{\max(d_{i1}, d_{1j}), \max(d_{i2}, d_{2j}), \ldots, \max(d_{in}, d_{nj})\}$.*

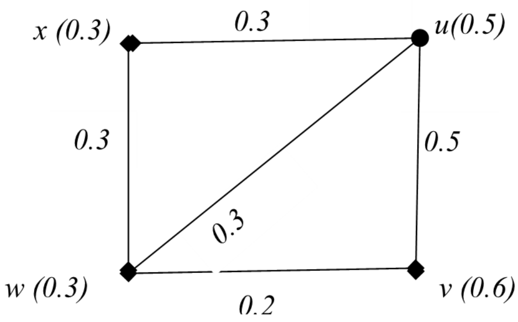


Figure 3.

The fuzzy distance matrix and its max-max composition are given below:

$$D_{vf} = \begin{bmatrix} 0 & 0.09 & 0.15 & 0.09 \\ 0.09 & 0 & 0.15 & 0.09 \\ 0.15 & 0.15 & 0 & 0.06 \\ 0.09 & 0.09 & 0.06 & 0 \end{bmatrix}, \qquad D_{vf} \circ D_{vf} = \begin{bmatrix} 0.15 & 0.15 & 0.15 & 0.15 \\ 0.15 & 0.15 & 0.15 & 0.15 \\ 0.15 & 0.15 & 0.15 & 0.15 \\ 0.15 & 0.15 & 0.15 & 0.09 \end{bmatrix}$$

Theorem 4.3. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph. The diagonal elements of the max-max composition of the fuzzy distance matrix of G with itself are the fuzzy eccentricities of the nodes.*

Theorem 4.4. *A connected fuzzy graph $G : (\sigma, \mu)$ is fuzzy self-centred if and only if all the entries in the principal diagonal of the max-max composition of the fuzzy distance matrix with itself are the same.*

Proof. The principal diagonal entries in the max-max composition of the fuzzy distance matrix with itself are the fuzzy eccentricities of the nodes. This means $e_{vf}(u)$ is same for all u in G. Then G is fuzzy self-centred. □

5. Conclusion

Fuzzy graphs are precise models of all kinds of networks. Distance is an important concept in the graph theory and Discrete Mathematics. In this paper, the authors made an attempt to generalize the concept of distance. The concept of fuzzy distance is relevant as it represents the net flow between a given pair of nodes of a fuzzy graph.

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