



# Journal of Materials and Engineering Structures

## Research Paper

### Reliability assessment of frame steel considering semi-rigid connections

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#### ARTICLE INFO

##### Article history :

Received : 10 July 2018

Revised : 9 January 2019

Accepted : 11 January 2019

##### Keywords:

Semi-rigid connections

Steel frame

Reliability assessment

Monte Carlo simulation

#### ABSTRACT

Initial stiffness and moment resistance are two parameters that formed the hardness of the connection between columns and beams. This problem has been studied based on empirical of many authors in the world. In this research been used initial stiffness and moment resistance models the proposed model of Aleksander K., for reliability assessment of frame steel under the internal force of semi-rigid joints when random input parameters. A stochastic model for the design of frame steel considering semi-rigid and Monte Carlo reliability analysis. This program is written by using the Matlab programming language. Then, various numerical tests are performed to illustrate the studied method.

## 1 Introduction

Steel frame with semi-rigid connections is always an interesting topic of the scientists all over the world. In 1934, Batho and Rowan proposed the straight-line common beam method to classify semi-rigid connection [1] Rathburn, in 1936, considered the stiffness of the connection in his research on the moment distribution method [2] Baker, William and Surochnikoff, in [3] investigated the influence of semi-rigid connections on the frame subjecting to the shake. Monforton and Wu [4, 5] were some of the first researchers applying the stiffness matrix method to analyze the plane frame with semi-rigid connection. In which, the stiffness matrix and nodal force vector of each element depend on the linear stiffness of the connection. Kim, S.E. and Choi, S.H proposed a novel method to analyze the spatial frame considering the material nonlinearity and large deflection effect [6]. The linear and nonlinear semi-rigid connection models were also interested by Hadianfar and Razani in 2003 [7].

In 1990, Kiureghian et al. assessed the reliability of the steel frame under the dynamic loads generated from the

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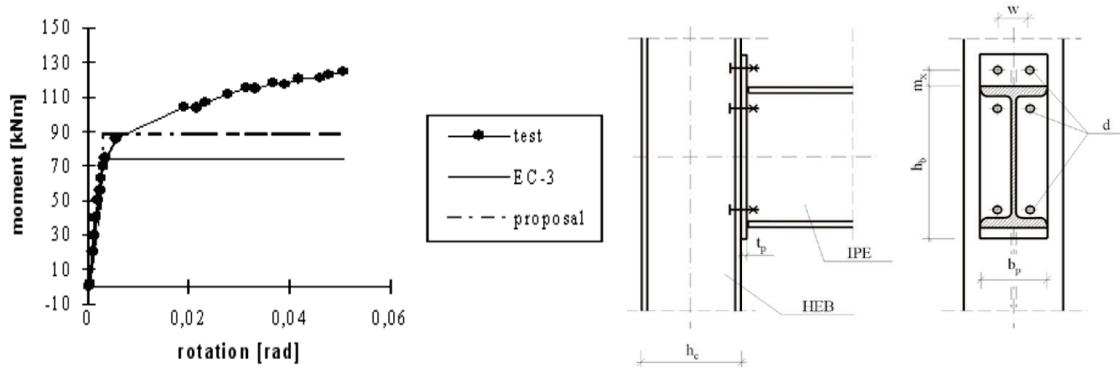
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earthquake El Centro in 1940 using  $\beta$  probability index method [8]. The same method was used in [9] by Hong et al. to study the steel frame according to the plastic limit state. Reliability of the steel structure under the corrosion was considered by Melchers in [10]. Hadianfard and Razani used the Monte Carlo simulation method to investigate the steel frame with flexible beam-column joint. In 2012, to study the reliability assessment of steel scaffold shoring structures for concrete formwork was considered HaoZhang et al in [11]. The rotational stiffness and the moment resisting of the connection were referred in [12] of Chen *et al.* This model of flexible joint, according to recommendation of Aleksander in [13], is not correct. This author proposed then a novel model based on the empirical tests and the Eurocode 3.

So can be noticed that reliability assessment of steel frame considering semi-rigid connections are important. However, semi-rigid models and reliability assessment methods are relatively limited. Therefore, this research aims to study reliability assessment of steel frame considering semi-rigid connections under durable conditions of the beam to column connections (joint) are elastic-plastic models, this value is determined through research of Aleksander K., [13] by Monte Carlo simulation.

## 2 Initial stiffness and moment resistance of joints steel

Initial stiffness and moment resistance are two parameters that formed the hardness of the connection between columns and beams. These two values were calculated using the proposed model of Aleksander K. *et al* [13]. The reliability of this model has been verified by comparing the experimental results according to Euro code 3 standard (EC. 3) (Figure 1).



**Fig. 1 – Validation of the Aleksander's model by comparison with EC.3 and experimental results [13]-**

The moment resistance of steel  $M_{Rd}$  and the initial stiffness of joints steel  $S_{j,ini}$  are determined as the following.

$$M_{Rd} = 7,4 \times 10^{-5} \times h_c^{0.62} \times h_b^{1.2} \times t_p^{0.4} \times d^{0.85} \quad (1)$$

$$S_{j,ini} = K_1 \times h_c^{0.44} \times h_b^{1.2} \times t_p^{0.35} \times d^{0.005} - K_2 \quad (2)$$

where  $S_{j,ini}$  is initial stiffness ( $kN.m/rad$ );  $S_j$  is the elastic stiffness of the connection ( $kN.m/rad$ ) which is determined as:

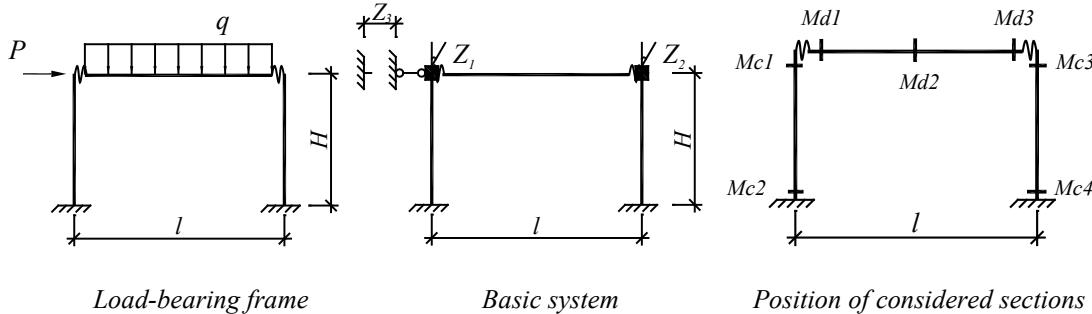
$$S_j = \frac{S_{j,ini}}{\eta} \quad (3)$$

$h_c$  (mm) is the height of the column section (HEB);  $h_b$  (mm) is the height of the beam section (IPE);  $t_p$  (mm) is the thickness of the end plate and  $d$  (mm) is the bolt diameter;  $K_1 = 1.5$  and  $K_2 = 19211$  are identified from experimental results [13].

## 3 Internal forces analysis frames of semi rigid connections

Consider a semi-rigid connections plane frame as shown in Figure 2. The internal forces of the frame are determined using the displacement method. The basic system, the position of considered sections are also presented in Figure 2 and the

equilibrium equations are shown by (4).



**Fig. 2 – Semi-rigid connections plane frame, basic system and position of considered sections**

$$\begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \begin{Bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{Bmatrix} + \begin{Bmatrix} R_{1P} \\ R_{2P} \\ R_{3P} \end{Bmatrix} = 0 \quad (4)$$

$$r_{11} = r_{22} = 4i + \frac{4S_j(3i + S_j)}{12i^2 + 8S_ji + S_j^2}; \quad r_{12} = r_{21} = 4i + \frac{2S_j^2i}{12i^2 + 8S_ji + S_j^2}; \quad i = \frac{EI_b}{L}$$

where  $r_{13} = r_{31} = -\frac{6i}{l}; \quad r_{23} = r_{32} = -\frac{6i}{l}; \quad r_{33} = \frac{12i}{l^2}$

$$R_{1P} = -\frac{1}{12} \frac{ql^2S_j(6i + S_j)}{12i^2 + 8S_ji + S_j^2}; \quad R_{2P} = \frac{1}{12} \frac{ql^2S_j(6i + S_j)}{12i^2 + 8S_ji + S_j^2}; \quad R_{3P} = -P$$

Moments of plane frame  $M_p$  and moments of joints  $M_1, M_2$  are determined as follows.

$$M_p = \sum \bar{M}_j \cdot Z_j + M_p^0; \quad M_1 = \frac{4EI_c}{H} Z_1 + \frac{6EI_c}{H^2} Z_3; \quad M_2 = \frac{4EI_c}{H} Z_2 - \frac{6EI_c}{H^2} Z_3 \quad (5)$$

with  $E$  is the Young modulus of the steel and  $I_b, I_c$  are respectively the moments of inertia of the cross section of beam and column.

#### 4 Validation of design computer program

The design program consists of two parts: structural analysis and safety condition verification. The first part will be validated by comparison with the result of SAP200 software. The second part was validated by the authors in [14]. The validation is performed in two extremal cases: articulation joints (stiffness of the joint is zero) and rigid joints (stiffness of the joint is infinitely high). Input parameters are shown in Table 1 and the results are shown in Table 2. We can observe that the error is very small in the case of button joint (<0,2%) and slightly increases in the other case (<2,5%). This result can be explained by the fact that in our program the rigid joint is modeled by setting a very high value to the stiffness of the joints and it gives the error. This result confirms the reliability of our program.

**Table 1 - Input parameters of the analysis**

Beam (cm)				Column (cm)				End plate (cm)	Bolt (cm)	Material (kN/cm <sup>2</sup> )	Load-bearing				
L	$h_{wb}$	$b_f$	$t_f$	$t_w$	H	$h_{wc}$	$b_f$	$t_f$	$t_w$	$t_p$	d	E	f	P(kN)	q(kN/cm)
500	30	20	2	2	400	30	20	2	2	2	1.6	2,1E4	21	100	0.05

**Table 2 - Validation of established program by comparison with Sap2000 software**

Element	Section	Articulation joints			Rigid joints		
		SAP2000	Program	error	SAP2000	Program	error
Columns	Mc1	-	-		7483,0	7531,8	0.65%
	Mc2	-20041,0	-20000	0.20%	-11515,9	-11352,0	1.42%
	Mc3	-	-	-	-8900,0	-9019,9	1.35%
Beams	Mc4	19958,9	20000	0.21%	12100,7	12096,0	0.04%
	Md1	-	-	-	7483,0	7531,8	0.65%
	Md2	-	-	-	-8900,0	-9019,9	1.35%
	Md3	1562,5	1562,5	0.0%	853,8	832,8	2.46%

## 5 Monte Carlo simulation method

Monte Carlo simulation is a method that uses the pseudo-random numbers to simulate the random property of the basic variables and then directly estimate the reliability based on the concept of larger number law. If the safe region is defined as  $f(\mathbf{X}) > 0$ , the failures probability of the system will be determined as follows:

$$P_f = \int I_{f(\mathbf{X})<0} f_{\mathbf{X}}(x) dx = E[I_{f(\mathbf{X})<0}] \quad (6)$$

Where

$$I_{f(\mathbf{X})<0} = \begin{cases} 1 & \text{khi } f(\mathbf{X}) < 0 \\ 0 & \text{khi } f(\mathbf{X}) \geq 0 \end{cases} \quad (7)$$

According to the statistical theory, if we have a number  $N$  of the sample of the random vector  $\mathbf{X}$ , a sample that consists of  $N$  values of  $I_{f(\mathbf{X})<0}$  can be obtained, and the expectation of  $I_{f(\mathbf{X})<0}$  can be approximated as the average of the sample as:

$$\hat{P}_f = E[I_{f(\mathbf{X})<0}] = \frac{1}{N} \sum_{i=1}^N I_{f(\mathbf{X})<0}^i \quad (8)$$

According Lemaire in [15], it is pointed out that the estimation in (8) is convergent and the confidence interval at 95% of  $P_f$  value can then be calculated as follows:

$$\hat{P}_f \left( 1 - 200 \sqrt{\frac{1 - \hat{P}_f}{N \hat{P}_f}} \right) \leq P_f \leq \hat{P}_f \left( 1 + 200 \sqrt{\frac{1 - \hat{P}_f}{N \hat{P}_f}} \right) \quad (9)$$

## 6 Reliability assessment joins of frame steel considering semi-rigid connections

### 6.1 Safe condition

According to Euro code 3 allows the uses initial stiffness for design structures. When used initial stiffness in joints frame apparition moment resistance. Safety conditions of joints of frames then are written in the following.

$$M_j \leq \frac{M_{j,Rd}}{n} \quad (10)$$

where  $M_j$  is the moment value of joints effect load-bearing;  $n$  is a safety factor

### 6.2 Deterministic model and uncertainty model

Deterministic model is the above internal forces analysis problem, in which the input parameters are those of geometry

$(H, L, h_c, b_b, t_b, h_c, b_c, t_c, t_p, d)$ , of Young modulus material ( $E$ ), of load ( $q$ ) and of flexible joint ( $K_1, K_2$ ). This model can be written in form with  $X = [H, L, h_c, b_b, t_b, h_c, b_c, t_c, t_p, d, q, E, \alpha, K_1, K_2]$ .

$$M_j = \Im(X) \quad (11)$$

Uncertainty model is constructed based on the deterministic model by taking into account the randomness of some input parameters. In this paper, we distinct two vector of input parameters: the first one of the parameters assumed to be deterministic  $X_1 = [H, L, h_c, b_b, t_b, h_c, b_c, t_c, t_p, d, K_1, K_2]$  and the second one of the parameters assumed to be random  $X_2(\omega) = [q(\omega), \alpha(\omega), E(\omega)]$  with  $\omega$  represents the randomness of the parameters. This model can be written in form.

$$M_j(\omega) = \Im(X_1, X_2(\omega)) \quad (12)$$

### 6.3 Reliability assessment of the steel frame by Monte Carlo simulation

By introducing the uncertainty model in the Monte Carlo simulation method, we obtain the scheme of the reliability assessment of steel frame as shown in Figure 3.

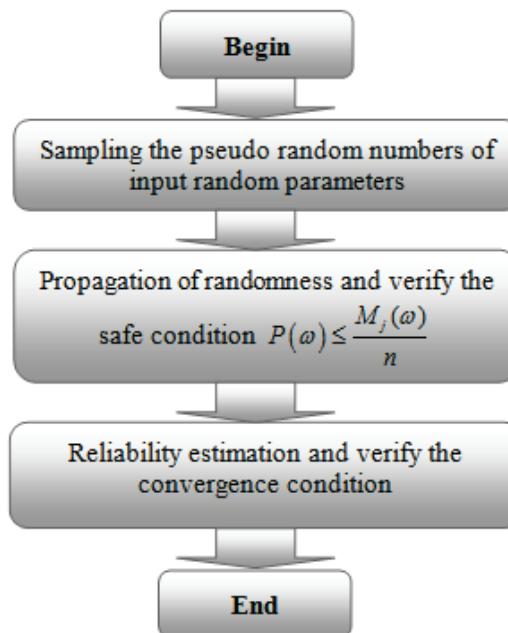


Fig. 3 – Scheme of the reliability assessment of steel frame using Monte Carlo simulation method

### 6.4 Convergence of the Monte Carlo simulation

Consider the steel frame as shown in figure 2 with the deterministic input parameters and the random input parameters respectively presented in Table 3 and Table 4. The compression load  $q(kN/m)$ , material ( $E$ ) and  $\alpha = qL/P$  assumed to be normal variable with mean  $\mu$  and coefficient of variation  $CV = \sigma / \mu$ .

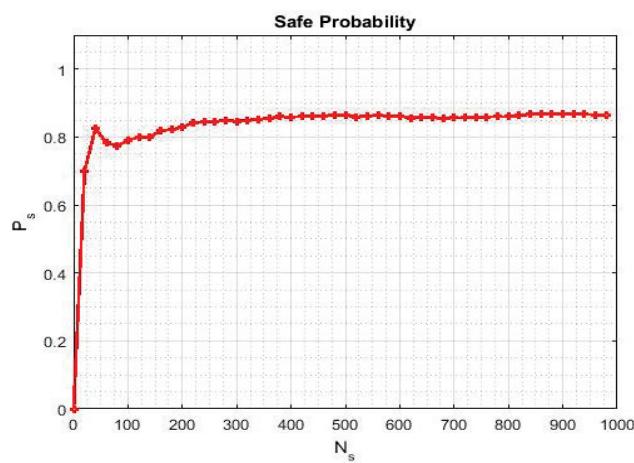
Table 3 - Deterministic input parameters

Beam (cm)				Column (cm)				End plate (cm)	Bolt (cm)	Material (kN/cm <sup>2</sup> )			
$L$	$h_{wb}$	$b_f$	$t_f$	$t_w$	$H$	$h_{wc}$	$b_f$	$t_f$	$t_w$	$t_p$	$d$	$E$	$f$
500	30	20	2	2	400	30	20	2	2	2	1.6	2.1E4	21

**Table 4 - Random input variables and their representative parameters**

<b>Random variable</b>	$E(kN/cm^2)$		$q(kN/cm)$		$\alpha = qL/P$	
Law of probability	<b>Norm</b>		<b>Norm</b>		<b>Norm</b>	
Representative parameters	$\mu_E$	$\mu_q$	$\mu_q$	$CV_E$	$\mu_\alpha$	$CV_\alpha$
	$2,1 \times 10^4$	10	1,30	0,15	0,01	0,15

Figure 4 shows the convergence of the safe probability of the steel frame in the Monte Carlo simulation to the value of 0,8315 or 83,15% after about 998 sampling in 4,83 minutes. The used convergence criteria of 2,5% justifies the confidence of the estimated reliability. This result also shows that although we have taken the safety factor is  $n = 1,1$  in the analysis but because of the randomness of some input parameters, the reliability of the structure is only of 83,15%. The assessment of the reliability of the structure thus is necessary.

**Fig. 4 – Convergence of the safe probability in the Monte Carlo simulation**

### 6.5 Effect of the coefficient of variation and safety factor

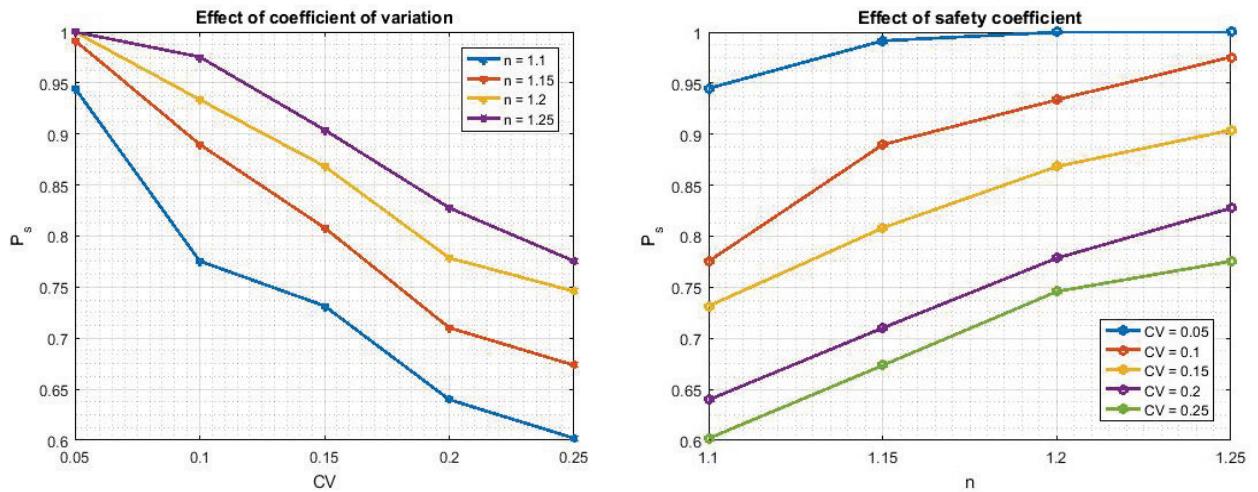
We know clearly that the variation of input random variables and the safety factor influence directly but inversely on the safe probability of the structure. Thus in order to clear the effect of these parameters, one reconsiders the above column with different coefficients of variation of the compression load  $CV = 0,05; 0,1; 0,15; 0,2; 0,25$  and different safety factors  $n = 1,1; 1,15; 1,2; 1,25$ . The results of the safe probability are listed in Table 5 and presented in figure 5.

**Table 5 - Effect of the coefficient of variation and effect of the safety factor on the safe probability**

<i>n</i>	<i>CV</i>				
	0,05	0,10	0,15	0,20	0,25
1,10	0,9449	0,7755	0,7316	0,6398	0,6020
1,15	0,9916	0,8898	0,8082	0,7102	0,6735
1,20	1,0000	0,9337	0,8684	0,7786	0,7459
1,25	1,0000	0,9755	0,9041	0,8276	0,7755

We can easily observe the effect inverse of the coefficient of variation and the safety factor in the Figure 5. The safe probability of the column decreases when the coefficient of variation increases, whereas it increases when the safety factors increases. This result seems to be obvious, but it has a very important significance. It shows that if there are many input random parameters or furthermore with the high randomness in the structural design or in the optimization problem, the use of the local coefficient such as the overload coefficient seems to be not sufficient. The structure can be in the dangerous state. In this case, it is necessary to determine a global safety factor, as is done in this study, to assure the absolute safety of

the structure. For example, in this test, if the coefficient of variation is 0,05, the global safety factor needs to be only 1,25 to obtain the safe probability 100%.

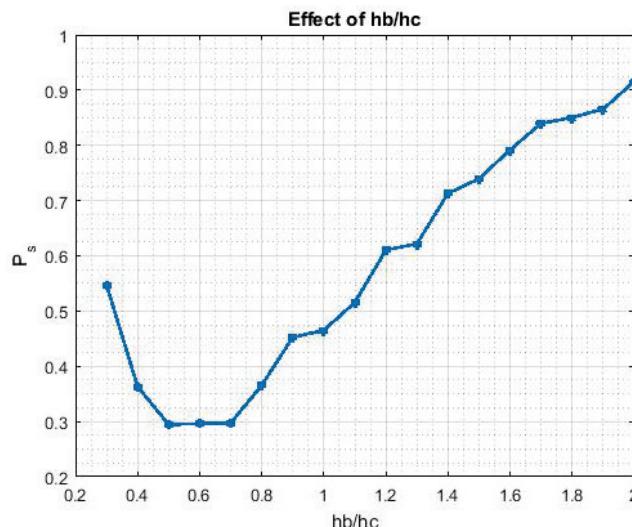


**Fig. 5 –Effect of the coefficient of variation (left) and effect of the safety factor (right) on the safe probability of the column**

### 6.6 Effect of the ratio $h_b/h_c$

In the section, the researchers investigated the impact of hardness ratio of beam and column to the reliability structure through the ratio of beam cross-section and column cross-section  $h_b/h_c$ . In [13] the value limit when applying the expression (1), (2) must satisfy the condition  $h_c = (140 \div 400)$  m and  $h_b = (160 \div 400)$  mm. Therefore ratio limits  $h_b/h_c$  in the research variable intervals  $h_b/h_c = (0,3 \div 2,0)$ . Deterministic parameters and random variable with a coefficient of variation  $\nu = 0,1$ ; safety coefficient  $n = 1,1$ .

Figure 6 and Table 6 shown Effect of  $h_b/h_c$  to the safe probability  $P_s$ . Figure 6 shown that when  $h_b/h_c = 0,3 \div 0,7$  then safe probability  $P_s$  are decreases. when  $h_b/h_c = 0,7 \div 2,0$  then safe probability  $P_s$  is an increase. Structures had Safe probability is the smallest value when  $h_b/h_c = 0,5 \div 0,7$ . From this result, we see that for this frame type and the loading should not choose a domain  $h_b/h_c = 0,5 \div 0,7$  and reasonable when  $h_b/h_c \approx 2,0$ .



**Fig. 6 –Effect of the Effect of  $h_b/h_c$  to the safe probability  $P_s$**

**Table 6 - Safe probability  $P_s$  when ratio  $h_b/h_c$** 

$h_b/h_c$	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1
$P_s$	0,5474	0,3632	0,2947	0,2960	0,2974	0,3658	0,4524	0,4653	0,5158
$h_b/h_c$	1,2	1,3	1,4	1,5	1,60	1,70	1,80	1,90	2,0
$P_s$	0,6105	0,6218	0,7132	0,7395	0,7921	0,8395	0,8507	0,8658	0,9158

## 7 Conclusion

This paper studies the reliability assessment of the steel frame with semi-rigid beam-column joint. The authors have successfully established with the Monte Carlo simulation method to construct the steel frame's reliability assessment program. The parametric tests are also performed to study the effect of the input parameters on the reliability of the structure. The obtained results showed the important significance of this research.

The results of this initial positive study are the premise for the authors to continue to expand the problem for other types of structures and to consider various random variable.

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