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SECTION 2. Applied mathematics. Mathematical modeling.

THE DEVELOPMENT OF THE LIBRARY IN MAPLE IMPLEMENTS THE ALGORITHMS OF THE METHOD OF INTERIOR POINT

***Abstract**: In this paper, we develop a library for Maple implementing algorithms for applying the method of internal points.*
***Key words**: Maple, interior point, library.*
***Language**: English*
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Introduction

The created library will be based on the previously used algorithms for the method of internal points. The name of the library InteriorPointUtemgaliev.MapleLib.

Set the names of algorithms:

- IP_Plot
- IP_OptimalAllocation
- IP_Slack
- IP_Slack_new
- IP_L3

Connection of the library is carried out by the command

```
read(`D:\InteriorPointUtemgaliev.MapleLib`) :
with(InteriorPoint);

    [IP_L3, IP_OptimalAllocation, IP_Plot, IP_Slack, IP_Slack_new]
```

Library code:

```
restart :
InteriorPoint := table( ) :

InteriorPoint[IP_Plot] :=proc(Equation, x1, x2) local n :
with(plots) :
plot3d(Equation, x[1] = x1, x[2] = x2, shading = zhue, axes = boxed, style
    = patchcontour, contours = 10);
end proc :
```

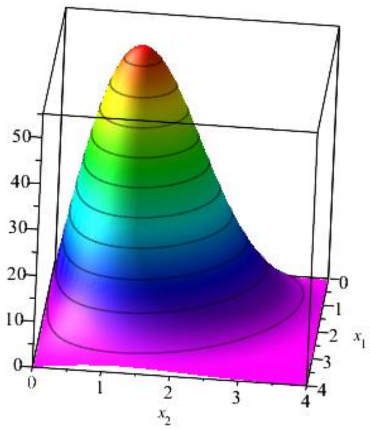
Code of the main program:

```
restart;
read(`D:\InteriorPointUtemgaliev.MapleLib`) :
with(InteriorPoint);
                    [IP_OptimalAllocation, IP_Plot]
```

Наша целевая функции приведена в виде параболоида:

```
F := 100·x[1]·x[2]*exp(-x[1]^2/3 - x[2]^2/3);
IP_Plot(F, 0..4, 0..4);
```

$$F := 100\, x_1\, x_2\, e^{-\frac{x_1^2}{3} - \frac{x_2^2}{3}}$$



Library code:

```
InteriorPoint[IP_OptimalAllocation] := proc(ob, con1, con2) local F, xx1, xx2, my1,
    my2, my3, my4, my5 :
with(plots) :
xx1 := evalf(solve(solve(convert(con1, equality), x[2]) = solve(convert(con2,
    equality), x[2]), x[1]), 5) :
xx2 := eval(solve(convert(con1, equality), x[2]), x[1] = xx1) :

my1 := plot(solve(convert(con1, equality), x[2]), x[1] = 0..10, color = red, thickness
    = 2) :
my2 := plot(solve(convert(con2, equality), x[2]), x[1] = 0..10, color = green,
    thickness = 2) :
my3 := plots:-contourplot(ob, x[1] = 0..5, x[2] = 0..5, color = black, contours = 10);
my4 := plots:-pointplot({[xx1, xx2]}, axes = boxed, symbol = solidcircle, symbolsize
    = 20) :
my5 := plots:-textplot([xx1, xx2, "Optimal Allocation"], align = {ABOVE, RIGHT},
    font = [times, bold, 16]) :
print([eval(ob, {x[1] = xx1, x[2] = xx2}), x[1] = xx1, x[2] = xx2]);
plots:-display({my1, my2, my3, my4, my5}, view = [0..4, 0..5]);

end proc :
```

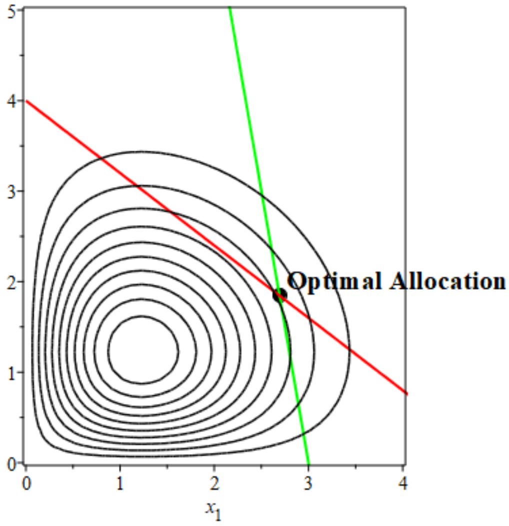
Code of the main program:

Теперь мы можем построить такую целевую функцию с соответствующими ограничениями и найти оптимальное решение следующим образом:

```
restart;
read(`D:\InteriorPointUtemgaliev.MapleLib`) :
with(InteriorPoint);
F := 100·x[1]·x[2]*exp((-x[1]^2)/3 - (x[2]^2)/3) :
ogr_1 := 4·x[1] + 5·x[2] ≤ 20 :
ogr_2 := 6·x[1] + 1·x[2] ≤ 18 :
IP_OptimalAllocation(F, ogr_1, ogr_2);
```

[IP_OptimalAllocation, IP_Plot, IP_Slack]

$[14.24508347, x_1 = 2.6923, x_2 = 1.846160000]$



Library code:

```
InteriorPoint[IP_Slack] := proc(ob, con1, con2) local F, xx1, xx2, my1, my2, my3,
   my4, my5, Slack1, Slack2, L :
xx1 := evalf(solve(solve(convert(con1, equality), x[2]) = solve(convert(con2,
   equality), x[2]), x[1]), 5) :
xx2 := eval(solve(convert(con1, equality), x[2]), x[1] = xx1) :

my1 := plot(solve(convert(con1, equality), x[2]), x[1] = 0 ..10, color = red, thickness
   = 2) :
my2 := plot(solve(convert(con2, equality), x[2]), x[1] = 0 ..10, color = green,
   thickness = 2) :

Slack1 := rhs(con1) − lhs(con1) :
Slack2 := rhs(con2) − lhs(con2) :

L := ob−λ·(ln(Slack1) + ln(Slack2)) :

my3 := plots:-contourplot(eval(L, λ = 0), x[1] = 0 ..10, x[2] = 0 ..10, color = black,
   contours = 10) :
print(Slack1); print(Slack2); print(L);
print(my3);
print(plots:-display({my1, my2, my3}, view = [0 ..4, 0 ..5]));
end proc :
```

Code of the main program:

Теперь решим вышеуказанную проблему, используя метод внутренней точки

Введем новую переменную slack

$$y \geq 0$$

неравенство

$$A \cdot x \leq b$$

можно преобразовать в форму

$$A \cdot x + y = b$$

Это означает, что наши ограничения могут быть записаны как:

$4 \cdot x[1] + 5 \cdot x[2] + 20;$
$6 \cdot x[1] + 1 \cdot x[2] + 10;$

$$4 x_1 + 5 x_2 + 20$$

$$6 x_1 + x_2 + 10$$

$4 \cdot x[1] + 5 \cdot x[2] + y = 20;$
$6 \cdot x[1] + 1 \cdot x[2] + y = 10;$

$$4 x_1 + 5 x_2 + y = 20$$

$$6 x_1 + x_2 + y = 10$$

restart;
read(`D:\InteriorPointUtemgaliev.MapleLib`) :
with(*InteriorPoint*);

$F := 100 \cdot x[1] \cdot x[2] * \exp\left(\frac{-x[1]^2}{3} - \frac{x[2]^2}{3}\right):$

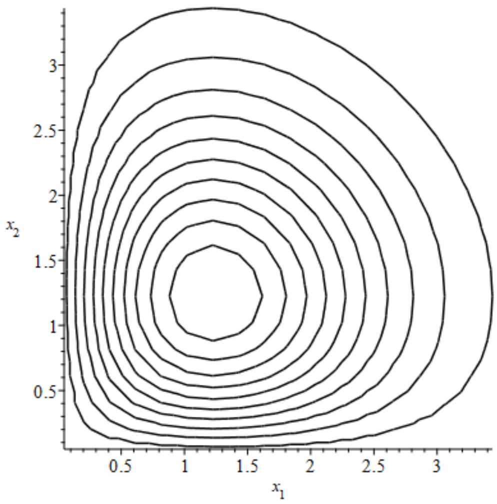
$ogr\_1 := 4 \cdot x[1] + 5 \cdot x[2] \leq 20:$
$ogr\_2 := 6 \cdot x[1] + 1 \cdot x[2] \leq 10:$
IP_Slack(*F*, *ogr_1*, *ogr_2*);

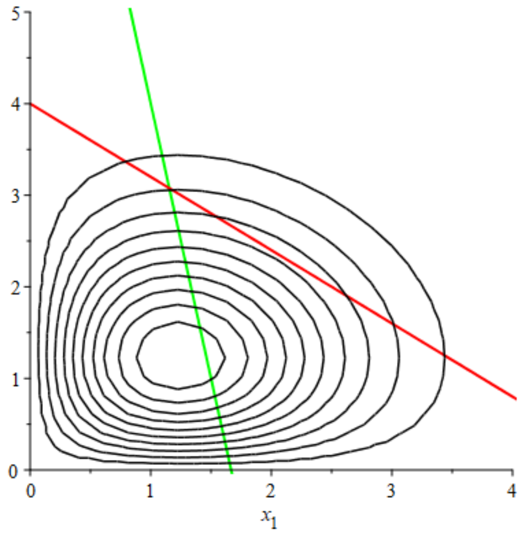
[*IP_OptimalAllocation*, *IP_Plot*, *IP_Slack*]

$$20 - 4 x_1 - 5 x_2$$

$$10 - 6 x_1 - x_2$$

$$100 x_1 x_2 e^{-\frac{x_1^2}{3} - \frac{x_2^2}{3}} - \lambda \left(\ln\left(20 - 4 x_1 - 5 x_2\right) + \ln\left(10 - 6 x_1 - x_2\right)\right)$$





Код библиотеки:

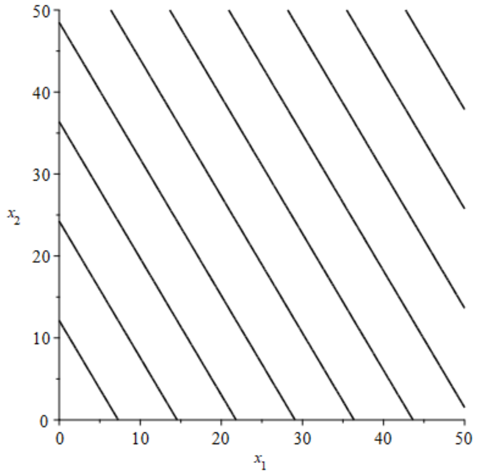
```
InteriorPoint[IP_Slack_new] :=proc(ob, con1, con2) local F, xx1, xx2, my1, my2,
   my3, my4, my5, Slack1, Slack2, L_new :
xx1 := evalf(solve(solve(convert(con1, equality), x[2]) = solve(convert(con2,
   equality), x[2]), x[1]), 5) :
xx2 := eval(solve(convert(con1, equality), x[2]), x[1] = xx1) :

my1 := plot(solve(convert(con1, equality), x[2]), x[1] = 0 ..10, color = red, thickness
   = 2) :
my2 := plot(solve(convert(con2, equality), x[2]), x[1] = 0 ..10, color = green,
   thickness = 2) :

Slack1 := rhs(con1) − lhs(con1) :
Slack2 := rhs(con2) − lhs(con2) :

L_new := (−lhs(con1) −lhs(con2)) / 2 −λ·(rhs(con1) − log(lhs(con1))
   + log(Slack2)) :
print(L_new);
my3 := plots:-contourplot(eval(L_new, λ = 0), x[1] = 0 ..50, x[2] = 0 ..50, color
   = black, contours = 10) :

print(my3);
print(plots:-display({my1, my2, my3}));
end proc :
```

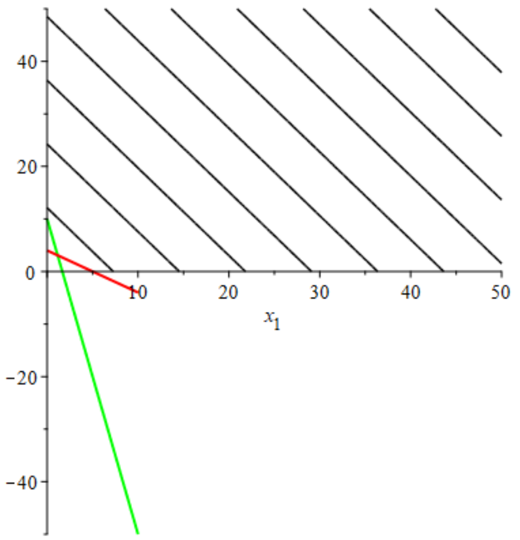
Code of the main program:

```
restart;
read(`D:\InteriorPointUtemgaliev.MapleLib`) :
with(InteriorPoint);
F := 100· x[1]·x[2] * exp(−x[1]^2/3 − x[2]^2/3) :
ogr_1 := 4·x[1] + 5·x[2] ≤ 20 :
ogr_2 := 6·x[1] + 1·x[2] ≤ 10 :
IP_Slack_new(F, ogr_1, ogr_2);
```

[IP_OptimalAllocation, IP_Plot, IP_Slack, IP_Slack_new]

$$-5\,x_1 - 3\,x_2 - \lambda\,\left(20 - \ln\left(4\,x_1 + 5\,x_2\right) + \ln\left(10 - 6\,x_1 - x_2\right)\right)$$



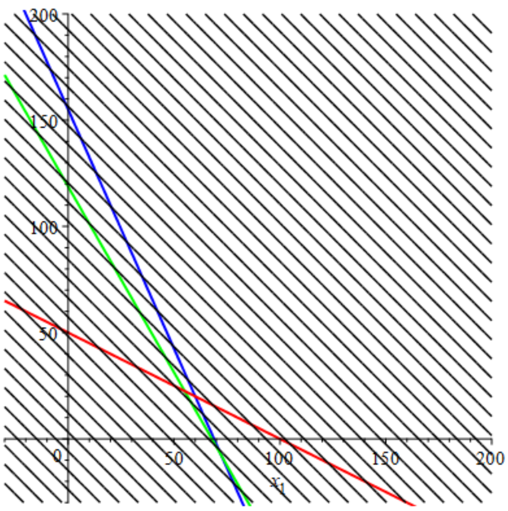


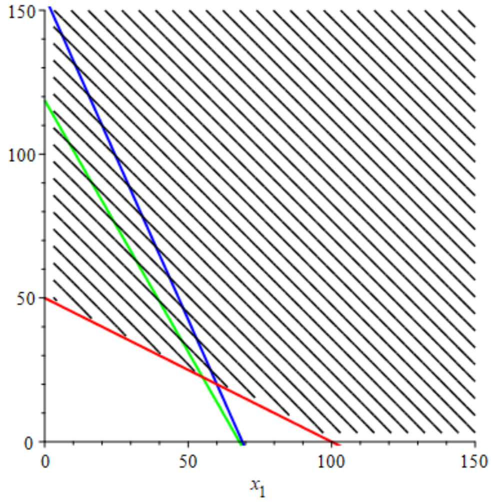
Library code:

```
InteriorPoint[IP_L3] :=proc(ob, λ0, con1, con2, con3, og) local my0, my1, my2, my3 :
my0 := plot(solve(convert(con1, equality), x[2]), x[1] = og, color = red, thickness
    = 2) :
my1 := plot(solve(convert(con2, equality), x[2]), x[1] = og, color = green, thickness
    = 2) :
my2 := plot(solve(convert(con3, equality), x[2]), x[1] = og, color = blue, thickness
    = 2) :

my3 := plots:-contourplot(eval(ob, λ = λ0), x[1] = og, x[2] = og, color = black,
    contours = 50) :
print(plots:-display( {my0, my1, my2, my3} , view = [og, og])) ;
end proc :
```

Code of the main program:

```
restart;
read(`D:\InteriorPointUtemgaliev.MapleLib`) :
with(InteriorPoint);
F := x[1] + x[2]-λ·(ln(x[1] + 2·x[2] − 100) + ln(x[1]) + log(x[2])) :
ogr_1 := -x[1]-2·x[2] + 100 ≤ 0 :
ogr_2 := -14 x[1] − 8·x[2] + 950 ≤ 0 :
ogr_3 := -9 x[1] − 4·x[2] + 620 ≤ 0 :
λ0 := 0 :
IP_L3(F, λ0 , ogr_1, ogr_2, ogr_3, -30 ..200);
λ0 := 0.1 :
IP_L3(F, λ0 , ogr_1, ogr_2, ogr_3, 0 ..150);

        [IP_L3, IP_OptimalAllocation, IP_Plot, IP_Slack, IP_Slack_new]
```





The library is compiled and saved by the command:

```
save(InteriorPoint, `D:\InteriorPointUtemgaliev.MapleLib`) :
```

Conclusion

As a result of the study, a review of the main methods of mathematical optimization for problems with constraints: the method of internal points, the problem of mathematical optimization, simplex method, projective gradient descent, ellipsoid method, the method of internal point, the problem of quadratic programming, the theoretical foundations of linear optimization. Algorithms of application of methods of internal points in Maple are considered. The existing methods used by the Maple optimization package are studied. Algorithms interior point method for Maple. Developed the library in Maple implements the algorithms of the method of interior point. The created library has been tested and can be used in solving optimization problems by the method of internal points.

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