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ANALYSIS OF TRANSVERSE STABILITY PARAMETERS OF HYBRID BUSES WITH ACTIVE TRAILERS

Summary. The objective of the article is to determine the transverse stability indexes of hinge-connected buses (HCBs) by applying the computation-analytical method. The transverse stability parameters of hybrid buses with active trailers are analysed. Based on these parameters (the angles of the roll and redistribution of loads on the sides), the analytical dependences are developed. The dependences describe the movement of the parts of HCBs in the vertical plane. Considering the action of longitudinal and transverse forces, the roll angles of the bus and the trailer were determined. The limiting angle of the side roll of the bus rollover was found to be $\gamma_{aMax} = 27.56^0 \gamma_{Max}$, and the trailer rollover to be $\gamma_{aMax} = 30.21^0$. The obtained transverse stability indexes of HCBs with a hybrid power plant testify to the compliance with the standard DSTU UN/ECE R 111-00.

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1. INTRODUCTION

The dynamic properties of motion, stability and vehicle handling, to a large extent, ensure the safety of traffic. At present, the problem of determining the conditions of dynamic system stability is sufficiently studied. However, the challenge is to define the nature of the system's behaviour in the state of instability and identify the causes of its occurrence.

The main parameters of any vehicle (indicators of its ability to perform its functions) are the overall dimensions, mass parameters, speed and dynamic characteristics of transportation, etc. Passenger capacity and turnover, acceleration dynamics, stability, handling and especially manoeuvrability are highly important factors in the operation of city buses. In most countries, according to UN/ECE Regulation No. 36, the overall length of single buses is limited to 12 m, although there are combinations with a length of up to 15 m and articulated ones up to 18 m [1].

The normalized indexes of manoeuvrability of large-class buses (HCBs) can be developed due to appropriate layout schemes and the use of directive (self-installing) wheels in the trailer section (trailer), which can be driven by an electric motor located in the coupling section. In [2], the required power of the electric motor, which is installed in the trailer section of the bus, is determined under the conditions of starting from the following: rest, rectilinear motion and circular motion. However, the retrofitting of the bus with an additional engine located in the trailer section requires verification of its stability in order to ensure a guaranteed level of its external passive safety.

The creation of requirements for guaranteeing the required level of the external passive safety of passenger vehicles in conditions of overturning began in 1973 after a terrible bus rollover accident. This led Hungary to raise this issue for the first time at the ECE Summit in Geneva [3]. It took 12 years to create the relevant regulatory requirements until the first revision of UN/ECE Regulation R66 appeared. The main problem was to create an adequate research method for the real rollover process. Based on this method, the strong spatial structure of the body can be easily distinguished from the weak one [4]. The reduction in the strength of the body spatial structure degrades passenger safety indexes [5].

A forklift truck or a truck crane acts as the test equipment for lifting the platform. It must ensure the simultaneous raising of axes with the difference in the platform inclination angles measured below these axes being less than 1°. In the case of separate platforms (Figure 1a, c-d), the synchronization of the lift is provided by two automobile cranes.

An additional requirement for the test equipment is the simultaneity of lifting all the model axes without rocking and dynamic impact until the model rolls over, with an angular speed up to 5°/s (0.087 rad/s). According to the results obtained in various testing laboratories, this index varies in the range from 0.64 to 5°/s.

The indexes of stability assessment are the critical values of motion parameters characterizing the dynamic stability and the positions characterizing the static stability of the car and the auto-train [3]. Recommended values of the vehicles' stability indexes and the methods of their determination are given in UN/ECE Regulation No. 107, MS ISO 4188-82, GOST 3163-76, GOST R 52302-2004, OST 37.001.471-88, OST 37.001.487-89, DSTU 3310 -96.

a)   b)   c)
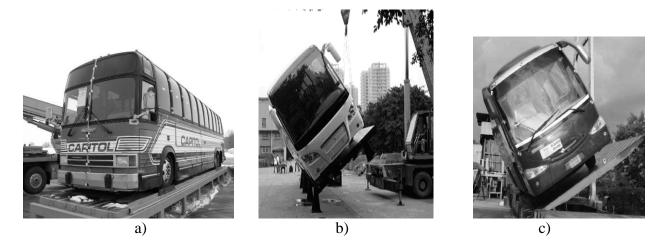
Fig. 1. General view of the test platform with the installed model of a bus:
a) Prevost Lemirage Motor Coach [8], b) Scania-Higer [9], c) IRIZAR PB Tour Bus [10]

Motor cars and small trucks can be easily tested by a turnover testing unit, while testing heavy trucks and especially auto-trains, as well as HCBs, incurs great difficulties, in particular, in the case of hinge-connected hybrid buses of a specifically large class with an active trailer. Thus, the purpose of the work is to determine the indexes of transverse stability of HCBs by the computation-analytical method [25].

2. MATERIALS AND METHODS

The application of the computation-analytical method for determining the parameters of the HCB's motion in the horizontal and vertical planes greatly simplifies the definition of these indexes [5].

Numerous studies have proven that there are mutual non-linear relationships between the variables characterizing auto-trains' movement in the horizontal and vertical planes; the relations in different cases of motion are different. The practice of studying the motion stability of both single vehicles and auto-trains in different regimes is extensive [6,7,24,26,27].

In this paper, the HCB's movement in the vertical plane along the angles of pitch (pitch, trim) and the roll are assumed to affect the lateral movement, mainly by changing the vertical loads on the wheels. The vertical reactions of the support surface are changed as well. In accordance with this concept, the movements were divided into lateral and longitudinal-transverse ones. Nowadays, the lateral movement is sufficiently studied [5-7], but the longitudinal-transverse motion should be investigated further. The spring and unsprung parts of the HCB's real construction are assumed to be connected with the help of elastic and damping elements, while the unsprung masses and the road are assumed to be connected with the help of tyres that possess both elastic and damping properties. At low velocities, the vertical displacements of spring and unsprung masses are carried out synchronously. In this case, there should be a static compression of the springs and tyres at low resistance of the shock absorbers [8]. Similarly, the spring masses are assumed to carry oscillations on the springs with some stiffness.

According to this approach, the forces acting on the support-coupling device do not affect the redistribution of loads on the sides of the HCB. The axis of the roll lies in the vertical

plane of symmetry. Therefore, a complex system such as an HCB can be considered as two systems that independently roll: a bus and a trailer section (trailer). Thus, the appearance and the analysis of the equations describing the roll are simplified.

Let us first formulate the equation of HCB parts' motion in the longitudinal and transverse planes [9]. For this purpose, the following notations are introduced: $2m_{ai}, 2m_{bj}$ - unsprung masses of the ith front and j^{th} rear suspensions; $c_{1i}, c_{2j}, c_{1iu}, c_{2ju}$ - respectively, radial stiffness of suspensions and tyres; f_{ai}, f_{bj} - static deflections of suspensions; λ^o_{ai} λ^o_{bj} - vertical deformations of tyres; F^o_{ai} F^o_{bj} - vertical reactions of the ith front and j^{th} rear supports on the bus body; z_0. l_{ai}, l_{bj} - the height of the location of corresponding points C, C_{ai} and C_{bj} (Figure 1) above the support surface in undeformed suspensions and tyres, φ - the angle of assembly of the train. In a state of static equilibrium, the applicates of these points are defined as in [9]; see Figure 2.

$$z^O_{ai} = l_{ai} - (f_{ai} + \lambda^O_{ai}) = z_o + a_i \times tg\psi_o,$$
$$z^o_{bi} = l_{bi} - (f_{bj} + \lambda^o_{bj}) = z_o - b_j \times tg\psi_o, \tag{1}$$

Based the condition of forces equilibrium (Figure 1), we find:

$$F^o_{ai} + m_{ai} \times q = F^o_{1iu}; \; F^O_{bj} + m_{bj} \times q = F^O_{2ju}, \tag{2}$$

where $F^o_{ai} = F^o_{ai}((f_{ai})$, $F^o_{bj} = F^o_{bj}(f_{bj})$, $F^o_{1iu} = F_{1iu}(\lambda^o_{ai})$, $F^o_{2ju} = F_{2ju}(\lambda^o_{bj})$ \hfill (3)

Since a piecewise linear approximation is assumed in the recorded functions, then:

$$F^o_{ai} = c_{1i} \times f_{ai}, \qquad F^o_{bj} = c_{2j} \times f_{bj}, \; F^o_{1uu} = c_{1iu} \times \lambda^o_{ai}, \qquad F^o_{2ju} = c_{2ju} \times \lambda f^o_{bj} \tag{4}$$
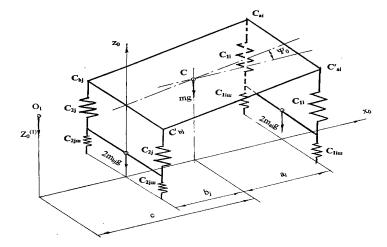

Fig. 2. On determining the state of the bus' static equilibrium

Then, based on (4) and (2), the expressions of the tyres' vertical deformations are deduced:

$$\lambda_{ai}^o = \frac{c_{1i} \times f_{ai} + m_{ai} \times g}{c_{1iu}} \qquad \lambda_{obj}^o = \frac{c_{2j} \times f_{bj} + m_{bj} \times g}{c_{2ju}} \qquad (5)$$

Based on (1) and (5), the formulae are developed thus:

$$f_{ai} = \frac{l_{ai} - \dfrac{m_{ai} \times g}{c_{1iu}} - z_o - a_i \times tg\psi_o}{1 + \dfrac{c_{1i}}{c_{1iu}}} \; ; \quad f_{bj} = \frac{l_{bj} - \dfrac{m_{bj} \times g}{c_{2ju}} - z_o + b_j \times tg\psi_o}{1 + \dfrac{c_{2j}}{c_{2ju}}} \qquad (6)$$

Substituting (5) with (4):

$$F_{ai}^o = q_{1i}(l_{ai} - \frac{m_{ai} \times g}{c_{1iu}} - z_o - a_i \times tg\psi_o)$$

$$F_{bj}^o = q_{2j}(l_{bj} - \frac{m_{bj} \times g}{c_{2ju}} - z_o - b_j \times tg\psi_o) , \qquad (7)$$

$$\text{where} \quad q_{1i} = \frac{c_{1i} \times c_{1iu}}{c_{1i} + c_{1iu}} \; ; \quad q_{2j} = \frac{c_{2j} \times c_{2ju}}{c_{2j} + c_{2ju}} \qquad (8)$$

The stiffness of two series-connected elastic elements, a suspension and tyres is given (Figure 2):

The equilibrium equation of the traction car body [9] (Figure 3) is:

$$\sum_K (\vec{F}_K)_{Zo} = 0; mom_{C\eta} \sum \vec{F}_K = 0$$

In an expanded form, it is presented as:

$$\sum_{i=1}^{n1}(F_{ai}^o + F_{ai}^{1o}) + \sum_{j=1}^{n2}(F_{bj}^o + F_{bj}^{1o}) - mg - Z_o^{(1)} = 0$$

$$\sum_{i=1}^{n1}(F_{ai}^o + F_{ai}^{1o})a_i - \sum_{j=1}^{n2}(F_{bj}^o + F_{bj}^{1o})b_j + Z_o c = 0, \qquad (9)$$

and

$$F_{ai}^o = F_{ai}^{1o} , \qquad F_{bj}^o = F_{bj}^{1o}$$

Substituting (7) with (9), the following formulae [9] are deduced:

$$z_o(\sum_{i=1}^{n1} q_{1i} + \sum_{j=1}^{n2} q_{2j}) + (\sum_{i=1}^{n1} a_i q_{1i} - \sum_{j=1}^{n2} b_j q_{2j})tg\psi_0 + \frac{Z_o^{(1)}}{2} =$$

$$= -\frac{mg}{2} + \sum_{i=1}^{n1} q_{1i}(l_{ai} - \frac{m_{ai}g}{c_{1iu}}) + \sum_{j=1}^{n2} q_{2j}(l_{bj} - \frac{m_{bj}g}{c_{2ju}});$$

$$z_o(\sum_{i=1}^{n1} a_i q_{1i} - \sum_{j=1}^{n2} b_j q_{2j}) + (\sum_{i=1}^{n1} a_i^2 q_{1i} + \sum_{j=1}^{n2} b_j^2 q_{2j}) tg\psi_0 - c\frac{Z_o^{(1)}}{2} =$$

$$= \sum_{i=1}^{n1} a_i q_{1i}(l_{ai} - \frac{m_{ai}g}{c_{1iu}}) - \sum_{j=1}^{n2} b_j q_{2j}(l_{bj} - \frac{m_{bj}g}{c_{2ju}})$$
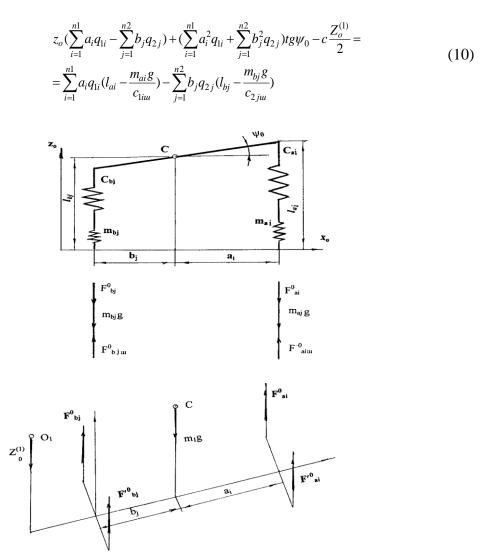
(10)

Fig. 3. Scheme of forces acting on the bus in the state of static equilibrium

The values Z_o, ψ_o and $Z_o^{(1)}$ are unknown in (10).

Let us derive the dynamic equations of a tractor. By analogy with (2), vertical reactions of the supporting surface (Figure 3) are found:

$$Z_{1i} = c_{1iu} \times \lambda_{ai} = F_{ai} + m_{ai}g,$$
$$Z_{2j} = c_{2ju} \times \lambda_{bj} = F_{bj} + m_{bj}g$$

(11)

where:

$$\lambda_{ai} = \frac{F_{ai} + m_{ai}g}{c_{1iu}}; \lambda_{bj} = \frac{F_{bj} + m_{bj}g}{c_{2ju}}$$

(12)

When the system deviates from the state of the static equilibrium, the restoring forces of the suspensions' elastic elements are equal to:

$$F_{ai} = c_{1i}(l_{ai} - z_{ai} - \lambda_{ai}) = c_{1i}(z^O_{ai} + f_{ai} - \lambda_{ai} + \lambda^O_{ai});$$

$$F_{bj} = c_{2j}(l_{bj} - z_{bj} - \lambda_{bj}) = c_{2j}(z^O_{bj} + f_{bj} - z_{bj} - \lambda_{bj} + \lambda^O_{bj}$$

(13)

After the exclusion of the vertical deformations of tyres $\lambda_{ai}, \lambda_{bj}$ (13) on the basis of (12), as well as the values l_{ai}, l_{bj} on the basis of (1), we obtain:

$$F_{ai} = c_{1i}f_{ai} + q_{1i}(z^o_{ai} - z_{ai}),$$

$$F_{bj} = c_{2j}f_{bj} + q_{2j}(z^o_{bj} - z_{bj})$$

(14)

Factoring in the known z_{ai}, z_{bj} or F_{ai}, F_{bj}, the vertical reactions on the wheels of the left side are written thus:

$$Z_{1i} = Z^o_{1i} + q_{1i}(z^o_{ai} - z_{ai}), (i = 1, 2, ...n)$$

$$Z_{2j} = Z^o_{2j} + q_{2j}(z^o_{bj} - z_{bj}), (j = 1, 2, ..n),$$

(15)

where:

$$Z^o_{1i} = c_{1i}f_{ai} + m_{ai}g, \qquad Z^o_{2j} = c_{2j}f_{bj} + m_{bj}g$$

(16)

By analogy with (15), the vertical reactions on the wheels of the right side are written as:

$$Z'_{1i} = Z^o_{1i} + q_{1i}(z^o_{ai} - z'_{ai}), (i = 1, 2, ...n)$$

$$Z'_{2j} = Z^o_{2j} + q_{2j}(z^o_{bj} - z'_{bj}), (j = 1, 2, ..n),$$

(17)

Vertical reactions of the road cloth are considered in three equations of the bus body (Figure 3):

$$m\ddot{z} = \sum_S (\vec{F}_S)_{Zo}; \qquad I\ddot{\psi} = mom_{C\eta}\sum_S \vec{F}_S; \qquad I_{Xo}\ddot{\gamma} = mom_{Cxo}\sum_S F_S,$$

where: I, I_{Xo} - moments of the tractor inertia about the transverse and longitudinal axes passing through the tractor's centre of mass (point C).

The following notations are entered:

$$\bar{x}_{1i} = x_{1i}\cos\theta_i + y_{1i}\sin\theta_i; \qquad \bar{y}_{1i} = -x_{1i}\sin\theta_i + y_{1i}\cos\theta_i;$$

$$\bar{x}'_{1i} = x'_{1i}\cos\theta'_i + y'_{1i}\sin\theta'_i; \qquad \bar{y}'_{1i} = -x'_{1i}\sin\theta'_i + y'_{1i}\cos\theta'_i.$$

Based on Figure 4, the following dynamic equations, which describe the vertical displacement of the tractor's centre of mass, rocking and roll, are developed [9]:

$$m\ddot{z} = -Z^{(1)} - mg + \sum_{i=1}^{n1}(F_{ai} + F'_{ai}) + \sum_{j=1}^{n2}(F_{bj} + F'_{bj});$$

$$I\ddot{\psi} = Z^{(1)}c - X^{(1)}(z_{o1} - z) - \sum_{j=1}^{n2}(F_{bj} + F_{bj}^{/})b_j - [\sum_{i=1}^{n1}(\bar{X}_{1i} + X_{1i}^{/}) + \sum_{j=1}^{n2}(X_{2j} + X_{2j}^{/})]z +$$

$$+\sum_{i=1}^{n1}[F_{ai}(a_i - \varepsilon\sin\theta_i) + F_{ai}^{/}(a_i + \varepsilon\sin\theta_i^{/})];$$

$$I_{Xo}\ddot{\gamma} = -Y^{(1)}(z_{o1} - z) + (H + \varepsilon)\sum_{j=1}^{n2}(F_{bj} - F_{bj}^{/}) + \sum_{i=1}^{n1}[F_{ai}(H + \varepsilon\cos\theta_i) - F_{ai}^{/}(H + \varepsilon\cos\theta_i^{/}) +$$

$$+[\sum_{i=1}^{n1}(\bar{Y}_{1i} + \bar{Y}_{1i}^{/}) + \sum_{j=1}^{n2}(Y_{2j} + Y_{2j}^{/})]z. \tag{18}$$
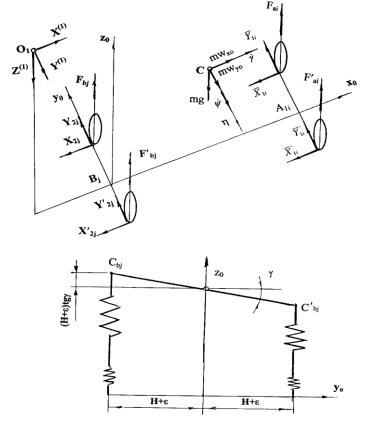

Fig. 4. Scheme of forces acting on the bus in the state of dynamic equilibrium

Taking into account expressions (6) and (7), the expressions for determining the reactions of suspensions of the bus body are presented [9]:

$$F_{ai} = F_{ai}^{o} + q_{1i}(z_{ai}^{o} - z_{ai}), \qquad F_{ai}^{/} = F_{ai}^{o} + q_{1i}(z_{ai}^{o} - z_{ai}^{/}),$$
$$F_{bj} = F_{bj}^{o} + q_{2j}(z_{bj}^{o} - z_{bj}), \qquad F_{bj}^{/} = F_{bj}^{o} + q_{2j}(z_{bj}^{o} - z_{bj}^{/}). \tag{19}$$

In the expression that determines the vertical reaction in the traction coupling unit, the static and dynamic components are defined:

$$Z^{(1)} = Z_o^{/} + \tilde{Z}_1 \tag{20}$$

Based on the static equations (6) and expressions (19) and (20), the dynamic equations (18) can be represented as [9,20]:

$$m\ddot{z} = -\tilde{z}^{(1)} + \sum_{i=1}^{n1} q_{1i}[2z_{ai}^o - (z_{ai} + z_{ai}^/)] + \sum_{j=1}^{n2} q_{2j}[2z_{bj}^o - (z_{bj} + z_{bj}^/)] \; ;$$

$$I\ddot{\psi} = c\tilde{Z}_1 + \sum_{i=1}^{n1} a_i q_{1i}[2z_{ai}^o - (z_{ai} + z_{ai}^/)] - \sum_{j=1}^{n2} b_j q_{2j}[2z_{bj}^o - (z_{bj} + z_{bj}^/)] -$$

$$-[\sum_{i=1}^{n1}(\bar{X}_{1i} + \bar{X}_{1i}^/) + \sum_{j=1}^{n2}(X_{2j} + X_{2j}^/)]z - X^{(1)}(z_{o1} - z) -$$

$$-\varepsilon \sum_{i=1}^{n1}\{F_{ai}^o(\sin\theta_i - \sin\theta_i^/) + q_{1i}[z_{ai}^o(\sin\theta_i - \sin\theta_i^/) - (z_{ai}\sin\theta_i - z_{ai}^/\sin\theta_i^/)]\};$$

$$I_{Xo}\ddot{\gamma} = -Y^{(1)}(z_{o1} - z) + (H + \varepsilon)\sum_{j=1}^{n2} q_{2j}(z_{bj}^/ - z_{bj}) + H\sum_{i=1}^{n1} q_{1i}(z_{ai}^/ - z_{ai}) +$$

$$+\varepsilon \sum_{i=1}^{n1}\{F_{ai}^o(\cos\theta_i - \cos\theta_i^/) + q_{1i}[z_{ai}^o(\cos\theta_i - \cos\theta_i^/) - (z_{ai}\cos\theta_i - z_{ai}^/\cos\theta_i^/)]\} + \qquad (21)$$

$$+[\sum_{i=1}^{n1}(\bar{Y}_{1i} + \bar{Y}_{1i}^/) + \sum_{j=1}^{n2}(Y_{2j} + Y_{2j}^/)]z.$$

By analogy with (1) and taking into account Figure 2, for the applicates of the point $C_{ai}, C_{ai}^/, C_{bj}, C_{bj}^/$, the following formulae are developed:

$$z_{ai} = z + a_i tg\psi + (H + \varepsilon\cos\theta_i)tg\gamma\cos\psi \, ,$$
$$z_{ai}^/ = z + a_i tg\psi - (H + \varepsilon\cos\theta_i^/)tg\gamma\cos\psi \, , \qquad (22)$$
$$z_{bj} = z - b_j tg\psi + (H + \varepsilon\cos\theta_i)tg\gamma\cos\psi \, ,$$
$$z_{bj}^/ = z - b_j tg\psi - (H + \varepsilon\cos\theta_i)tg\gamma\cos\psi \, .$$

According to (22):
$$z_{ai} + z_{ai}^/ = 2(z + a_i tg\psi) + \varepsilon(\cos\theta_i + \cos\theta_i^/)tg\gamma\cos\psi \; ;$$
$$z_{ai} - z_{ai}^/ = -[2H + \varepsilon(\cos\theta_i + \cos\theta_i^/)]tg\gamma\cos\psi \; ;$$
$$z_{bj} + z_{bj}^/ = 2(z - b_j tg\psi);$$
$$z_{bj}^/ - z_{bj} = -2(H + \varepsilon)tg\gamma\cos\psi \, . \qquad (23)$$

In accordance with the considered method and Figure 5, the equation of the trailer body motion is written as:
$$m_2\ddot{z}_{C2} = -Z^{(1)} - m_2 g + F_{as} + F_{as}^/ \; ;$$
$$I_2\ddot{\psi}_2 = Z^{(1)}d - (\bar{X}_s + X_s^/)z_{C2} + (F_{as} + F_{as}^/)a_s;$$
$$I_{X2}\ddot{\gamma}_2 = -Y^{(1)}(z_{o2} - z_{C2}) + (F_{as} - F_{as}^/)(H + \varepsilon) + [\bar{Y}_{as} + \bar{Y}_{as}^/]z_{C2}, \qquad (24)$$

where:
$$\bar{X}_s = X_s\cos\phi - Y_s\sin\phi; \qquad \bar{Y}_s = Y_s\sin\phi + Y_s\cos\phi.$$
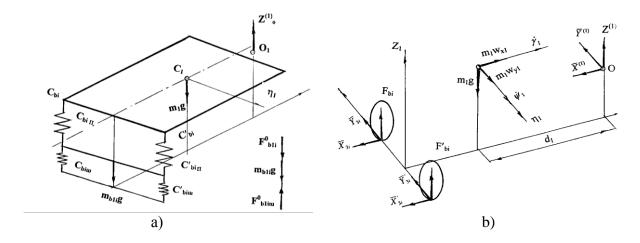
a)   b)

Fig. 5. Scheme of forces acting on the trailer in the state of the static (a) and dynamic equilibrium (b)

Equations (21) and (24) describe the movement of an auto-train in a vertical plane (longitudinal and transverse) and possess six unknowns: the generalizing coordinates z, ψ, γ, ψ_2, γ_2 and the dynamic component $\tilde{Z}^{(1)}$ of the vertical reaction in the traction coupling unit. After the exclusion of the latter, an equation for determining the generalizing coordinates of the auto-train is deduced. In this case, based on generalizing coordinates, the expressions for z_{C1}, z_{C2}, z_{o1}, z_{o2} (Figure 5) are written

$$h_{o1} = z_o - c\sin\psi + h\cos\psi ; \qquad z_{o1} = z - c\sin\psi + h\cos\psi ;$$
$$h_{o2} = h_{C1} - c_1 \sin\psi_2 - h_2 \cos\psi_2; \qquad z_{o2} = z_{C1} - c_1 \sin\psi_2 - h_2 \cos\psi_2; \qquad (25)$$
$$z_{C2} = z - c\sin\psi + h\cos\psi + h_3 \cos\psi_2 - d_2 \cos\psi_2 .$$

The value $\tilde{Z}^{(1)}$ is derived from the first equation (24). Considering (25):

$$\tilde{Z}^{(1)} = m_2 \ddot{z} - m_2 \ddot{\psi}(c \times \cos\psi + h\sin\psi) - m_2 \dot{\psi}_2^2 \times (h_3 \cos\psi_2 - d_2 \sin\psi_2). \qquad (26)$$

The final motion equations of unsprung masses of an auto-train are written as:
- by variable z

$$(m_1 + m_2)\ddot{z} + (m_1 + m_2)\ddot{\psi}(c \times \cos\psi + h\sin\psi) + m_2 \ddot{\psi}_2(d_2 \cos\psi_2 + h_3 \sin\psi_2) =$$
$$= (m_1 + m_2)\dot{\psi}_2^2(h\cos\psi - c\sin\psi) + m_2(h_3 \cos\psi_2 - d_2 \sin\psi_2)\dot{\psi}_2^2 + 2\sum_{i=1}^{n1} q_{1i}[z_{ai}^o - z -$$
$$-a_i tg\psi - \frac{\varepsilon}{2}(\cos\theta - \cos\theta^{/})\cos\psi tg\gamma] + 2\sum_{j=1}^{n2} q_{2j}(z_{bj}^o - z - b_j tg\psi) +$$
$$+2\sum_{\rho=1}^{n3} c_{b1\rho u}[h_{C1} - z_{C1} - b_{1\rho}(tg\psi_{1o} - tg\psi_1)] + 2\sum_{s=1}^{n4} q_{b2s}[z_{b2s}^o - z_{C2} - b_{2s} tg\psi_2 -$$
$$-\frac{\varepsilon_2}{2}\cos\psi_2 tg\gamma_2].$$

- by variable ψ

$$-(m_1+m_2)c\ddot{z}+[I+(m_1+m_2)c(c\times\cos\psi+h\sin\psi)]\ddot{\psi}+m_2c\ddot{\psi}_2(d_2\cos\psi_2+h_3\sin\psi_2)=$$

$$=-c(m_1+m_2)\dot{\psi}_2^2(h\cos\psi-c\sin\psi)-m_2c(h_3\cos\psi_2-d_2\sin\psi_2)\dot{\psi}_2^2+2\sum_{i=1}^{n1}a_iq_{1i}[z_{ai}^o-z-$$

$$-a_itg\gamma-\frac{\varepsilon}{2}\cos\psi tg\gamma]-2\sum_{j=1}^{n2}b_jq_{2j}(z_{bj}^o-z-b_jtg\psi)-2c\sum_{\rho=1}^{n3}c_{b1\rho uu}(h_{C1}-z_{C1})-$$

$$-2c\sum_{s=1}^{n4}q_{b2s}[z_{b2s}^o-z_{C2}-b_{2s}tg\psi_2-\frac{\varepsilon_2}{2}\cos\psi_2tg\gamma_2]-X^{(1)}(h\cos\psi-c\sin\psi)$$

$$-z[\sum_{i=1}^{n1}(X_{11}+X_{11}^/)+(X_2+X_2^/);$$

- by variable γ

$$I_{Xo}\ddot{\gamma}=-Y^{(1)}(h\cos\psi-c\sin\psi)-2H\sum_{i=1}^{n1}q_{1i}[H+\frac{\varepsilon}{2}(\cos\theta_i+\cos\theta_i^/)\cos\psi tg\gamma-$$

$$-2\sum_{j=1}^{n2}q_{2j}(H+\varepsilon)^2\cos\psi tg\gamma+z[(Y_{1i}+Y_{1i}^/)+(Y_{2j}+Y_{2j}^/)]$$

$$+\varepsilon\sum F_{ai}^o(\cos\theta-\cos\theta^/)+q_{1i}[z_{ai}^o(\cos\theta-\cos\theta^/)]+Z_{ai}^/\cos\theta^/-Z_{ai}\cos\theta;$$

- by variable ψ_2

$$-m_2d_2\ddot{z}+m_2d_2(c\times\cos\psi+h\sin\psi)]\ddot{\psi}+[I_2+m_2d_2(d_2\cos\psi_2+h_3\sin\psi_2)]\ddot{\psi}_2=$$

$$=-m_2d_2)\dot{\psi}_2^2(h\cos\psi-c\sin\psi)-m_2d_2(h_3\cos\psi_2-d_2\sin\psi_2)\dot{\psi}_2^2-$$

$$-(h_3\cos\psi_2-d_2\sin\psi_2)X^{(2)}-2(b_{2s}+d_2)q_{b2s}\times[z_{b2s}^o-z_{C2}-b_{2s}tg\psi_2-$$

$$-\frac{\varepsilon}{2}(\cos\theta_{2s}-\cos\theta_{2s}^/)\times\cos\psi_2tg\gamma_2];$$

- by variable γ_2

$$I_{X2}\ddot{\gamma}_2=z_{C2}(Y_s+Y_s^/)+(h_3\cos\psi_2-d_2\sin\psi_2)Y^{(2)}-2H_2\cos\psi_2tg\gamma_2\times q_{b2s}. \tag{27}$$

The equations (27) describing the motion of the HCB parts in the vertical plane (longitudinal and transverse) are applied in order to find the indexes of the HCB's transverse stability. In the case of steady motion, the problem is reduced to the analysis of finite equations (the values of the roll angles and the redistribution of loads along the sides are constant). For this purpose, the equation of dynamic equilibrium relative to the points K, K_2, K_4 is developed; see Figure 6.

The formulae are developed [9]:

$$mom_{Kx0}^F=-P_n(0,5H+\varepsilon-h\sin\gamma_o)+F(z_o+\varepsilon tg\gamma+h\cos\gamma)+(F_A^1+F_{B1}^1)H=0$$

$$mom_{K2x2}^F=-P_{n2}(0,5H_2+\varepsilon_2-h_2\sin\gamma_2)+F_2(z_{o2}+\varepsilon_2tg\gamma_2+h_2\cos\gamma_2+\sum_{j=1}^{3}F_{2j}^1H_2=0$$
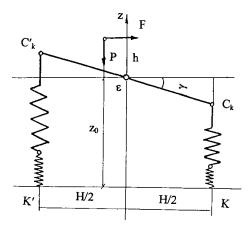
Fig. 6. On determining the forces acting on the bus (trailer) in a steady motion

In the expanded form:

$$-P_n(0,5H + \varepsilon - h\sin\gamma_o) + F(z_o tg\gamma + h\cos\gamma) + H\{q_A^1(l_A - m_A g / C_{u1} - z_o$$
$$+0,5Htg\gamma) + q_{B1}^1[l_{B1}^1 - m_{B1}g / (C_{u11} + C_{u12}) - z_o + 0,5H_1 tg\gamma]\} =$$
$$= [F\varepsilon - 0,5H^2(q_A^1 + q_{B1}^1)]tg\gamma + P_n h\sin\gamma - P_n(0,5H + \varepsilon) + F(z_o + h\cos\gamma) +$$
$$+H[(l - z_o)(q_A^1 + q_{B1}^1) - q_A^1 m_A g / C_{u1} - q_{B1}^1 m_{B1}g / (C_{u11} + C_{u12})] = 0$$
$$[F_2\varepsilon_2 - 0,5H_2^2(q_{B21}^1 + q_{B2}^1)]tg\gamma_2 + P_{n2}h_2\sin\gamma_2 - P_{n2}(0,5H_2 + \varepsilon_2) + F_2(z_{o2} + h_2\cos\gamma_2) +$$
$$+H_2[(l_2 - z_{o2})(q_{B21}^1 + q_{B2}^1) - q_{B21}^1 m_{B21}g / C_{u21} - q_{B2}^1 m_{B2}g / (C_{u22} + C_{u23})] = 0$$

(28)

If we assume that $tg\gamma_o \approx \gamma_o$, $tg\gamma_2 \approx \gamma_2$, $H_1 \approx H$, then the other equations of a system (17) and (19) can be applied to determine the static values of the roll angles of the train's parts:

$$\gamma_o = \frac{-P_n(0,5H + \varepsilon) + H[(l - z_o)(q_A + q_{B1}) - q_A m_A g / C_{u1} - q_{B1}m_{B1}g / (C_{u11} + C_{u12})]}{0,5H^2(q_A + q_{B1})} =$$
$$= \{l - z_o - <[0,5P_n + P_n\varepsilon / H + q_A m_A g / C_{u1} + q_{B1}m_{B1}g / (C_{u11} + C_{u12})] / (q_A + q_{B1}) >\} \times \frac{2}{H},$$

$$\gamma_o^{(2)} = \frac{-P_{n2}(0,5H_2 + \varepsilon_2) + H_2[l_2 - z_{o2}^{(2)})(q_{B21} + q_{B2}) - q_{B21}m_{B21}g / C_{u21} - q_{B2}m_{B2}g / (C_{u22} + C_{u23})]}{0,5H_2^2(q_{B21} + q_{B2})} =$$

(29)

$$\{l_2 - z_o^2 - <[0,5P_{n2}\varepsilon / H_2 + q_{B21}m_{B21}g / C_{u21} + q_{B2}m_{B2}g / (C_{u22} + C_{u23})] / (q_{B21} + q_{B22}) >\}\frac{2}{H_2}$$

In order to define the dynamic values of the roll angles γ, γ_2, the difference between the corresponding equations (29) and the last system equations (17) should be considered. Then,

$$mom_{KX0}^F - mom_{Kxo} = F\varepsilon tg\gamma - 0,5H^2(q_A + q_{B1})(tg\gamma - tg\gamma_o) + F(z_o + h\cos\gamma) +$$
$$+Ph(\sin\gamma - \sin\gamma_o) = \gamma[F\varepsilon - 0,5H^2(q_A + q_{B1})] + 0,5H^2\gamma_o(q_A + q_{B1}) + F(z_o + h) = 0,$$

$$mom_{K2X2}^{F} - mom_{K2z2} = F_2\varepsilon_2 tg\gamma_2 - 0,5H_2^2(q_{B21} + q_{B2})(tg\gamma_2 - tg\gamma_o^{(2)}) + F_2(z_o^{(2)} +$$
$$+ h_2\cos\gamma_2) + P_2h_2(\sin\gamma_2 - \sin\gamma_o^{(2)}) = \gamma_2[F_2\varepsilon_2 - 0,5H_2^2(q_{B21} + q_{B2})] +$$
$$+ 0,5H_2^2\gamma_o^{(2)}(q_{B21} + q_{B2}) + F_2(z_o^{(2)} + h_2) = 0,$$

Thus:

$$\gamma = \frac{\gamma_o 0,5H^2(q_A + q_{B1}) + F(Z_o + h)}{-F\varepsilon + 0,5H^2(q_A + q_{B1})};$$

$$\gamma_2 = \frac{\gamma_o^{(2)}0,5H_2^2(q_{B21} + q_{B2}) + F_2(Z_o^{(2)} + h_2)}{-F_2\varepsilon_2 + 0,5H_2^2(q_{B21} + q_{B2})};$$

(30)

The dynamic components of the vertical reactions in the supports, which are defined by the roll angles γ and γ_o, γ_2 and $\gamma_o^{(2)}$ (loading and unloading), are determined:

- for the left side

$$\Delta G_1 = q(\gamma-\gamma_o)H/2; \quad \Delta G_{11} = q_{11}(\gamma-\gamma_o)H_1/2; \quad \Delta G_2 = q_2(\gamma_2-\gamma_o^{(2)})H_2/2;$$

- for the right side

$$\Delta G^1{}_1 = q^1(\gamma-\gamma_o)H/2; \quad \Delta G^1{}_{1i} = q^1{}_{11}(\gamma-\gamma_o)H_1/2; \quad \Delta G^1{}_2 = q^1{}_2(\gamma_2-\gamma_o^{(2)})H_2/2.$$

Consequently, the dynamic loads, with consideration of the side redistribution, can be written as:

$$G_1 = G_1^o - \Delta G_1; \quad G_{11} = G_{11}^o - \Delta G_{11}; \quad G_2 = G_2^o - \Delta G_2;$$
$$G_1^1 = G_1^{o1} - \Delta G_1^1; \quad G_{11}^1 = G_{11}^{o1} - \Delta G_{11}^1; \quad G_2^1 = G_2^{o1} - \Delta G_2^1$$

(31)

Equations (31) are the basis for calculating the values of loading and unloading of the wheels of the bus and the trailer.

The determination of the stability indexes of buses, including the articulated ones, is based on some assumptions: the bus is fully loaded; the mobility of passengers is absent; and the entire unsprung mass acts as a solid body [8].

Output data for calculating the stability of an articulated bus (Figure 7) are accepted: the total weight of the auto-train - 25,600 kg; the load on the support-coupling device of the tractor - 1,990 kg; the total stiffness of the suspension of the bus front wheels - 640 kN/m, rear wheels - 950 kN/m, trailer wheels - 840 kN/m, tyre stiffness - 1,250 kN/m; geometric parameters of the auto-train: length - 17,500 mm, width - 2,460 mm, height - 3,585 mm; geometric parameters of the bus: length - 10,000 mm, width - 2,460 mm, height - 3,585 mm; geometric parameters of the connected part: length - 7,500 mm, width - 2,460 mm; the wheel track of the tractor - 1,850 mm, the trailer - 1,850 mm; spring wheels of a tractor: front wheels - 750 mm, rear wheels - 1,250 mm; the connected part - 1,200 mm; the suspension stiffness of the bus front wheels C_{p1}=320 kN/m, rear wheels - 480 kN/m, trailer wheels - 430 kN/m; the type and size of tyres of the car-tractor and semi-trailer: 12.00 - R20, the static radius of the wheel - 0.525 m, the stiffness - 1,250 kN/m [10].
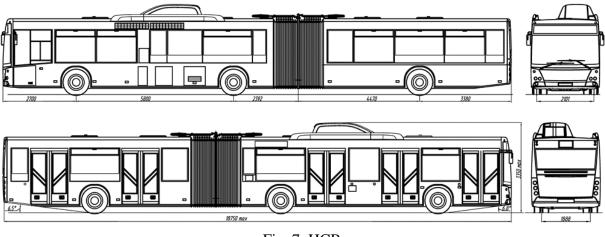
Fig. 7. HCB

In Figure 8, the results from calculating the roll of the bus body and the trailer during the circle motion are shown. In Figure 9, the values of loading and unloading of the bus wheels while performing the same manoeuvre are presented.

Based on the above-mentioned dependences, during the circle motion, the HCB's body roll and the side loading exceed the indexes of the trailer coupling part. This is explained by the lower position of the trailer's centre of mass ($h_g = 1.21$ m) compared with the bus ($h_g = 1.53$ m) [10].

The limiting angle of the lateral roll of rollover γ_{Max} characterizes the HCB's stability when moving along a slope. According to the calculations, this angle is:
- for the bus $\gamma_{aMax} = 27.56^0$.
- for the trailer $\gamma_{aMax} = 30.21^0$.

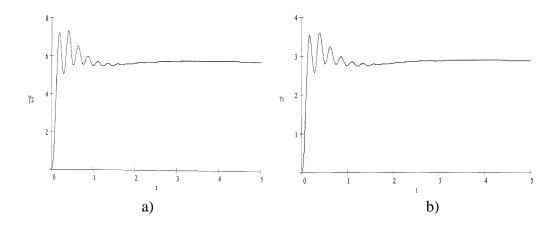a)   b)

Fig. 8. Change in the roll angle of the bus (a) and trailer (b) during the circle motion of the auto-train: R=25 m; v=10 m/s

These angles considerably exceed the possible slopes on highways, that is, the transverse stability of the HCB is greatly ensured, especially for the coupling section, due to the low position of the centre of mass.

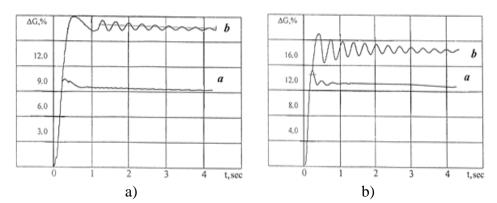a)   b)

Fig. 9. Loading the wheels of the bus outer side during the circle motion of the auto-train, while factoring in the roll (a) and the effects of longitudinal and lateral reactions (b): R=25 m; v=10 m/s

4. CONCLUSIONS

In order to ensure safe traffic conditions, which are regulated by international and national rules and standards, the dynamic parameters, in particular, the stability and handling of the vehicles, are studied.

The parameters of the transverse stability of hybrid buses with active trailers are analysed in accordance with the objective of the study. In this paper, the movement of the parts of HCBs in the vertical plane along the angles of rocking (pitch, trim) and the roll are assumed to affect the lateral movement mainly by changing the vertical loads on the wheels. The vertical reactions of the bearing surface are changed as well. The limiting angle of the side roll of the bus rollover is found to be $\gamma_{a\max} = 27.56°\gamma_{\max}$, and the trailer rollover to be $\gamma_{a\max} = 30.21°$. Therefore, the transverse stability of HCBs with a hybrid power plant is ensured, since, according to the standard DSTU UN/ECE R 111-00, this angle should not exceed 25°.

In addition to technical factors, when talking about safety, it is important to remember about aspects related to bus passengers and their behavior [21-23].

References

1.   ДСТУ UN/ECE R 36-03:2002. 2002. Єдині технічні приписи щодо офіційного затвердження пасажирських транспортних засобів великої місткості стосовно загальної конструкції (Правила ЄЕК ООН №36-03: 1993, IDT). [In Ukrainian: GOST UN/ECE R 36-03:2002. 2002. Uniform Technical Prescriptions Concerning the Approval of Passenger Vehicles of Large Capacity in Relation to the Overall Design (UN/ECE Regulations №36-03: 1993, IDT).] Kiev: State Committee of Ukraine for Technical Regulation and Consumer Policy.
2.   ДСТУ UN/ECE R 111-00: 2002. 2002. Єдині технічні приписи щодо офіційного затвердження автоцистерн категорій N і O стосовно їх стійкості проти перекидання. [In Ukrainian: DSTU UN/ECE R 111-00: 2002. 2002. Uniform Technical Prescriptions Concerning the Approval of Tank Vehicles Categories N and O with Respect to their Resistance to Rollover.] Kiev: State Committee of Ukraine for Technical Regulation and Consumer Policy.

3. ДСТУ 3310-96. 1996. Засоби транспортні дорожні. Стійкість. Методи визначення основних параметрів випробуваннями. [In Ukrainian: GOST 3310-96. 1996. Transport Vehicles. Stability. Methods of Identifying Key Parameter Tests.] Kiev: State Standard of Ukraine.

4. Vansauskas V., M. Bogdevičius. 2009. "Investigation into the Stability of Driving an Automobile on the Road Pavement with Ruts". *Transport* 24(2): 170-179.

5. Matolcsy M. 2007. "The Severity of Bus Rollover Accidents. *ESV Paper* 989.

6. European Commission Council Regulation (EC). 1998. EC No 2135/98 of 24 September 1998 Amending Regulation (EEC) No 3821/85 on Recording Equipment in Road Transport and Directive 88/599/EEC Concerning the Application of Regulations (EEC) No 3820/84 and (EEC) No 3821/85 Official Journal L 274, 09/10/1998: 1-21.

7. Matolcsy M. 2001. "Standard Accident as the Basis of Vehicle Approval Test". *International EAEC Congress, Bratislava, Section BO1, Safety*: 9.

8. ECE Regulation 66 Test of Motor Coach Roof Strength 1991. 2007. *Prevost Lemirage Motor Coach/Report Number: 66-MGA-2007-002 R*: 237.

9. EOBUS. 2008. "Scania Higer A80 Passed European Rollover Test". Available at: http://www.eobus.com/news/105.htm.

10. Irizar PB. 2017. "Tour Bus". Available at: http://www.irizar.com.

11. Pacejka H. 2005. *Tyre and Vehicle Dynamics.* Elsevier.

12. Sachs H., M. Singh. 1977. "Automobile Stability - A Study of the Domain of Attraction". *Vehicle System Dynamics* 6(2-2): 169-177.

13. Антонов Д. А. 1984. *Расчет устойчивости движения многоосных автомобилей*. [In Russian: Antonov D. 1984. *Calculation of Stability of Motion of Multiaxle Cars*.] Moscow: Mashinostroenie.

14. Rice R.S., W.F. Milliken. 1980. "Static Stability and Control of the Automobile Utilizing the Moment Method". *SAE Technical Paper No. 800847*.

15. Segel L. 1966. "On the Lateral Stability and Control of the Automobile as Influenced by the Dynamics of the Steering System". *Journal of Engineering for Industry* 88(3): 283-294.

16. Segel L. 1957. "Research in the Fundamentals of Automobile Control and Stability". *SAE Technical Paper No. 570044.*

17. Sakashita K., T. Okada. 1964. "Effects of Dynamical Features of Steering System on Automobile Stability and Control, Predicted by Linearized Theory". *Journal of Society of Automotive Engineers of Japan* 18(4): 268-273.

18. Фаробин Я.Е., В.С. Шупляков. 1983. *Оценка эксплуатационных качеств автопоездов для международных перевозок*. [In Russian: Farobyn Ya., V. Shuplyakov. 1983. *Evaluation of Operation Qualities of Car Trains for International Transit*.] Moscow: Transport.

19. Vuchic V.R. 2002. "Bus Semirapid Transit Mode Development and Evaluation". *Journal of Public Transportation* 5(2): 71-85.

20. Moulin L.S., A.A. Da Silva, M.A. El-Sharkawi, R.J. Marks. 2004. "Support Vector Machines for Transient Stability Analysis of Large-scale Power Systems". *IEEE Transactions on Power Systems* 19(2): 818-825.

21. Czech P. 2017. "Physically disabled pedestrians - road users in terms of road accidents". In: E. Macioszek, G. Sierpiński, eds., Contemporary Challenges of Transport Systems and Traffic Engineering. *Lecture Notes in Network Systems* 2: 157-165. Cham, Switzerland: Springer.

22. Czech P. 2017. "Underage pedestrian road users in terms of road accidents". In: G. Sierpiński, ed., Intelligent Transport Systems and Travel Behaviour. *Advances in Intelligent Systems and Computing* 505: 75-85. Cham Switzerland: Springer.
23. Sokolovskij E., O. Prentkovskis. 2013. "Investigating traffic accidents: the interaction between a motor vehicle and a pedestrian". *Transport* 28(3): 302-312.
24. Dyakov I., O. Prentkovskis. 2008. "Optimization problems in designing automobiles". *Transport* 23(4): 316-322.
25. Schmidt M., S. Voss. 2017. "Advanced systems in public transport". *Public Transport* 9(1-2): 3-6.
26. Stopka O., B. Sarkan, M. Chovancova, L.M. Kapustina. 2017. "Determination of the appropriate vehicle operating in particulat urban traffic conditions". *Komunikacie* 2: 18-22.
27. Andoga R., F. Adamcik jr., J. Hrabowsky, T. Vaispacher. 2016. "A Hybrid Diagnostic System for a Small Turbojet Engine". *Nase More* 63(3): 86-92.
28. Mbara T.C. 2016. "Tuk-tuk, 'new kid on the block' in Johannesburg: Operational and user travel characteristics, competition and impacts". *Journal of Transport and Supply Chain Management* 10(1)(a214): 1-9. DOI: https://doi.org/10.4102/jtscm.v10i1.214.