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SECTION 1. Theoretical research in mathematics.

MONTE CARLO METHOD IN THE PROBLEM OF REMOTE SENSING

Abstract: *The paper considers the integral equation of radiation transfer. Considered the problem of efficient modeling of propagation radiation in layered-homogeneous medium. Under these assumptions have been obtained approximate calculation formulas to estimate the values of functionals, having a physical means of the intensity of radiation and it's derivative by dispersion coefficient. Further on is used the traditional method of addressing backward problems, based on the Newton-Kantorovich method.*

Key words: *Monte-Carlo methods, integral equations, remote sensing, radiation transfer.*
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Introduction

In environmental research one of the most important is the task of optical remote sensing of the parameters of a continuous medium. Under these In meeting such challenges is widely and effectively used methods is the Monte Carlo [1],[2],[3], based on the probabilistic interpretation of the kernel of the integral equation of transfer of radiation

$$f(\vec{x}) = \int_X k(\vec{x}', \vec{x}) f(\vec{x}')d\vec{x}' + \psi(\vec{x}), \qquad (1)$$

where $X = D \times \Omega \times [0,T]$ - the phase space of coordinates $\vec{r} = (x, y, z) \in D \subset R^3$, lines $\vec{\omega} = (\mu, \beta) \in \Omega = [-1,1] \times [0, 2\pi]$, $\mu = \cos\theta$, $\theta \in [0, \pi]$, and time $t \in [0,T]$; $\vec{x} = (\vec{r}, \vec{\omega}, t) \in X$, $\vec{x}' = (\vec{r}', \vec{\omega}', t') \in X$;

$f(\vec{r}, \vec{\omega}, t)$ - the density of the collision of photons with environmental elements;

$\psi(\vec{r}, \vec{\omega}, t)$ - the density distribution of sources;

$k(\vec{x}', \vec{x})$ - the density of the transition photon from «condition» $\vec{x}'$ at «condition» $\vec{x}$.

Applications are important functionals of the form

$$I_\varphi = (f, \varphi) = \int_X f(x)\varphi(x)dx \qquad (2)$$

from the solution $f(x)$ equation (1).

It's known [4], that $\sup_{x \in X} \int_X |k(x', x)dx' < 1$ and under the conditions $\psi, \varphi \in L_1(x)$, the equation (1) has a single solution in the class of functions $L_1(X)$, submitted convergent series of the Neumann:

$$f(\vec{x}) = \sum_{i=0}^{\infty} K^i \psi = \psi(\vec{x}) + \sum_{i=1}^{\infty} \int_X ... \int_X \psi(\vec{x}_0) k(\vec{x}_0, \vec{x}_1) ... k(\vec{x}_{i-1}, \vec{x}) d\vec{x}_0 ... d\vec{x}_{i-1}.$$

Everywhere in a further sign of the vector on variables $\vec{x}, \vec{x}_i$, $i \geq 0$, we will drop out. We describe the basic idea of the Monte Carlo methods [5]. Let points $x_0, x_1, ..., x_n, ...$ - random and form a homogeneous Markov chain with the probability density distribution $\psi(x)$ «initial state» x_0 and probability density «transition» $k(x_{i-1}, x_i)$ from «condition» x_{i-1} at «condition» x_i. Then the linear functional (2) by solving the equation (1) is a $M\xi$ - the mathematical expectation of a random variable

$\xi=\sum_{i=0}^{\infty}\varphi(x_i)$. Since $I_\varphi=(f,\varphi)=M\xi$, the task now is, to calculate $M\xi$. For this special formulas in a computer simulated sample values $\xi_1,\xi_2,...,\xi_N$ a random variable ξ and calculates the sum $S_N=\frac{1}{N}\sum_{j=1}^{N}\xi_j$. According to the law of large numbers, $M\xi\approx S_N$ for sufficiently large values of N.

In real physical problems such Markov chains can be identified the process of the spread of elementary particles in a medium and behind $x_0,x_1,...,x_n,...$ to take the point of collision of these particles with the elements of the environment [6]. Let the medium homogeneous, that is a constant value of the scattering coefficient σ_s^* absorption σ_a^* and full attenuation $\sigma^*=\sigma_s^*+\sigma_a^*$. Required to determine σ_s^* at known and fixed σ_a^*. Not being interested in the specific form and the physical meaning of functions $\varphi_k(x,\sigma)=\sigma\exp(-\sigma\,|\vec r_{sur.}-\vec r\,|)F_1$, where $\sigma=\sigma_s+\sigma_a^*$, $\vec r_{sur.}$ – the radius vector of the point of collision on the surface of the medium, F_1 – not depend on σ_s, $k=1,...,m$, will consider functionals of the form

$$I_k(\sigma)=\sum_{j=0}^{\infty}\int_X...\int_X\psi(x_0)\prod_{i=0}^{j-1}k(x_i,x_{i+1},\sigma)\varphi_k(x_j,\sigma)dx_0..dx_{j-1}dx_j,\quad(3)$$

where $k(x_i,x_{i+1},\sigma)=\sigma\exp(-\sigma\,|\vec r_{i+1}-\vec r_i\,|)F_2$, F_2 also not depend on σ_s.

Label by I_k^* the values of the functionals, measured experimentally. Suppose $I_k(\sigma^*)=I_k^*$.

Then to find the exact value of the scattering coefficient we obtain the following system of nonlinear equations [7]:

$$I_1(\sigma)=I_1^*,...,I_m(\sigma)=I_m^*.\quad(4)$$

To solve the resulting system using well-known Newton-Kantorovich method [8]. We write the linearized system:

$$\frac{\partial I_k(\sigma^0)}{\partial\sigma_s}(\sigma_s-\sigma_s^0)=I_k^*-I_k(\sigma^0),\ (5)$$

where σ_s^0 - some prognostic value of the scattering coefficient, $\sigma^0=\sigma_s^0+\sigma_a^*$.

We introduce describe $a_k=\frac{\partial I_k(\sigma^0)}{\partial\sigma_s}$, $\Delta\sigma_s=(\sigma_s-\sigma_s^0)$.

The resulting system is generally incompatible, to deal with this problem it involve the least squares method and arrive at the equation

$$\sum_{k=1}^{m}a_k^2\Delta\sigma_s=\sum_{k=1}^{m}a_k[I_k^*-I_k(\sigma^0)].$$

Next we construct successive approximations. Let $\sigma_s^{(p)}$ – the current approximation of the scattering coefficient. Then the following approximation $\sigma_s^{(p+1)}$ is how

$$\sigma_s^{(p+1)}=\sigma_s^{(p)}+\left\{\sum_{k=1}^{m}(a_k^{(p)})^2\right\}^{-1}\sum_{k=1}^{m}a_k^{(p)}\left[I_k^*-I_k(\sigma^{(p)})\right],$$

where $\sigma^{(p)}=\sigma_s^{(p)}+\sigma_a^*$.

The whole question now boils down to, to calculate the value at each iteration

$I_k^{(p)}=I_k(\sigma^{(p)})$

and $a_k^{(p)}=\frac{\partial I_k(\sigma^{(p)})}{\partial\sigma_s}=\left.\frac{\partial I_k(\sigma)}{\partial\sigma_s}\right|_{\sigma=\sigma^{(p)}}$.

To do this, we rewrite (3) as

$$I_k(\sigma)=\sum_{j=0}^{\infty}\int_X...\int_X\psi(x_0)\prod_{i=0}^{j-1}k(x_i,x_{i+1},\sigma^0)R_{jk}(\sigma)dx_0...dx_{j-1}dx_j,$$

where

$$R_{jk}(\sigma)=\varphi_k(x_j,\sigma)\prod_{i=0}^{j-1}\frac{k(x_i,x_{i+1},\sigma)}{k(x_1,x_{i+1},\sigma^0)},$$

σ^0 - a constant value for the parameter σ. It is easy to see that

$$\frac{\partial}{\partial\sigma_s}R_{jk}(\sigma^0)=\frac{\partial}{\partial\sigma_s}\varphi_k(x_j,\sigma^0)+\varphi_k(x_j,\sigma^0)\frac{\partial}{\partial\sigma_s}\prod_{i=0}^{j-1}\frac{k(x_i,x_{i+1},\sigma)}{k(x_i,x_{i+1},\sigma^0)}\bigg|_{\sigma=\sigma^0}=$$

$$=\frac{\partial}{\partial\sigma_s}\varphi_k(x_j,\sigma^0)+\varphi_k(x_j,\sigma^0)\sum_{i=0}^{j-1}\frac{\partial}{\partial\sigma_s}\ln k(x_i,x_{i+1},\sigma^0)=\varphi_k(x_j,\sigma^0)w_j(\sigma^0),$$

where

$$w_j(\sigma^0)=\frac{\partial}{\partial\sigma_s}\ln\varphi_k(x_j,\sigma^0)+\sum_{i=0}^{j-1}\frac{\partial}{\partial\sigma_s}\ln k(x_i,x_{i+1},\sigma^0)\,. \quad (6)$$

Here's were we obtain the desired estimate for the derivative of the intensity

$$\frac{\partial I_k(\sigma^0)}{\partial\sigma_s}=\sum_{j=0}^{\infty}\int_X\ldots\int_X\psi(x_0)\prod_{i=0}^{j-1}k(x_i,x_{i+1},\sigma^0)\frac{\partial}{\partial\sigma_s}R_{jk}(\sigma^0)dx_0..dx_j=$$
$$=M\left\{\sum_{j=0}^{\infty}\varphi_k(x_j,\sigma^0)w_j(\sigma^0)\right\}. \quad (7)$$

For the most intensity known local estimate [4]:

$$I_k(\sigma^0)=M\left\{\sum_{j=0}^{\infty}\varphi_k(x_j,\sigma^0)\right\} \quad (8)$$

As $\varphi_k(x_j,\sigma^0)=\sigma\exp(-\sigma\,|\,\vec r_{sur.}-\vec r_j\,|)F_1$ and F_1 not depend on σ_s, it

$$\frac{\partial}{\partial\sigma_s}\ln\varphi_k(x_j,\sigma^0)=\frac{1}{\sigma^0}-|\,\vec r_{nов.}-\vec r_j\,|)\ .$$

Similarly

$$\frac{\partial}{\partial\sigma_s}\ln k(x_j,x_{i+1},\sigma^0)=\frac{\partial}{\partial\sigma_s}\ln\{\sigma\exp(-\sigma\,|\,\vec r_{i+1}-\vec r_i\,|\,F_2\}\big|_{\sigma_s=\sigma_s^0}=\frac{1}{\sigma^0}-|\,\vec r_{i+1}-\vec r_i\,|\cdot$$

Consequently,

$$w_j(\sigma^0)=\frac{j+1}{\sigma^0}-\sum_{i=0}^{j-1}|\,\vec r_{i+1}-\vec r_i\,|-|\,\vec r_{nов.}-\vec r_j\,|. \quad (9)$$

In the work [9] in the case of convergence of the Neumann series to the solution (1) proved the finiteness of the average number of States of the Markov chain, in other words, Markov chain terminates with probability 1 through the end and the random number of transitions γ. Further according to the laws of distribution $\psi(x)$ and $k(x',x)$, simulated N different trajectories (Markov chains):

$x_0^{(l)},x_1^{(l)},\ldots,x_{\gamma(l)}^{(l)}$, $l=1,2,\ldots,N$, where $\gamma(l)$ - random number, chain which terminates with number l. Along each path construct the sum:

$$\xi_l^{(p)}(k)=\sum_{j=0}^{\gamma(l)}\varphi_k(x_j^{(l)},\sigma^{(p)}), \quad (10)$$

$$\eta_l^{(p)}(k)=\sum_{j=0}^{\gamma(l)}\varphi_k(x_j^{(l)},\sigma^{(p)})w_j^{(l)}(\sigma^{(p)}) \quad (11)$$

where

$$w_j^{(l)}(\sigma^0)=\frac{j+1}{\sigma^{(p)}}-\sum_{i=0}^{j-1}|\,\vec r_{i+1}^{(l)}-\vec r_i^{(l)}\,|-|\,\vec r_{nов.}^{(l)}-\vec r_j^{(l)}\,|,$$

$\vec{r}_0^{(l)}, \vec{r}_1^{(l)}, \ldots, \vec{r}_j^{(l)}$ - the collision point l- th simulated trajectory.

Put now

$$S_1 = \frac{1}{N}\sum_{l=1}^{N} \xi_l^{(p)}(k), \qquad S_2 = \frac{1}{N}\sum_{l=1}^{N} \eta_l^{(p)}(k),$$

$$D_1 = \frac{1}{N}\sum_{l=1}^{N} \left(\xi_l^{(p)}(k)\right)^2, \qquad D_2 = \frac{1}{N}\sum_{l=1}^{N} \left(\eta_l^{(p)}(k)\right)^2.$$

Then we obtain the following approximate formulas, based on the law of large numbers [10]:

$$I_k(\sigma^{(p)}) \approx S_1 \quad \text{и} \quad \frac{\partial I_k(\sigma^{(p)})}{\partial \sigma_s} \approx S_2.$$

An unbiased estimates for the errors of these approximations are finite and have a look $\sqrt{N(D_1 - S_1^2)/(N-1)}$, $\sqrt{N(D_2 - S_2^2)/(N-1)}$ accordingly.

Conclusion

As seen from (10) и (11) estimates the intensity and its time derivative are performed on the same trajectories and differ only by a factor $w_j^{(l)}(\sigma^{(p)})$.

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