International Journal for Quality Research 13(1) 221–234
ISSN 1800-6450

**Yun-Cheng Huang**
**Chien-Chuan Cheng**
**Yi-An Ding**

# THE COMPARATIVE STUDY ON EXPECTED TOTAL QUALITY COST BETWEEN TRADITIONAL SINGLE SAMPLING PLAN AND ECONOMICAL DESIGN

**Article info:**
Received 07.06.2018
Accepted 25.09.2018

UDC – 303.446:005.6
DOI – 10.24874/IJQR13.01-13

***Abstract:*** *In the quality inspection practice of the consumer electronics industry, MIL-STD-105E sampling table is viewed as the basis for sampling plans. This traditional quality inspection plan determine the sample size and reject rule based on the size of lot, consumer's and producer's risk and average quality level (AQL). Traditional sampling plan does not consider internal and external quality costs. However, quality costs were considered in many previous researches, but the comparison between traditional and economical design of single sampling plan is rare from now. This paper discusses the sampling test before the receiving inspection which is vendor simulated buyers. Includes the costs of inspection, rework, replacement, and external failure cost are considered. We compare the quality economical design with traditional single sampling plan under the total quality cost. This paper can be regarded as a reference for future studies and practical applications.*

***Keywords:*** *MIL-STD-105E, Economical design, Single sampling plan, Quality costs*

## 1. Introduction

The company improves its corporate image through by good product quality. In addition, in the setting of the quality management system, it is also necessary to arrange relevant departments, members, and operating procedures to face periodic audits from customers or third-party. Customers can also increase their confidence in product management through audits. Therefore, companies often invest in the cost of relevant quality inspections, whether they are inspections on the process or on the product (Schilling & Neubauer, 2009). In the practice of sampling inspection, products are sampled for inspections prior to shipment. Besides to simulating the incoming inspection from customer, the company also increases the product quality through the sampling inspection method, and meet the customer requirement of the Open Box Audit. There are the agreement of Average Quality Level (AQL) as the standard for sampling inspection (Collins et al., 1973). Use the MIL-STD-105E sampling table to determine the sample sizes and the maximum number of permissible defects (Nadeem & Velasco, 1993). However, the true product yield rate is still limited to 100% inspections. If the product features are small in size and large in number, they cannot be implemented in such a high-cost way as full inspection. If the ratio of sampling size and batched sizes is too small, it is easy because the sampling ratio is

[1] Corresponding author: Yi-An Ding
Email: yian0213@gmail.com

too small to fully grasp the defective rate of the batch of products. In addition, with the traditional single sampling method, only the two parties discussed and accepted each other's risks, and no actual consideration was made of the internal and external quality costs.

This paper takes the seller (producer) as the main decision-making for the inspection. The producer considers the cost of inspection, external failure, rework, and revenue of scrap to instruct mathematical model of the total quality cost. Based on minimum total cost of the quality, the sampling sizes and the number of permissible defects are determined and compare with traditional single sampling and as a reference for the company when selecting a sampling inspection plan.

The purpose of this paper is to compare the sampling plan under the economic design with the traditional single-sampling method as reference for the sampling inspection plan in terms of the difference in cost. Finally, use a practical example of consumer electronics product, simulation analysis of various scenarios is conducted and specific conclusions are made. The results of this paper can be used as reference for future research and practical applications.

The followings are the research limitations of this paper:

1) The seller is required to conduct a sampling inspection of the products shipped by the buyer and the buyer no longer conducts a sample inspection.
2) The product actual defective rate is based on the test of goodness of fit from historical data or assuming compliance with a specific distribution.
3) This paper does not apply to destructive testing.
4) The rejected batch was disposed of in 100% inspection, and the defective product was disposed for rework or scrapped. The defective products of acceptance batch was also the same disposed.
5) Assume that the inspectors do not have the inspection bias.
6) The quality inspection method is an attribute inspection.

## 2. Literature

This section mainly discusses related researches of the sampling plan, and explains the differences between this paper and the literature. The literature is divided into two parts for discussion. The first part is about the use of sampling inspection plans, and the second part is the relevant literature for the construction of the sample plan that the quality costs were considered. Comparison of literature differences can be seen on table 1.

### 2.1. The use of sampling inspection plan

Brooks (1989) reported the development and use of a computer program which may be used to design single sampling plans using either the binomial or Poisson distribution. The program also finds alternate plans with smaller sample size, and gives a measure of the proximity of such alternate plan to optimality. Some rudimentary artificial intelligence techniques are employed in the search and selection of optimal plans and the near-optimal alternative plans. Sultan (1994) presented a developing model to compute the optimum design for a double sampling plan. This model was demonstrated with an application. Also, a sensitivity analysis was carried out to show the effect of change of various parameters on the solution. Pearn and Wu (2006) developed a new sampling plan based on the exact sampling distribution rather than approximation. Practitioners can use the proposed sampling plan to determine accurate number of product items to be inspected and the corresponding critical acceptance value, to make reliable decisions. They also tabulated the required sample size n and the corresponding critical acceptance value for various α-risks, β-risks, and the levels of lot or process fraction of defectives that correspond to acceptable and rejecting quality levels. Jamkhaneh et al. (2010) presented the acceptance single sampling plan when the fraction of nonconforming items is a fuzzy number and being modeled based on the fuzzy Poisson distribution. They have shown that the operating characteristic (OC) curves of the plan are like a band having high and low bounds whose width depends on the ambiguity proportion parameter in the lot when that sample size and acceptance numbers is fixed. Dumičić and Žmuk (2012) showed that some intentional manipulations by using different sampling plans are possible. Hald (1960) reviewed present sampling inspection plans for attributes placing particular emphasis on their underlying assumptions. A model optimum sampling plans are derived which minimize the average costs. Qin et al. (2015) developed a three-step solution procedure that effectively reduces the solution time for larger size problems commonly seen in assembly lines. The proposed optimization model provides insightful implications for quality management. Rezaei (2016) constructed the economic order quantity (EOQ) models that consider imperfect items. The numerical examples show that by considering the sampling inspection plans, the buyer gains more profit compared to the traditional EOQ, and EOQ models with full inspection. Wu et al. (2017) proposed two types of variables quick switching sampling (VQSS) system based on the process capability index Cpk are proposed. The one is under a normal inspection and the other is under a tightened inspection. The performance of the two types of VQSS system are compared with the single sampling plan through the operating characteristic (OC) curve and the average sample number (ASN) required for inspection. Lee et al. (2018) develops a modified sampling plan that considers preceding lot information. By minimizing the average sample number while satisfying the quality levels demanded by both the producer and the consumer, the plan parameters can be obtained for product acceptance determination.

### 2.2. The sampling plan construction under quality costs consideration

Kobilinsky and Bertheaub (2005) determined a cost control function based on the number of groups and the total number of samples to obtain the least costly acceptance sampling plan and ensure that the risk rate of consumers and producers were below predetermined thresholds of both parties. Haji and Haji (2004) consider a special sampling plan, which has been developed for continuous production processes. The objective is to derive the total cost includes the costs of inspection, reworks, and defective items returned by the customers, and the minimum cost policy for the sampling plan. Torng et al. (2009) provided cost model modified by adding the statistical constraints to develop the design model of DS X-bar chart for the optimization of design parameters-sample size, control limit coefficient, warning limit coefficient and sampling interval. Nezhad and Nasab (2011) introduced a control policy for the acceptance sampling problem. Decision was made based on the number of defectives items in an inspected batch. The objective of the model is to find a constant control level that minimizes the total costs, including the cost of rejecting the batch, the cost of inspection and the cost of defective items. The optimization is performed by approximating the negative binomial distribution with Poisson distribution and using the properties of binomial distribution. Bouslah et al. (2016) consider the preventive maintenance and quality control for a stochastic production system subject to both reliability and quality deteriorations. The main objective is to optimize the production lot size, the inventory threshold, the sampling plan parameters and the overhaul threshold by minimizing the total incurred cost.

**Table 1.** The Table of Literature comparison

| Author \ Feature | MIL-STD-105E | Sampling Plan | Quality risk | Quality costs | External failure cost | Scrap revenue |
|---|---|---|---|---|---|---|
| Brooks (1989) | | V | V | | | |
| Sultan (1994) | | V | V | | | |
| Alireza and Rasoul (2004) | | | V | V | V | |
| Haji and Haji (2004) | | V | V | V | | |
| Kobilinsky and Bertheaub (2005) | | | V | V | | |
| Pearn and Wu (2006) | | V | V | | | |
| Torng et al. (2009) | | V | V | V | | |
| Jamkhaneh et al. (2010) | | V | V | | | |
| Nezhad and Nasab (2011) | | V | V | V | | |
| Dumičić and Žmuk (2012) | | V | V | | | |
| Qin et al. (2015) | | V | V | | | |
| This paper | V | V | V | V | V | V |

## 3. Model construction

This paper defines an inspection plan under an economic design. Based on considering all quality-related costs, including sampling inspection cost, external failure cost, rework cost, and scrap revenue to discuss the difference between traditional single-sampling plans and economic design in the total expected quality costs. By designing the Visual Studio 2010 program, we establish a mathematical model of total expected quality costs of the traditional single sampling plan and economic design under the sampling plan, and discuss the difference between traditional single sampling plans and the economic design under various situations. The mathematical model derivation and solution process in this article are explained later.

### 3.1. Symbol definition

$TQC(n, c)$: The total expected quality costs function given n, c, $P_{real}$ and $P_{critical}$
$N$: Delivery batch or production lot
$n$: Sample size, $n$ =1, 2, 3…
$x$: The number of defective products in the sample (a random variable)
$\hat{p}$: Sample defective rate
$\hat{p}_U$: Upper limit of defective rate in sample size
$p_{real}$: Real defective rate
$q_{real}$: Real yield rate, $p_{real} = 1 - q_{real}$
$p_{critical}$: Upper limit of real defective rate in lot
$q_{critical}$: Lower limit of real yield rate in lot, $P_{critical} = 1 - q_{critical}$
$\alpha$: The probability of Type I error for given $p_{critical}$
$\beta$: The probability of Type II error for given $p_{real} > p_{critical}$
$C_I$: The inspection cost per unit
$C_L$: The external failure cost per unit, refers to damages caused by defective products used by the customer
$C_U$: The make-up cost per unit
$C_{rework}$: Rework cost per unit
$INC_{scrap}$: The scrap income per unit
$p_I$: The proportion of the total expected number in the delivery batch
$p_{out}$: The scrap rate of defects
$TC_I$: The total expected inspection cost
$TR_{scrap}$: The total expected scrap revenue
$TC_L$: The total expected external failure cost
$TC_U$: The total expected make-up cost

$E(T_n)$: The total expected number of inspected products in delivery batch
$E(T_x)$: The expected number of defective products in the delivery batch
$f(x)$: Probability density function of $x$
$f(p_{real})$: Probability density function of $p_{real}$

### 3.2. Mathematical model of traditional single sampling plan

The traditional single-sampling plan assumes that the number of defective products detected from the sample size $n$ is a random variable $X$. From the product lot $N$, $n$ samples are randomly taken out with non-replacement method. The probability that the number of defective products contained in the sample follows the hypergeometric distribution, and when $N \geqq 10n$, the binomial distribution can be used. Furthermore, if $p \leqq 0.1$, then Poisson distribution can be used to approximate the binomial distribution (Bai, 2010). As above, we can assume $X \sim Poisson(\mu)$, and the real defective rate ($P_{real}$) can be obtained from historical data, Therefore, the probability density function we can obtain as follows:

$X \sim Poisson(\mu)$

$$\Rightarrow \mathrm{f(x)} = \frac{e^{-\mu}}{x!} \cdot \mu^x, x = 0, 1, 3, \ldots \quad (1)$$

And the expected value of $X$ ($E(X)$) is as follows:

$\mathrm{E(X)} = \mu$

Where $\mu$ is the average number of defects in the sample size $n$.

The benefits and loss of producer and buyer are considered for traditional single sampling plan. In other words, the producer risk α and consumer risk $\beta$ are agreed to determine the sample size $n$ and the acceptance number of defects allowed $c$, as a rule for accept or reject. It shows in Figure 1.

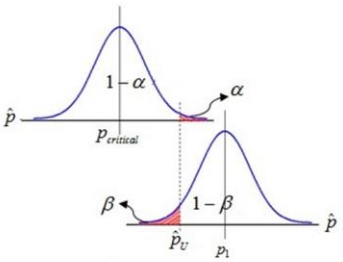


**Figure 1.** Producer risk α and consumer risk $\beta$

The formula of the upper limit of defect specification ($\hat{p}_U$) is as follows:

$$\hat{p}_U = p_{critical} + Z_{1-\alpha} \cdot \sqrt{\frac{p_{critical} \cdot q_{critical}}{n}} = p_1 - Z_{1-\beta} \cdot \sqrt{\frac{p_1 \cdot q_1}{n}}$$

$$\Rightarrow \frac{1}{\sqrt{n}} \cdot \left(Z_{1-\alpha} \cdot \sqrt{p_{critical} \cdot q_{critical}} + Z_{1-\beta} \cdot \sqrt{p_1 \cdot q_1}\right) = p_1 - p_{critical}$$

$$\therefore n = \left\lceil \left(\frac{Z_{1-\alpha} \cdot \sqrt{p_{critical} \cdot q_{critical}} + Z_{1-\beta} \cdot \sqrt{p_1 \cdot q_1}}{p_1 - p_{critical}}\right)^2 \right\rceil \quad (2)$$

$\lceil \quad \rceil$ means unconditional carrying.

The acceptance number of defects allowed (c) is as follows:

Let $\hat{p}_U = \frac{c}{n}$

$$= p_{critical} + Z_{1-\alpha} \cdot \sqrt{\frac{p_{critical} \cdot q_{critical}}{n}}$$

$$= p_{real} - Z_{1-\beta} \cdot \sqrt{\frac{p_{real} \cdot q_{real}}{n}}$$

$$\therefore \mathrm{c} = \mathrm{n} \cdot p_{critical} + Z_{1-\alpha} \cdot \sqrt{n \cdot p_{critical} \cdot q_{critical}} \quad (3)$$

We know $c = n \cdot \hat{p}_U$ , if $n \cdot \hat{p}_U$ is not integer, then round down to integer, we can write $\mathrm{c} = \lfloor n \cdot \hat{p}_U \rfloor$. Based on the derivation as above, we can determine the sample size ($n$) and acceptance number of defects allowed ($c$). If $\mathrm{x} \leq \mathrm{c}$, then accepted the delivery batch; if, $\mathrm{x} > \mathrm{c}$ then rejected it.

### 3.3. The mathematical model of sampling plan under economic design

The inspection plan under economic design considers the quality costs including inspection cost, external failure cost, the make-up cost, rework cost and scrap revenue to determine the sample sizes ($n$) and the number of defective products allowed ($c$) that minimized the total expected cost of quality, however, the scrap revenue is an income item.

Assume the number of defects in the sample is a random variable $X$ and follows Poisson distribution, i.e. $X \sim Poisson(\mu)$ (Bai, 2010), the formula is as follows:

$$f(x,\mu) = \frac{e^{-\mu}}{x!} \cdot \mu^x, x = 0, 1, 2, 3, K, n$$

$$0 < \mu < \infty$$

Where $\mu = n \cdot p_{real}$

The acceptance rule is as follows:

$$\begin{cases} if\ 0 \leq x \leq c, & \text{Accept} \\ \text{if } c<x \leq n, & \text{Reject} \end{cases}$$

Based as above, the total expected number of inspected products in delivery batch is as follows:

$$\mathrm{E}(T_n) = n \cdot \sum_{0}^{c} \frac{e^{-\mu}}{x!} \cdot \mu^x + N \cdot \sum_{c+1}^{n} \frac{e^{-\mu}}{x!} \cdot \mu^x,$$

$$x = 0, 1, 2, 3, k, n \qquad (4)$$

The total expected number of defects in the inspected sample ($E(X)$) is as follows:

$$\mathrm{E(X)} = \mathrm{n} \cdot p_{real}$$

Besides, the proportion of the total expected number in the delivery batch ($p_I$) is as follows:

$$p_I = \frac{E(T_n)}{N} \qquad (5)$$

The total expected number of defects is not inspected in delivery batch is as follows:

$$N \cdot p_{real} - N \cdot p_{real} \cdot p_I = N \cdot p_{real} \cdot (1 - p_I)$$

This study discusses the inspection plan under economic design. The quality costs are mainly considered. For given delivery batch ($N$), real yield rate ($p_{real}$) and the upper limit of defective rate ($p_{critical}$), the sample size ($n$) and the acceptance numbers of defective products allowed ($c$) are determined based on the minimized total expected quality costs. The various quality cost items are described as follows:

$$TC_I = E(T_n) \cdot C_I \qquad (6)$$

$$TC_L = \left(N \cdot p_{real} \cdot (1 - p_I)\right) \cdot C_L \qquad (7)$$

$$TC_U = C_U \cdot p_{out} \cdot E(T_n) \cdot p_{real} \qquad (8)$$

$$TC_{rework} = C_{rework} \cdot (1 - p_{out}) \cdot E(T_n) \cdot p_{real} \qquad (9)$$

Total scrap revenue = the sales income per unit scrap × the numbers of scraps, where the scrap revenue means the benefit from recycle or resale. This is a deduction item from total quality cost. The formula is as follows:

$$TR_{scrap} = INC_{scrap} \cdot p_{out} \cdot E(T_n) \cdot p_{real} \qquad (10)$$

Total quality cost is the sum of the total inspection cost, total external failure cost, total scrap cost, the total make-up cost and total rework cost, and the total scrap revenue is deducted. The formula is as follows:

$$\mathrm{TQC(n,c)} = \int_{\hat{p}^L}^{\hat{p}^U} (TC_I + TC_L + TC_U + TC_{rework} - TR_{scrap}) \cdot f(\hat{p}) \cdot d\hat{p} \qquad (11)$$

So, the complete mathematical model is as follows:

**Min:** $TQC(n,c)$

**s.t.:**

$$\mathrm{E}(T_n) = n \cdot \left( \sum_{x=0}^{c} \frac{e^{-n \cdot p_{real}} \cdot (n \cdot p_{real})^x}{x!} \right) + N \cdot \left( \sum_{x=c+1}^{n} \frac{e^{-n \cdot p_{real}} \cdot (n \cdot p_{real})^x}{x!} \right)$$

$$p_I = \frac{E(T_n)}{N}$$

$$X \sim Poisson(\mu), \mu = n \cdot p_{real}$$

$$1 \leq n < N$$

$$0 \leq c \leq N$$

$$\hat{p} \sim f(\hat{p})$$

According to the established mathematical model, using the optimization theory and numerical integration method, find out the optimal combination of decision variables that can make the TQC have the minimum costs under the constraint conditions. The other part that needs to be explored is the probability density function, which can use the historical data and find the *p.d.f* of $\hat{p}$ through goodness-of fit test. The commonly used goodness-of fit test is the K-S test. However, if there is no historical data, it is generally assumed that $\hat{p}$ follows a given distribution.

This study uses the production data from company to find out the $(n^*, c^*)$ satisfied the constraints and make *TQC*($n$,$c$) minimum. Next, we compare the traditional single sampling plan with the economic design sampling plan under various values of $p_{real}$. What is better at *TQC* and what is worse. And explores the possible causes, and puts forward specific conclusions and recommendations. The complete model is as follows:

$$TQC(n,c) = \int_{\hat{P}L}^{\hat{P}U} \left[TC_I + TC_L + TC_U + TC_{rework} - TR_{scrap}\right] \cdot f(\hat{P}) \cdot d\hat{P}$$

$$= \int_{\hat{P}L}^{\hat{P}U} \left\{ C_I \cdot \left[ n \cdot \sum_{x=0}^{c} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} + N \cdot \sum_{x=c+1}^{n} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} \right] + C_L \right.$$

$$\cdot \left[ N \cdot \hat{P} \cdot \left( 1 - \left( \frac{n \cdot \sum_{x=0}^{c} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} + N \cdot \sum_{x=c+1}^{n} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!}}{N} \right) \right) \right]$$

$$C_U \cdot p_{out} \cdot \left[ \hat{P} \cdot \left( n \cdot \sum_{x=0}^{c} \frac{e^{-n\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} + N \cdot \sum_{x=c+1}^{n} \frac{e^{-n\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} \right) \right] + C_{rework} \cdot \hat{P} \cdot (1 - p_{out}) \cdot \left[ n \cdot \sum_{x=0}^{c} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} + N \cdot \sum_{x=c+1}^{n} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} \right] - INC_{scrap} \cdot p_{out} \cdot \left( n \cdot \sum_{x=0}^{c} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} + N \cdot \sum_{x=c+1}^{n} \frac{e^{-n\cdot\hat{P}} \cdot (n\cdot\hat{P})^x}{x!} \right) \cdot \hat{P} \left. \right\} \cdot f(\hat{P}) \cdot d\hat{P} \quad (12)$$

### 3.4. Sensitivity Analysis

After explaining the model of the sampling inspection plan under economic design, sensitivity analysis was conducted to further understand the impact of various parameter changes on the total quality cost. As shown in the previous section, the total cost of quality under economic design is as follows:

$$TQC(n,c) = \int_{\hat{P}L}^{\hat{P}U} \left(TC_I + TC_L + TC_U + TC_{rework} - TR_{scrap}\right) \cdot f(\hat{P}) \cdot d\hat{P}$$

$$TQC(n,c) = \int_{\hat{P}L}^{\hat{P}U} \left\{ (C_I \cdot E(T_n)) + \left(C_L \cdot N \cdot \left(1 - \frac{E(T_n)}{N}\right) \cdot \hat{P}\right) + \left(C_U \cdot p_{out} \cdot E(T_n) \cdot \hat{P}\right) + \left(C_{rework} \cdot (1 - p_{out}) \cdot E(T_n) \cdot \hat{P}\right) - \left(INC_{scrap} \cdot p_{out} \cdot E(T_n) \cdot \hat{P}\right) \right\} \cdot f(\hat{P}) \cdot d\hat{P} \quad (13)$$

Where the total expected number of inspection is as follows:

$$E(T_n) = n \cdot \sum_{0}^{c} \frac{e^{-\mu}}{x!} \cdot \mu^x + N \cdot \sum_{c+1}^{n} \frac{e^{-\mu}}{x!} \cdot \mu^x$$

$$E(T_n) = n \cdot F_{Poisson}(n,c) + N \cdot \left(1 - F_{Poisson}(n,c)\right) \quad (14)$$

Where $F_{Poisson}(n,c) = \sum_{0}^{c} \frac{e^{-\mu}}{x!} \cdot \mu^x$

Therefore, the following sensitivity analysis is performed by partial $TQC(n,c)$ with respect to each parameter.

1) The effect of change in production lot ($N$) on total expected quality costs:

$$\frac{\partial TQC(n,c)}{\partial N} =$$

$$\int_{\hat{P}^L}^{\hat{P}^U} \left\{ \begin{array}{l} C_I \cdot [(1 - F_{Poisson}(n,c))] + \\ C_L \cdot \hat{P} + C_L \cdot \hat{P} \cdot (1 - F_{Poisson}(n,c)) + \\ C_U \cdot p_{out} \cdot \hat{P} \cdot (1 - F_{Poisson}(n,c)) + \\ C_{rework} \cdot \hat{P} \cdot (1 - p_{out}) \cdot (1 - F_{Poisson}(n,c)) - \\ INC_{scrap} \cdot \hat{P} \cdot p_{out} \cdot (1 - F_{Poisson}(n,c)) \end{array} \right\} \cdot f(\hat{P}) \cdot d\hat{P} \quad (15)$$

If $\partial TQC(n,c)/\partial N \leq 0$, it means the production lot is inversely proportional to the total expected quality costs; If $\frac{\partial TQC(n,c)}{\partial N} > 0$, it means the production lot is proportional to the total expected quality costs.

2) The effect of change in inspection cost per unit ($C_I$) on total expected quality costs:

$$\frac{\partial TQC(n,c)}{\partial C_I} =$$

$$\int_{\hat{P}L}^{\hat{P}U} \{n \cdot F_{Poisson}(n,c) + N \cdot (1 - F_{Poisson}(n,c))\} \cdot f(\hat{P}) \cdot d\hat{P} \geq 0 \quad (16)$$

It means the increase of inspection cost per unit will cause the increase of total expected quality costs. The two have changed in the same direction.

3) The effect of change in external failure cost per unit ($C_L$) on total expected quality costs:

$$\frac{\partial TQC(n,c)}{\partial C_L} = \int_{\hat{P}^L}^{\hat{P}^U} \{N \cdot \hat{P} - \hat{P} \cdot (n \cdot F_{Poisson}(n,c) + N \cdot (1 - F_{Poisson}(n,c)))\} \cdot f(\hat{P}) \cdot d\hat{P} \geq 0 \quad (17)$$

It means the increase of external failure cost will cause the increase of total expected quality costs. The two have changed in the same direction.

4) The effect of change in make-up cost per unit ($C_U$) on total expected quality costs:

$$\frac{\partial TQC(n,c)}{\partial C_U} = \int_{\hat{P}^L}^{\hat{P}^U} \{p_{out} \cdot \hat{P} \cdot n \cdot F_{Poisson}(n,c) + p_{out} \cdot \hat{P} \cdot N \cdot (1 - F_{Poisson}(n,c))\} \cdot f(\hat{P}) \cdot d\hat{P} \geq 0 \quad (18)$$

It means the increase of make-up cost per unit will cause the increase of total expected quality costs. The two have changed in the same direction.

5) The effect of change in rework cost per unit ($C_{rework}$) on total expected quality costs:

$$\frac{\partial TQC(n,c)}{\partial C_{rework}} = \int_{\hat{P}^L}^{\hat{P}^U} \hat{P} \cdot (1 - p_{out}) \cdot [n \cdot F_{Poisson}(n,c) + N \cdot (1 - F_{Poisson}(n,c)] \cdot f(\hat{P}) \cdot d\hat{P} \geq 0 \quad (19)$$

It means the increase of rework cost per unit will cause the increase of total expected quality costs. The two have changed in the same direction.

(6) The effect of change in scrap revenue per unit ($INC_{scrap}$) on total expected quality costs:

$$\frac{\partial TQC(n,c)}{\partial INC_{scrap}} = -\int_{\hat{P}^L}^{\hat{P}^U} \{\hat{P} \cdot p_{out} \cdot (n \cdot F_{Poisson}(n,c) + N \cdot (1 - F_{Poisson}(n,c))\} \cdot f(\hat{P}) \cdot d\hat{P} \leq 0 \quad (20)$$

It means the increase of scrap revenue per unit will cause decrease of total expected quality costs. The two have an inverse relationship.

## 4. Case Analysis

This section takes an example to discuss the difference between the traditional single sampling plan and the sampling plan under economic design on the basis of the total quality cost. The traditional single sampling plan based on the agreement of risks by producer and buyer and used MIL-STD-105E to determine the sample sizes and the number of defective products allowed. The production data and various parameters of the delivery batch are shown in Table 2. Furthermore, the sensitivity analysis of various cost parameters is also performed.

**Table 2.** Production data

| Item | Value | Item | Value |
|---|---|---|---|
| $N$ | 5000 | $C_{rework}$ | 15 |
| $C_I$ | 5 | $p_{out}$ | 0.05 |
| $C_L$ | 100 | $p_{real}^U$ | 0.02 |
| $C_U$ | 70 | $p_{real}^L$ | 0 |
| $C_{reback}$ | 5 | $p_{critical}$ | 0.65% |

### 4.1. The cost analysis of MIL-STD-105E sampling table

The industry generally uses MIL-STD-105E as the basis for sampling inspection. MIL-STD-105E is divided into two steps. The first step determines the level of inspection first. When there is no other requirement, level II is generally used. Therefore, the number of inspections confirmed is based on the number *N*. The second step is to confirm the inspection code, according to the $AQL$ (Average Quality Level, that is $p_{critical}$) to check the table to confirm the number of samples *n* and the allowable number of defects *c*. Following the production information in Table 2, confirm the sampling number and allowable number, as shown in Table 3, *N* = 5000; $AQL$ = 0.65%. According to Table 3, the inspection code is “*L*”, and then according to Table 4, the sampling number $n$ = 200 and the permissible number $c$ = 3.

In accordance with the above MIL-STD-105E table, we can find (n, c) = (200,3) and put the result into the economic model of the sample test, the total quality cost is calculated as follows:

$$TQC(200{,}3) = \int_0^{0.02}\{5\cdot[200\cdot F_{Poisson}(200{,}3) + 5000\cdot(1-F_{Poisson}(200{,}3)] + 100\cdot\left[5000\cdot\hat{P}\cdot(1-\frac{200\cdot F_{Poisson}(200{,}3)+5000\cdot(1-F_{Poisson}(200{,}3)}{5000}\right] + 70\cdot 0.05\cdot[\hat{P}\cdot(200\cdot F_{Poisson}(200{,}3)+5000\cdot(1-F_{Poisson}(200{,}3)\big] + 15\cdot p_{real}\cdot(1-0.05)\cdot\left[200\cdot F_{Poisson}(200{,}3)+5000\cdot\left(1-F_{poison}(200{,}3)\right)\right] - 5\cdot 0.05\cdot[200\cdot F_{Poisson}(200{,}3)+5000\cdot(1-F_{Poisson}(200{,}3)]\cdot\hat{P}\}\cdot f(\hat{P})\cdot d\hat{P} = 9{,}447$$

## 4.2. The cost analysis of traditional single sampling plan

The traditional single sampling plan mainly considers the risk rate of producers and consumers to determine the sample size and the number of permissible defects. In general, the statistically significant level α is set to 5% and β is 10%. On the other hand, the criticality rate of the product is determined by both the seller and the buyer as AQL = 0.65%. Substituting it into the following formula yields:

$$n = \left\lceil\left(\frac{Z_{1-\alpha}\cdot\sqrt{p_{critical}\cdot q_{critical}} + Z_{1-\beta}\cdot\sqrt{p_1\cdot q_1}}{p_1 - p_{critical}}\right)^2\right\rceil = \left\lceil\left(\frac{(1.6456\times\sqrt{0.0065\times 0.9935)} + (1.2883\times\sqrt{0.001\times 0.999)}}{0.001-0.0065}\right)^2\right\rceil = 532$$

$$c = \lfloor(n\cdot p_{critical} + Z_{1-\alpha}\cdot\sqrt{n\cdot p_{critical}\cdot q_{critical}})^2\rfloor = \left\lfloor(532\times 0.001 + 1.6456\times\sqrt{532\times 0.0065\times 0.9935})^2\right\rfloor = 3$$

Substituting (n, c) = (532, 3) into the mathematical model of total quality cost under economic design, the total cost of quality for the traditional single-spot inspection program is 18,285. The calculation process is as follows:

$$TQC(532{,}3) = \int_0^{0.02}\{5\cdot[532\cdot F_{Poisson}(532{,}3) + 5000\cdot(1-F_{Poisson}(532{,}3)] + 100\cdot\left[5000\cdot\hat{P}\cdot(1-\frac{532\cdot F_{Poisson}(532{,}3)+5000\cdot(1-F_{Poisson}(532{,}3)}{5000}\right] + 70\cdot 0.05\cdot[\hat{P}\cdot(532\cdot F_{Poisson}(532{,}3)+5000\cdot(1-F_{Poisson}(532{,}3)\big] + 15\cdot\hat{P}\cdot(1-0.05)\cdot[532\cdot F_{Poisson}(532{,}3)+5000(1-F_{Poisson}(532{,}3))\big] - 5\cdot 0.05\cdot[532\cdot F_{Poisson}(532{,}3)+5000\cdot(1-F_{Poisson}(532{,}3)]\cdot\hat{P}\}\cdot f(\hat{P})\cdot d\hat{P} = 18{,}285$$

## 4.3. Cost analysis of sampling plan under economic design

After considering the minimization of the total costs of quality, use a computer program to solve the problem and use the production data in Table 2. After solving the problem, it is known as (1, 1), and the total costs of quality is 5,055, which represents the number of samples of the inspection batch is 1, the number of permissible defectives is also 1, which can minimize the total costs of quality, because the production data in Table 2, there is not much difference between the external failure cost and the internal scrapping cost. If the external cost of failure is adjusted from 100 to 2,000, then the number of allowed defects will be 0 and the total costs of quality is 25,354. Table 5 compares the cost of each sampling inspection model.

**Table 3.** MIL-STD-105E Normal-Sampling Code List (Zhan, 2012)

| Lot size | | | Special level | | | | Normal level | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | S-1 | S-2 | S-3 | S-4 | I | II | III |
| 2 | to | 8 | A | A | A | A | A | A | B |
| 9 | to | 15 | A | A | A | A | A | B | C |
| 16 | to | 25 | A | A | B | B | B | C | D |
| 26 | to | 50 | A | B | B | C | C | D | E |
| 51 | to | 90 | B | B | C | C | C | E | F |
| 91 | to | 150 | B | B | C | D | D | F | G |
| 151 | to | 280 | B | C | D | E | E | G | H |
| 281 | to | 500 | B | C | D | E | F | H | J |
| 501 | to | 1200 | C | C | E | F | G | J | K |
| 1201 | to | 3200 | C | D | E | G | H | K | L |
| 3201 | to | 10000 | C | D | F | G | J | **L** | M |
| 10001 | to | 35000 | C | D | F | H | K | M | N |
| 35001 | to | 150000 | D | E | G | J | L | N | P |
| 150001 | to | 500000 | D | E | G | J | M | P | Q |
| 500001 | to | over | D | E | H | K | N | Q | R |

**Table 4.** MIL-STD-105E Normal – Single Sampling Plan (Zhan, 2012)

| Sample code | Sample size | AQL | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | .025 | .040 | .065 | .10 | .15 | .25 | .40 | .65 |
| | | AcRe | AcRe | AcRe | AcRe | AcRe | AcRe | AcRe | AcRe |
| A | 2 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| B | 3 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| C | 5 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| D | 8 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| E | 13 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ |
| F | 20 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 0 1 |
| G | 32 | ↓ | ↓ | ↓ | ↓ | ↓ | ↓ | 0 1 | ⇕ |
| H | 50 | ↓ | ↓ | ↓ | ↓ | ↓ | 0 1 | ⇕ | ⇕ |
| J | 80 | ↓ | ↓ | ↓ | ↓ | 0 1 | ⇕ | ⇕ | **1 2** |
| K | 125 | ↓ | ↓ | ↓ | 0 1 | ⇕ | ⇕ | 1 2 | 2 3 |
| L | 200 | ↓ | ↓ | 0 1 | ⇕ | ⇕ | 1 2 | 2 3 | 3 4 |
| M | 315 | ↓ | 0 1 | ⇕ | ⇕ | 1 2 | 2 3 | 3 4 | 5 6 |
| N | 500 | 0 1 | ⇕ | ⇕ | 1 2 | 2 3 | 3 4 | 5 6 | 7 8 |
| P | 800 | ⇕ | ⇕ | 1 2 | 2 3 | 3 4 | 5 6 | 7 8 | 10 11 |
| Q | 1250 | ⇕ | 1 2 | 2 3 | 3 4 | 5 6 | 7 8 | 10 11 | 14 15 |
| R | 2000 | 1 2 | 2 3 | 3 4 | 5 6 | 7 8 | 10 11 | 14 15 | 21 22 |

Ac: Acceptance number
Re: Reject number

**Table 5.** The cost comparison for each sampling inspection model

| $C_L$ / Model of sampling plan | $C_L = 100$ | $C_L = 2000$ |
|---|---|---|
| MIL-STD-105E | $TQC$(200,3)=9,447 | $TQC$(200,3)=73,868 |
| Traditional single sampling plan | $TQC$(532,3)=18,285 | $TQC$(532,3)=33,136 |
| Economical design | $TQC$(1,1)=5,055 | $TQC$(727,0)=25,354 |

From Table 3, it can be seen that the sampling plan under economic design is the best in the performance of total quality cost, but when the external failure cost is close to the internal scrap cost, for example, the customer complains that the defective product can agree to be replaced directly with the good product. The result of the mathematical model of the sampling plan under the economic design mentioned above is (1, 1), but it will not be implemented in inspection practice. Therefore, the direct acceptance method may be adopted, in other words, when the internal cost and external failure cost are similar, the direct acceptance method can be adopted; otherwise, if the external failure cost is much higher than the internal scrap cost, the mathematical model of the sampling plan under the economic design is obtained with n = 727, c = 0, it means that no defective products are allowed to occur, so as to avoid huge losses of external failure cost.

### 4.4. Sensitivity Analysis

This section analyzes the sensitivity of various cost parameters. Under all other parameters are fixed, the effect of single parameter changes on the total quality cost is discussed.

1) The effect of delivery batch changes on total quality cost.

From Table 6, we can see that given n = 200, c = 3, = 5, = 100, and when N increases, the total cost of quality will also increase, showing the two have positive relationship. When the delivery batch is doubled, the total cost of quality increases by about 92%.

2) The effect of unit inspection cost changes on total quality cost.

From Table 7, we can see that when the other cost parameters are given (n = 200, c = 3, $C_L$= 100), and $C_I$ increases, the total cost of quality will also increase, showing the positive correlation between the two. When the unit inspection cost doubles, the total quality cost increases by approximately 61%.

3) The effect of external failure cost per unit changes on total quality cost.

From Table 8, we can see that given other cost parameters are fixed (n = 200, c = 3, $C_I$= 5), when the external failure cost per unit increases, the total cost of quality will also increase. The two are in the same direction of change. When the external failure cost per unit is doubled, the total cost of quality also increases by approximately 36%.

4) The effect of rework cost per unit changes on total quality cost.

From Table 9, we can see that when given the other cost parameters are fixed (n = 200, c = 3, $C_I$= 5, $C_L$= 100), the higher the unit cost of rework, the total cost of quality will also increase. Although the two have positive correlation, but when the rework cost per unit is doubled, the total quality cost will only increase by about 1.84%.

5) The effect of scrap revenue per unit changes on total quality cost.

From Table 10, we can see that given the other cost parameters are fixed (n = 200, c = 3, $C_I$= 5, $C_L$= 100), when the scrap revenue per unit increases, the total cost of quality will decrease. A negative relationship exists. When the scrap revenue per unit is doubled, the total quality cost will only decrease by approximately 0.04%.

**Table 6.** The effect of the delivery batch changes on total quality cost

| $N$ | 5,000 | 10,000 | 15,000 | 20,000 | 25,000 | 30,000 |
|---|---|---|---|---|---|---|
| $TQC(200,3)$ | 9,447 | 18,200 | 26,953 | 35,705 | 44,458 | 53,211 |
| Degree of change | Basis | 92.65% | 92.65% | 92.65% | 92.65% | 92.65% |

**Table 7.** The effect of unit inspection cost changes on total quality cost

| $C_I$ | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| $TQC(200,3)$ | 9,447 | 15,214 | 20,981 | 26,748 | 32,515 | 38,282 |
| Degree of change | Basis | 61.05% | 61.05% | 61.05% | 61.05% | 61.05% |

**Table 8.** The effect of external failure cost per unit changes on total quality cost

| $C_L$ | 100 | 200 | 300 | 400 | 500 | 600 |
|---|---|---|---|---|---|---|
| $TQC(200,3)$ | 9,447 | 12,838 | 16,229 | 19,619 | 23,010 | 26,400 |
| Degree of change | Basis | 35.89% | 35.89% | 35.88% | 35.88% | 35.88% |

**Table 9.** The effect of rework cost per unit changes on total quality cost

| $C_{rework}$ | 15 | 30 | 45 | 60 | 75 | 90 |
|---|---|---|---|---|---|---|
| $TQC(200,3)$ | 9,447 | 9,684 | 9,920 | 10,157 | 10,393 | 10,630 |
| Degree of change | Basis | 2.51% | 2.51% | 2.51% | 2.51% | 2.51% |

**Table 10.** The effect of scrap revenue per unit changes on total quality cost

| $C_{reback}$ | 5 | 10 | 15 | 20 | 25 | 30 |
|---|---|---|---|---|---|---|
| $TQC(200,3)$ | 9,447 | 9,443 | 9,439 | 9,435 | 9,431 | 9,427 |
| Degree of change | Basis | -0.04% | -0.04% | -0.04% | -0.04% | -0.04% |

From the analysis as above, the total quality cost will increase if the delivery batch, inspection cost per unit and external failure cost increase. However, the total quality cost is inversely related to the scrap revenue per unit.

## 5. Conclusions

During the production process, due to assignable or non-assignable cause, the defects will occur. Therefore, in order to ensure product quality, the use of sampling plans has become quite widespread in the industry, and many of them follow MIL-STD-105E as a sampling plan standard, but this sampling plan did not consider various quality cost factors. Therefore, this study proposes a sampling inspection program under the economic design. Finally, the comparison of the three (MIL- STD-105E sampling plan, sampling plan under risks of producer and consumer, and sampling plan under economic design of this study) are performed. The specific conclusions of this study are as follows:

1) The research presents a sampling plan under economic design with minimum total quality cost by considering various quality costs and scrap revenue.
2) If the make-up cost is close to the external failure cost, the sampling plan under economic design is ($n$*, $c$*) = (1, 1), indicating that the customer agrees that defective products can be exchanged by good products and therefore the delivery batch can be accepted directly. In the

other hand, when the inspection cost of the unit is far lower than the external failure cost, the sampling plan needs to use c = 0, which means that the defective sample will not be allowed to appear in the sample. When there is a defective product, a full inspection is required.

3) From the sensitivity analysis results, we can see that the total quality cost will increase with the increase of each cost item. Among them, the impact of the number of delivery batch ($N$), unit inspection costs ($C_I$) and external failure costs ($C_L$) will be more significant than other cost parameters.

## References:


Bai, C. Q. (2010). *Statistical Methods*. Taiwan: Chinese Society for Quality.

Bouslah, B., Gharbi, A., & Pellerin, R. (2016). Integrated production, sampling quality control and maintenance of deteriorating production systems with AOQL constraint. *Omega*, *61*, 110-126. doi: 10.1016/j.omega.2015.07.012

Brooks, G. H. (1989). Acceptance sampling design by computer — Optimality concerns. *Computers & Industrial Engineering*, *17*(1-4), 508-513.

Collins, R. D. Jr., Case, K. E., & Bennett, G. K. (1973). The effects of inspection error on single sampling inspection plans. *International Journal of Production Research*, *11*(3), 289-298. doi: 10.1080/00207547308929972

Dumičić, K., & Žmuk, B. (2012). Decision making based on single and double acceptance sampling plans for assessing quality of lots. *Business Systems Research*, *3*(2), 27-40. doi: 10.2478/v10305-012-0010-4

Haji, A., & Haji, R. (2004). The optimal policy for a sampling plan in continuous production in terms of the clearance number. *Computers & Industrial Engineering*, *47*(2-3), 141-147. doi: 10.1016/j.cie.2004.06.005

Hald, A. (1960). The Compound Hypergeometric Distribution and a System of Single Sampling Inspection Plans Based on Prior Distributions and Costs. *Technometrics*, *2*(3), 275-340.

Jamkhaneh, E. B., Sadeghpour-Gildeh, B., & Yari, Gh. (2010). Acceptance Single Sampling Plan by Using of Poisson Distribution. *The Journal of Mathematics and Computer Science*, *1*(1), 6-13. doi: 10.22436/jmcs.001.01.02

Kobilinsky, A., & Bertheau, Y. (2005). Minimum cost acceptance sampling plans for grain control, with application to GMO detection. *Chemometrics and Intelligent Laboratory Systems*, 75(2), 189-200. doi: 10.1016/j.chemolab.2004.07.005

Lee, A. H.I., Wu, C. W., & Wang, Z. H. (2018). The construction of a modified sampling scheme by variables inspection based on the one-sided capability index. *Computers & Industrial Engineering*, *122*, 87-94. doi: 10.1016/j.cie.2018.05.037

MIL-STD-105E (1989). *Military standard: sampling procedures and tables for inspection by attributes.* Washington, DC: US Department of Defense (DoD).

Nadeem, M., & Velasco, T. (1993). Knowledge-based approach to quality control assurance using bar code identification systems. *Computers & Industrial Engineering*, *25*(1-4), 41-44.

Nezhad, M. S. F., & Nasab, H. H. (2011). Designing a Single Stage Acceptance Sampling Plan Based on the Control Threshold Policy. *International Journal of Industrial Engineering & Production Research*, *22*(3), 143-150.

Pearn, W. L., & Wu, C. W. (2006). Critical acceptance values and sample sizes of a variables sampling plan for very low fraction of defectives. *Omega*, *34*(1), 90-101. doi: 10.1016/j.omega.2004.08.001

Qin, R., Cudney, E. A., & Hamzic, Z. (2015). An optimal plan of zero-defect single-sampling by attributes for incoming inspections in assembly lines. *European Journal of Operational Research*, *246*(3), 907-915. doi: 10.1016/j.ejor.2015.05.054

Rezaei, J. (2016). Economic order quantity and sampling inspection plans for imperfect items. *Computers & Industrial Engineering*, *96*, 1-7. doi: 10.1016/j.cie.2016.03.015

Schilling, E. G., & Neubauer, D. V. (2009). *Acceptance sampling in quality control*. CRC Press.

Sultan, T. I. (1994). Optimum design of sampling plans in electronic industry. *Microelectronics Reliability*, *34*(8), 1369-1373. doi: 10.1016/0026-2714(94)90152-X

Torng, C. C., Lee, P. H., & Liao, N. Y. (2009). An economic-statistical design of double sampling $\bar{\mathbf{X}}$ control chart. *International Journal of Production Economics*, *120*(2), 495-500.

Wu, C. W., Lee, A. H.I., Liu, S. W., & Shih, M. H. (2017). Capability-based quick switching sampling system for lot disposition. *Applied Mathematical Modelling*, *52*, 131-144. doi: 10.1016/j.apm.2017.07.050

Zhang, Y. C. (2012). *Sampling Inspection.* Chinese Society for Quality, Taiwan.

**Yun-Cheng Huang**
Department of Industrial Management, National Pingtung University of Science and Technology, Taiwan,
Pingtung
ychuang@mail.npust.edu.tw

**Chien-Chuan Cheng**
Department of Industrial Management, National Pingtung University of Science and Technology, Taiwan,
Pingtung
eddiecheng8078@gmail.com

**Yi-An Ding**
Department of Industrial Management, National Pingtung University of Science and Technology, Taiwan,
Pingtung
yian0213@gmail.com