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APPLICATION OF THE DGPS METHOD FOR THE PRECISE POSITIONING OF AN AIRCRAFT IN AIR TRANSPORT

Summary. This article presents research results concerning the determination of the position of a Cessna 172 aircraft by means of the DGPS positioning method. The position of the aircraft was recovered on the basis of P1/P2 code observations in the GPS navigation system. The coordinates of the aircraft were designated due to the application of the Kalman forward-filtering method. The numerical calculations were conducted using RTKLIB software in the RTKPOST module. In the scientific experiment, the authors used research materials from the test flight conducted by a Cessna 172 aircraft in the area of Dęblin in the Lublin Voivodeship in south-eastern Poland. The research experiment exploited navigation data and GPS observation data recorded by the geodetic Topcon Hiper Pro receiver mounted in the cockpit of the Cessna 172 and installed on the REF1 reference station. The typical accuracy for recovering the position of the Cessna 172 with the DGPS method exceeds in the region of 2 m. In addition, the authors specify the parameters of availability, integrity and continuity of GNSS satellite positioning in air navigation. The obtained findings of the scientific experiment were compared with the International Civil Aviation Organization's (ICAO's) technical standards.

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1. INTRODUCTION

The DGPS positioning technique refers to a differential measurement approach, which can be executed both in near real time and in postprocessing. The DGPS measurement method requires a rover receiver and a base reference station from the user. In near real time, the coordinates of the rover receiver's antenna are determined on the basis of differential corrections sent via the NTRIP protocol in the RTCM format from the service of the reference station's network [12]. In the case of calculations during postprocessing, the coordinates of a rover receiver are determined on the basis of registered raw GNSS satellite observations by the rover receiver and the base reference station. In the DGPS measurement technique, mostly single-frequency (or dual-frequency) code observations are used from one or more GNSS navigation systems [3].

In practice, the DGPS positioning technique allows for reducing or eliminating a number of systematic errors in GNSS satellite measurements. Systematic errors that are related to the satellite clock and the receiver are completely eliminated in the DGPS method. In this way, it is possible to remove the satellite clock error correction, the receiver clock error correction, the relativistic satellite clock correction, TGD hardware delay for the satellite and the RDCB instrumental bias for the receiver. On the other hand, the impact of the ionosphere correction and the troposphere correction is reduced at the differentiation stage for the observation equations of the mathematical model [2]. It is worth mentioning that, in the DGPS measurements, it is crucial to determine the characteristics of the antenna of the rover receiver and the base reference station.

The DGPS measurement method is used for positioning in static and kinematic modes. In the kinematic mode, the method of DGPS positioning provides, for example, the designation of the precise position of the aircraft in air navigation [6]. The recovery of a reliable aircraft position affects the improvement in the safety of air operations in airspace. In addition, the technique of DGPS positioning is important in the development of aircraft approach procedures for landing with the use of the GNSS system in air transport [7].

The aim of this investigation is to recover the possibility of aircraft coordinates using the DGPS positioning method in air navigation. In the test research, we recovered the position of a Cessna 172 aircraft by executing a test flight around the airfield in Dęblin. The position of the aircraft was recovered using RTKLIB software in the RTPOST module. Satellite data were used for the numeric calculations, which were obtained from a Topcon Hiper Pro received mounted on board the Cessna 172 and installed as a physical reference station at the military airfield in Dęblin. The calculations were made in the postprocessing mode for the GPS code observations.

2. RESEARCH METHODOLOGY

The basic observation equations in the DGPS positioning method rely on the use of the operation of double difference of GPS code observations, as follows [4,8]:

$$\begin{cases} \nabla\Delta P_{AB,1}^{ij} = \rho_{AB}^{ij} + \nabla\Delta I_{AB,1}^{ij} + \nabla\Delta T_{AB}^{ij} + \nabla\Delta M_{P1} \\ \nabla\Delta P_{AB,2}^{ij} = \rho_{AB}^{ij} + \nabla\Delta I_{AB,2}^{ij} + \nabla\Delta T_{AB}^{ij} + \nabla\Delta M_{P2} \end{cases} \tag{1}$$

where:

∇ is the operator of the double difference for code measurements, which allows for comparing the difference in code measurements for two satellites tracked by two receivers

Δ is the operator of a single difference for code measurements, which allows for determining the difference in code measurements for two satellites tracked by one receiver

$\vec{AB}$ is the vector in the space between the base station ($\vec{A}$) and the rover receiver ($\vec{B}$) mounted on board the aircraft

$P_{AB,1}^{ij}$ is the value of the double code difference (expressed in metres) on the vector $\vec{AB}$ between the satellites i and j on the L1 frequency in the GPS system

$P_{AB,2}^{ij}$ is the value of the double code difference (expressed in metres) on the vector $\vec{AB}$ between the satellites i and j on the L2 frequency in the GPS system

ρ_{AB}^{ij} is the geometric distance of the vector $\vec{AB}$ for the double code difference (expressed in geocentric coordinates XYZ)

$I_{AB,1}^{ij}$ is the value of the ionosphere delay on the L1 frequency for the double code difference

$I_{AB,2}^{ij}$ is value of the ionosphere delay on the L2 frequency for the double code difference

$I_{AB,2}^{ij} = \gamma \cdot I_{AB,1}^{ij}$ is the relationship of the ionosphere delay on the L1 and L2 frequency

$\gamma = \left(\frac{f_1}{f_2}\right)^2$ is the scaling coefficient

f_1 is the L1 frequency in the GPS system

f_2 is the L2 frequency in the GPS system

T_{AB}^{ij} is the value of the troposphere delay for the double code difference

M_{P1} is the multipath effect and noise measurement at the L1 frequency for the code measurements

M_{P2} is the multipath effect and noise measurement at L2 frequency for the code measurements

The observation equations (1) were recorded for the code observations P1/P2 for the carrier frequencies L1/L2 in the GPS navigation system. In Equation (1), the unknown parameters are the coordinates of the aircraft involved in the geometrical distance factor. The parameters of the ionosphere and troposphere delays are expressed by deterministic models. The values of the multipath effect are expressed on the basis of empirical models. The observation model from Equation (1) is usually solved in two stages, using Kalman filtering; see below [1]:

a) Process of “prediction”:

$$\begin{cases} x_p = A_0 \cdot x_0 \\ P_p = A_0 \cdot P_0 \cdot A_0^T + Q_o \end{cases} \tag{2}$$

where:

A_0 is the matrix of coefficients
x_0 is the estimation of the values of the designated parameters a priori from the previous step
P_0 is the estimation of the values of covariance a priori from the previous step
x_p is the prediction of the state value
P_p refers to the predicted covariance values
Q_0 is the variance matrix of the noise of the measurement process

b) Process of "correction":

$$\begin{cases} K_k = P_p \cdot H^T \cdot \left(H \cdot P_p \cdot H^T + R\right)^{-1} \\ x_k = x_p + K_k \cdot \left(z - H \cdot x_p\right) \\ P_k = \left(I - K_k \cdot H\right) \cdot P_p \end{cases} \tag{3}$$

where:
R is the covariance matrix of measurements
H is the matrix of partial derivatives
K_k is the Kalman gain matrix
z is the vector of measured values
I is the unit matrix
x_k refers to the parameters determined a posteriori
P_k is the covariance matrix of parameters determined a posteriori

The Kalman filtering process is performed sequentially for all measured epochs registered by the GNSS receiver mounted on board the aircraft. Additionally, in the stochastic process of developing the GPS observations, the accuracy of positioning the aircraft is also determined. It should be emphasized that the designated coordinates of the aircraft and their accuracies are expressed in the geocentric coordinates XYZ.

3. RESEARCH EXPERIMENT

The verification of applying the DGPS technique in air navigation was carried out in an air experiment using a Cessna 172 aircraft. The air experiment was conducted on a military airfield in Dęblin and in the surrounding area. The test flight on the Cessna 172 was made in the morning, from 09:39:03 to 10:35:03, according to the time of the GPS navigation.

Figure 1 shows the trajectory of the Cessna 172 on the horizontal plane. The coordinates of the aircraft were expressed using BLh ellipsoidal coordinates (B: latitude, L: longitude, h: ellipsoidal height). In order to transform the coordinates of the aircraft from the XYZ geocentric coordinate frame into the BLh ellipsoidal frame, the Helmert transformation was used [15, 18]. Figure 1 shows the location of the reference station REF1, which was used to recover the precise trajectory of the flight of the Cessna 172 in postprocessing. The base station REF1 was mounted and installed on the roof of the Air Force Academy in Dęblin. The basis of the technical infrastructure of the reference station REF1 was the dual-frequency Topcon Hiper Pro receiver, which recorded code, phase and Doppler observations from the navigations systems (GPS and GLONASS). The frequency of registering GNSS satellite observations on the memory card of the receiver equalled 1 s. The target reference coordinates of the REF1 base station in the BLh ellipsoidal frame were as follows:

– Latitude: 51° 33’ 19.92606” N
– Longitude: 21° 52’ 08.72275” E
– Ellipsoidal height: 152.069 m

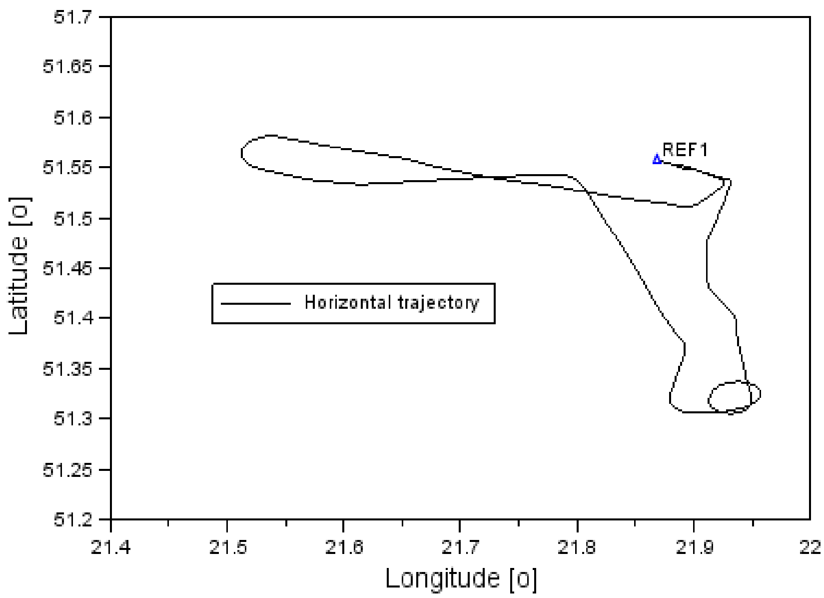

Fig. 1. The horizontal trajectory of the flight of the Cessna 172

During the flight test, the geodetic receiver Hiper Pro was installed on board the Cessna 172 (see Figure 2). The aim of the Topcon Hiper Pro rover receiver was to collect raw GNSS observations in order to recover the coordinates of the aircraft in postprocessing. The frequency of data registration in the rover receiver was also equal to 1 s. Furthermore, the SAMSET system, which monitored the position of the aircraft in near real time, was installed on board the Cessna 172.

Fig. 2. The GNSS receiver in the pilot’s cabin of the Cessna 172

The simultaneous synchronization of GNSS observations from the Topcon Hiper Pro rover receiver and the receiver of the REF1reference station allowed for the designation of the Cessna 172's position, as well as the determination of the positioning accuracy. The coordinates of the aircraft were designated on the basis of a single baseline (spatial vector $\vec{AB}$), i.e., baseline (vector) REF1-Cessna 172. In order to recover the aircraft coordinate system, the authors used P1/P2 code observations for the DGPS positioning method (see Chapter 1). In order to determine the accurate coordinates of the aircraft and its precision, we used RTKLIB software. For this purpose, we used the computational module "DGPS/DGNSS", stored in the RTKPOST library. For the sake of performing the calculations, the "DGPS/DGNSS" module in RTKPOST library was configured as below [17]:

- GNSS system: GPS system
- GNSS observations: code observations P1/P2 in the GPS system
- Construction of the observation equations: double difference for code observations in the GPS system
- Data source of the ephemeris GPS satellites: GPS navigation data message
- Source of the GPS observation: RINEX 2.11 file
- Method for determining the coordinates of the GPS satellites: based on the parameters of the Kepler orbit
- Correction of the pseudorange from the satellite to the receiver antenna: applied
- Effect of the earth's rotation: applied
- Sagnac effect: applied
- Correction of the satellite clock: eliminated
- Relativistic effects: eliminated
- TGD hardware delay: eliminated
- Receiver hardware delay: eliminated
- Troposphere model: Saastamoinen
- Ionosphere correction model: Klobuchar
- Source of ionosphere correction: GPS navigation data message
- Receiver antenna phase centre: based on the ANTEX IGS08 file
- Elevation angle: 10°
- Observation weighting: applied
- A priori standard deviation of code observations: 1 m
- Initial values of aircraft coordinates: based on the RINEX file header
- Frame of coordinates: geocentric XYZ and ellipsoidal BLh (ultimately ETRF '89)
- Method of calculations: Kalman forward-filtering
- Positioning method: DGPS/DGNSS
- Positioning mode: kinematic
- Computational mode: postprocessing
- Interval of calculations: 1 s
- Blunder error detection in GPS measurements: RAIM module algorithm
- Number of iterations in the measurement epoch: five
- Maximum value of the DOP coefficient: 30
- Final recording of coordinates: coordinates in the XYZ geocentric frame and the BLh ellipsoidal frame
- Correction of the receiver clock: eliminated

- Geodynamic and tidal effects: applied
- Rover receiver: Topcon Hiper Pro mounted in a Cessna 172 aircraft
- Base receiver: Topcon Hiper Pro fixed at the REF1 reference station

4. RESEARCH RESULTS

The examination of the use of GNSS satellite technology in air navigation is focused on determining four basic positioning parameters: availability, accuracy, continuity and integrity. The availability parameter is determined based on the visibility of the GNSS constellation during the measurement session. In addition, when tracking the GNSS satellite constellation, no break must appear in the satellite positioning due to the lack of navigation data and observation data. Therefore, monitoring the available satellites of a given constellation of the GNSS system (e.g., the GPS system) is of crucial importance. In accordance with Annex 10 to the Convention on International Civil Aviation, entitled "Radio Communication", Volume I "Radio Navigation Aids", a typical parameter value of the availability of the GPS system is 0.99 (99%) [9]. This means, de facto, that, during the executed air test, the continuity of tracking a GPS constellation equals at least 0.99 of the duration of the whole flight. Thus, the lack of data or GPS system failure may occur only in the case of 1% of the duration of the flight test. Figure 3 shows the number of available GPS satellites during the executed test flight in Dęblin on 1 June 2010.

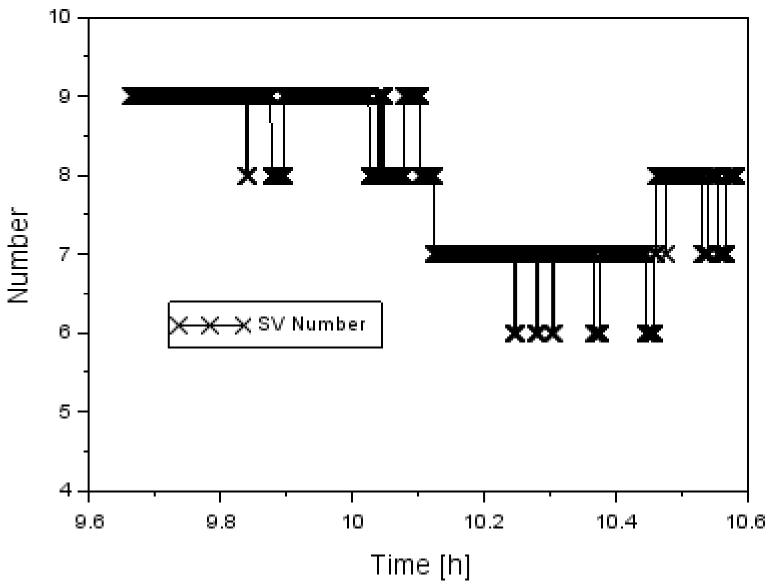

Fig. 3. Number of satellites in the GPS constellation

Based on Figure 3, it can be concluded that the number of available GPS satellites during the test flight ranged from five to nine. Therefore, when executing the test flight, the tracking of the GPS constellation was still available. Likewise, navigation data were not missing. Therefore, the availability parameter of the constellation of GPS satellites was above 0.99 (99%), which complies with the ICAO requirements. It should be added that the number of available GPS satellites in Figure 3 de facto expresses the total number of GPS satellites tracked jointly by the rover receiver mounted on board the Cessna 172 and the REF1 reference station.

An important parameter in determining the quality of satellite positioning is the accuracy of the set position. The accuracy parameter in the GNSS measurements is represented by the values of the standard deviation for the designated coordinates of the aircraft. In this case, the accuracy of the set position of the Cessna 172 aircraft can be referred to geocentric XYZ coordinates, as below:

$$mX = \sqrt{P_k(1,1)}; \quad mY = \sqrt{P_k(2,2)}; \quad mZ = \sqrt{P_k(3,3)} \tag{4}$$

where:

mX is the accuracy of the aircraft position along the X-axis
mY is the accuracy of the aircraft position along the Y-axis
mZ is the accuracy of the aircraft position along the Z-axis

or adequately expressed in the coordinates of the ellipsoidal BLh, as follows [13]:

$$mB = \sqrt{\mathrm{m}_{\mathrm{BLh}}(1,1)}; \quad mL = \sqrt{\mathrm{m}_{\mathrm{BLh}}(2,2)}; \quad mh = \sqrt{\mathrm{m}_{\mathrm{BLh}}(3,3)} \tag{5}$$

where:

$\mathrm{m}_{\mathrm{BLh}}$ is the covariance matrix in the geodetic frame (BLh), $\mathrm{m}_{\mathrm{BLh}} = \mathrm{R} \cdot P_{\mathrm{k}} \cdot \mathrm{R}^{\mathrm{T}}$
R is the transition matrix from the geocentric (XYZ) to the geodetic frame (BLh)
mB is the standard deviation in latitude
mL is the standard deviation in longitude
mh is the standard deviation in ellipsoidal height

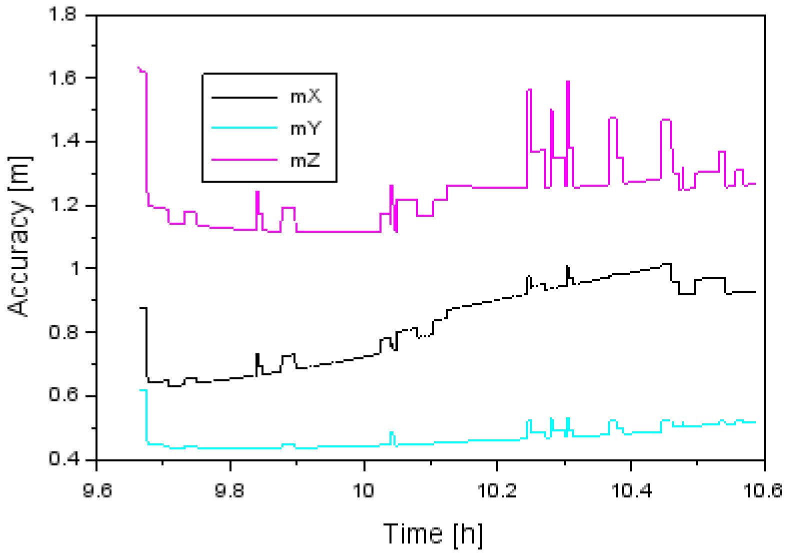

Fig. 4. The accuracy of the Cessna 172 aircraft in the XYZ geocentric frame

Figure 4 shows the values of positioning accuracy of the Cessna 172 in the XYZ geocentric frame; see Equation (4). The average positioning accuracy along the X-axis is 0.826 m and the median value is equal to 0.874 m. Furthermore, the dispersion of the obtained results of the parameter *mX* ranges from 0.630 m to 1.018 m. It should be noted that, for approximately 96% of the results, the parameter *mX* is less than 1 m. The average positioning accuracy along the Y-axis is 0.466 m and the median value is equal to 0.458 m. Moreover, the dispersion of the obtained results of the parameter *mY* ranges from 0.436 m to 0.619 m. It is worth stressing that, for 100% of the results, the *mY* parameter is less than 0.7 m. In addition, for approximately 82% of the results, the *mY* parameter is included in the range from 0 m to 0.5 m. The average positioning accuracy along the Z-axis equals 1.229 m and the median value is equal to 1.251 m. Furthermore, the dispersion of the obtained results of the *mZ* term is included in the range from 1.113 m to 1.634 m. For approximately 82% of the results, the *mZ* term is in the range from 0 m to 1.3 m. On the other hand, for about 100% of the results, the *mZ* parameter is included in the range from 0 m to 1.7 m.

Figure 5 shows the values of positioning accuracy in the BLh ellipsoidal frame; see Equation (5). The average positioning accuracy of geodetic latitude B is equal to 0.905 m and the median value equals 0.917 m. Furthermore, the dispersion of the obtained results of the parameter *mB* ranges from 0.768 m to 1.119 m. It should be noted that, for approximately 77% of the results, the parameter *mB* is less than 1 m. The average positioning accuracy of geodetic longitude L is 0.553 m and the median value is equal to 0.544 m. Moreover, the dispersion of the obtained results of the parameter *mL* ranges from 0.483 m to 0.681 m. It is worth stressing that, for approximately 72% of the results, the *mL* parameter is less than 0.6 m. Furthermore, for 100% of the results, the parameter *mL* is the range from 0 m to 0.7 m. The average positioning accuracy of ellipsoidal height h equals 1.135 m, whereas the median value is approximately equal to 1.118 m. Besides, the dispersion of the obtained results of the parameter *mh* ranges from 1.027 m to 1.546 m. For approximately 92% of the results, the parameter *mh* is included in the range from 0 m to 1.3 m. On the other hand, about 100% of the results of the parameter *mh* is included in the range from 0 m to 1.6 m.

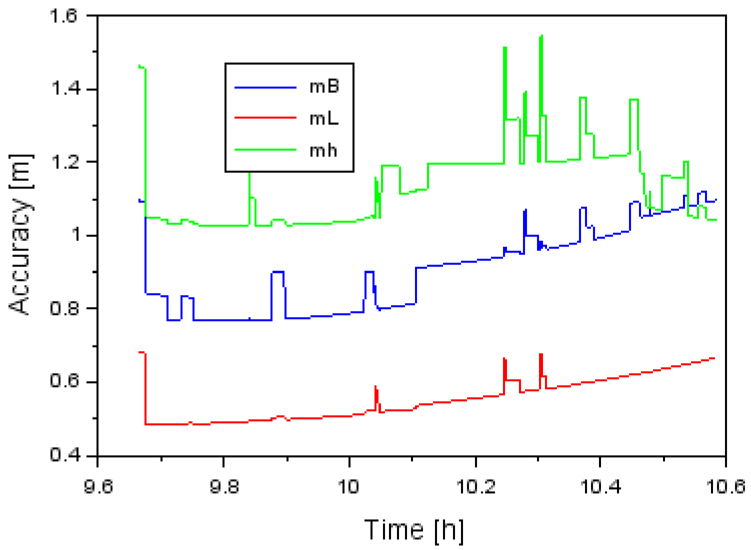

Fig. 5. The accuracy of the Cessna 172 aircraft in the BLh geodetic frame

Annex 10 to the Convention on International Civil Aviation, entitled “Air Communication”, Volume I “Radio Navigation Aids”, specifies the technical standards for the parameter of accuracy of satellite positioning using the GPS navigation system in civil aviation [9]. The ICAO imposed the framework for commissioning the GPS system on civilian users in aviation. The ICAO’s accuracy standards are matched with air operations for a specific flight plane of an aircraft in civil aviation. For navigation on the horizontal plane, the accuracy of flight navigation LNAV ranges from 9 m to 17 m with a confidence level of 95%. For navigation on the vertical horizontal plane, the accuracy of flight navigation VNAV ranges from 15 m to 37 m with a confidence level of 95%. On the basis of the conducted investigation and obtained results with regard to the positioning accuracy of the Cessna 172, it can be concluded that the boundary limitations of the ICAO technical standards were met. The lowest values of accuracy for the horizontal coordinates B and L are 1.2 and 0.7 m, respectively, which do not exceed the boundary accuracy for the ICAO's technical standards. On the other hand, the lowest accuracy for the ellipsoidal height is 1.6 m, which does not exceed the accuracy limit in the ICAO's technical standards for navigation on the vertical plane, either.

Figure 6 shows the positioning accuracy of the Cessna 172 in three-dimensional (3D) space. The position error of the aircraft in 3D space is defined using the MRSE parameter. The mathematical formula defining the MRSE parameter is as follows [16]:

$$MRSE = \sqrt{mB^2 + mL^2 + mh^2} \tag{6}$$

The average value of the MRSE parameter is 1.555 m and the statistical value of the median equals 1.601. The dispersion of the obtained MRSE parameter values ranges from 1.372 m to 1.955 m. It is worth noting that, for approximately 43% of the results, the MRSE parameter is less than 1.3 m, while, for approximately 84% of the results, the MRSE parameter is in the range from 0 m to 1.4 m. Besides, for approximately 100% of the results, the MRSE parameter achieves an accuracy of up to 2 m.

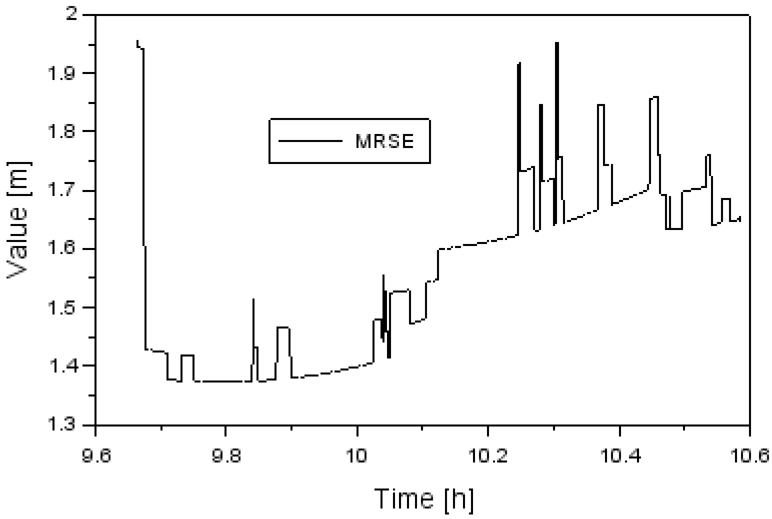

Fig. 6. The values of the MRSE parameter

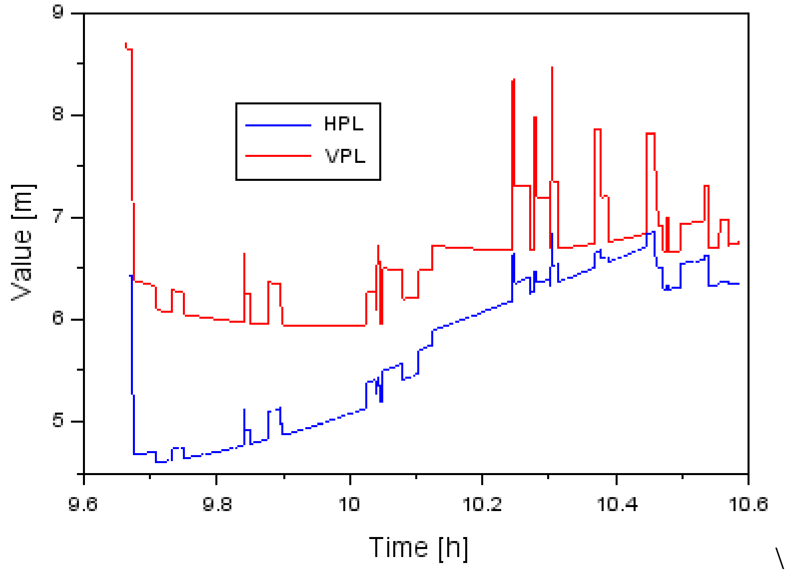

Fig. 7. The values of the HPL/VPL parameters

The parameter of integrity for the DGPS satellite positioning in civil aviation is of the utmost importance during the execution of air operations in airspace. In civil aviation, the integrity parameter is specified by means of safety levels. In practice, the levels of safety are referenced to navigation on both the horizontal and the vertical planes. On the horizontal plane, the safety level is determined by the HPL parameter, and by means of the VPL parameter on the vertical plane. The approximate values of the HPL and VPL safety parameters are determined on the basis of Equation (7) [10]:

$$\begin{cases} HPL = k_{HPL} \cdot \sqrt{mB^2 + mL^2} \\ VPL = k_{VPL} \cdot mh \end{cases} \tag{7}$$

where:

$k_{HPL} = 6$ for the horizontal plane

$k_{VPL} = 5.33$ for the vertical plane [5]

Figure 7 shows the obtained values for the HPL and VPL parameters. The average value of the HPL parameter is 5.697 m and the median value is equal to 5.912 m. In addition, the dispersion of the obtained results of the HPL parameter ranges between 4.599 m and nearly 6.861 m. It should be noted that the values of the HPL parameter for the entire duration of the research experiment have an upward tendency. The average value of the VPL parameter is 6.553 m and the statistical value of the median is equal to 6.669 m. Moreover, the dispersion of the obtained results of the VPL parameter ranges from 5.934 m to almost 8.711 m.

Annex 10 to the Convention on International Civil Aviation, entitled "Air Communication", Volume I "Radio Navigation Aids", specifies the technical standards for the parameter of integrity for satellite positioning using the GNSS navigation system in civil aviation [9]. The integrity values for GNSS satellite positioning in civil aviation were specified for the selected type of aircraft approach for landing. Within the framework of the ICAO's technical standards, there are three types of aircraft approach to landing with the GNSS sensor:

– Non-precision approach (NPA)
– Approach procedures with vertical guidance (APV)
- Precision approach (PA)

In Poland, the largest civilian passenger and transport airports have implemented technical regulations for the NPA with the GNSS sensor. The framework for the operation and application of the GNSS sensor for this approach was introduced by the Polish Air Navigation Services Agency. It must be underlined that, with regard to the NPA, the accuracy of determining the position of the aircraft on the horizontal plane is equal to 220 m. In addition, the integrity of GNSS satellite positioning is 556 m for the horizontal plane. The NPA with the GNSS sensor does not take into account the technical standards for conducting navigation on the vertical plane [5].

The obtained values of the HPL and VPL parameters can be used directly or indirectly to determine the continuity of GNSS satellite positioning in civil aviation. The parameter of continuity specifies and defines the gaps in tracking down a moving object with the use of GNSS satellite techniques. Furthermore, the parameter of continuity indicates possible failure and a lack of data from the GNSS positioning system. The mathematical formula to determine the parameter of continuity is as follows [11]:

$$\begin{cases} HPL < HAL \\ VPL < VAL \end{cases} \tag{8}$$

where:

HAL is the maximum alert value on the horizontal plane

VAL is the maximum alert value on the vertical plane

The parameter values of HAL and VAL specify the maximum alert levels for the integrity of GNSS positioning during the selected type of aircraft approach for landing. The continuity parameter is exceeded when the HPL is larger than HAL, or when VPL is larger than VAL. Within the framework of the NPA using GNSS, the HAL value is 556 m, whereas the VAL value is unspecified. Therefore, the HPL value does not exceed the HAL limit for navigation on the horizontal plane for research data from the flight test. Consequently, the continuity of the GNSS navigation solution on the horizontal plane is preserved and maintained. On the other hand, the continuity of GNSS positioning on the vertical plane for the NPA approach cannot be compared.

5. DISCUSSION

In this section, the obtained trajectory of an aircraft from a DGPS application was verified and compared with results from the DGLONASS solution. The aircraft position was estimated using the DGLONASS method in the RTKPOST library within the RTKLIB software package. The Kalman filter solution was applied as a stochastic scheme of designation for the aircraft coordinates in the RTKLIB program. The coordinates of the aircraft were recovered with an interval of 1 s using the DGLONASS method. The aircraft coordinates from the DGLONASS solution are referenced to the ETRF'89 frame, similar to the DGPS method.

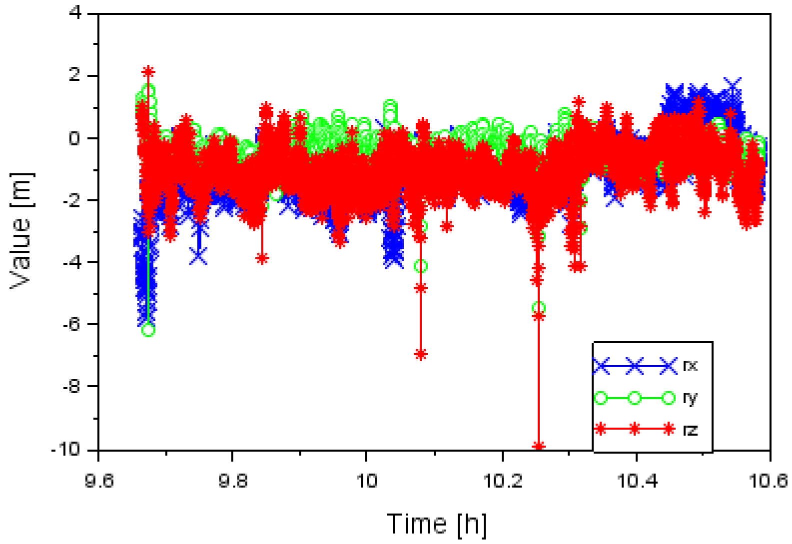

Fig. 8. The difference in the XYZ geocentric coordinates of the aircraft between the DGPS and DGLONASS solutions

The difference in the aircraft coordinates in the geocentric XYZ frame between the DGPS and DGLONASS solutions was calculated as follows:

$$
\begin{aligned}
rx &= x_{DGPS} - x_{DGLO} \\
ry &= y_{DGPS} - y_{DGLO} \\
rz &= z_{DGPS} - z_{DGLO}
\end{aligned} \tag{9}
$$

where:

x_{DGPS} is the x coordinate of the aircraft based on the DGPS solution – see Equation (1)

x_{DGLO} is the x coordinate of the aircraft based on the DGLONASS solution

y_{DGPS} is the y coordinate of the aircraft based on the DGPS solution – see Equation (1)

y_{DGLO} is the y coordinate of the aircraft based on the DGLONASS solution

z_{DGPS} is the z coordinate of the aircraft based on the DGPS solution – see Equation (1)

z_{DGLO} is the z coordinate of the aircraft based on the DGLONASS solution

Figure 8 presents the values of the (rx, ry, rz) coordinates based on a comparison of the DGPS and DGLONASS solutions. The mean difference for the x coordinate of the aircraft equals -0.941 m and the root mean square (RMS) bias [14] equals 0.821 m. In addition, the dispersion of results for parameter rx is between -5.805 m and +1.688 m. The mean difference for the y coordinate of the aircraft equals -0.435 m, with an RMS bias of about 0.457 m. Moreover, the dispersion of results for the ry parameter is between -6.149 m and +1.517 m. The mean difference for the coordinate from the aircraft equals -1.041 m and the RMS bias is about 0.733 m. In addition, the dispersion of results for the rz term is between -9.894 m and +2.110 m.

6. CONCLUSIONS

This article analysed the applicability of the DGPS positioning method in determining aircraft coordinates in air navigation. To this end, we recovered the position of a Cessna 172 aircraft in postprocessing. The calculations were carried out using RTKLIB software in the RTKPOST module by exploiting GPS code observations. The mathematical model for designating the aircraft position was based on the use of observation equations for dual code differences. In the research experiment, we used the P1/P2 code observations recorded by the geodetic receivers mounted on board the Cessna 172 and the Topcon Hiper Pro receiver installed at the reference station REF1. The materials for research came from a test flight, which was conducted at the military airfield in Dęblin. Within the framework of the conducted research, we recovered the trajectory of the Cessna 172 using the Kalman forward-filtering forward base solution. The article also analysed the GNSS satellite positioning for determining the parameters of availability, accuracy, integrity and continuity in civil aviation. The parameter of availability of the GPS satellite constellation was 100%, which facilitated a continuous navigation solution for the position of the Cessna 172 aircraft. The accuracy of the designated coordinates of the Cessna 172 was higher than 1.7 m in the XYZ geocentric frame and higher than 1.6 m in the ellipsoidal BLh frame. Furthermore, we set the MRSE parameter, whose accuracy was higher than 2 m. The integrity of GNSS satellite positioning in civil aviation was defined by the HPL and VPL parameters. The values of the HPL parameter did not exceed 7 m, and 9 m with regard to the VPL parameter. The continuity of GNSS satellite positioning was maintained, with no failure on the part of the navigation solution for the position of the Cessna 172. It should be added that the HPL values did not exceed the border limits of HAL positioning for air operations on the horizontal plane. In the external control, the geocentric coordinates (x, y, z) of the aircraft were compared with the results of the DGLONASS technique. On the basis of the comparison between the DGPS and DGLONASS solutions, it was found that the values of RMS bias for the (x, y, z) aircraft coordinates were less than 1 m.

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