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Research Article

Computational modeling and constructal design method applied to the geometric evaluation of stiffened thin steel plates considering symmetry boundary condition

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Article Info	Abstract
	The present article presents a geometric evaluation of stiffened plates employing the symmetry boundary condition. The computational models were developed in the software ANSYS, based on the Finite Element Method, using the SHELL281 finite element. As expected, the numerical simulations considering the symmetry demonstrated a gain in computational processing because of the possibility of developing the models in a quarter of a plate rather than simulating the entire plate, without accuracy loss. The geometric configurations are obtained through the application of Constructal Design. For this, some geometric parameters are varied (degrees of freedom) and others are kept fixed (restrictions). After that, these plates with different geometries are numerically simulated and their mechanical behaviors (deflection and stress) are compared to each other through the Exhaustive Search Technique. Then, it is possible to define the stiffened plate that has the best mechanical behavior. For this, a reference plate without stiffeners was used. In this plate part of its volume of material was transformed into stiffeners by reducing the thickness of the same, thus maintaining the value of the length and the value of the width as constants. The objective here is to determine the geometric configurations that minimize the maximum deflection and the maximum von Mises stress of the stiffened plates. The obtained results showed that, by transforming part of the volume of the reference plate into stiffeners, structural stiffness gains can be achieved in the stiffened plates when compared with the values of deflection and stress reached by the reference plate.

1. Introduction

Plates or panels are flat and three-dimensional structural components that may be subject to transverse loads that cause them to deflect. These structures, when have a thickness ten times smaller than their length or width, can be classified as thin plates. Thus, the analysis of displacements, deformations and stresses will go from a three-dimensional problem to

a two-dimensional problem, according to Kirchhoff's Thin Plate Theory (Bhaskar & Varadan [1]).

According to Szilard [2], some hypotheses must be respected for the use of this theory, being:

- The material should be homogeneous, isotropic and linear elastic.
- The plate, in its initial state, should be flat.
- The middle surface of the plate remains unrestricted during the deflection.
- The transverse deflections w (x,y) must not exceed one-tenth the thickness of the plate.
- The shear deformations γ_{yz} and γ_{xz} must be very small and neglected.

In order to improve the stiffness of the thin plates, there are attached to them reinforcements called stiffeners. These reinforcements are usually positioned longitudinally, transversely or in both directions relative to the plate. So, investigations involving the mechanical behavior of the stiffened plates were performed, such as: Orozco [3] made a comparison between two methodologies, Orthotropic Plate Method and FEM, of stiffened plates submitted to a transverse load and determined which of the two methodologies presents the best behavior of the dimensionless deflection and stress parameter curves; Silva [4] determined the influence of the eccentricity on the structural behavior in stiffened plates through the use of computational models developed in the ANSYS software; Bhaskar & Pydah [5] presented an analytical solution for stiffened plates that consider the shear force and rotational inertia in order to quantify the individual contribution of the plate and the stiffeners in the total deformation of the structure; and Troina [6] developed 3D and 2D computational models of stiffened plates subjected to uniform transverse loading in ANSYS and evaluated the influence of stiffener height by stiffener thickness and transformation of a volume fraction of material into stiffeners in a central deflection analysis.

Normally, the researches considered the whole plate as the computational domain, not using the symmetry boundary condition. The symmetry, when possible, allows to work with half or a quarter of the plate as computational domain. The symmetry boundary condition can be adopted if the structural component presents symmetry of: geometry, loading, support conditions and material properties. When applying the symmetry boundary condition it is assumed that the out-of-plane translations and in-plane rotations are set to zero (ANSYS [7]).

Therefore, the present article presents a geometric evaluation by means a computational model using the symmetry boundary condition in a quarter of stiffened plates subjected to a uniform transverse loading and clamped in its edges. The computational model to numerically simulate its mechanical behavior was elaborated in the software ANSYS 18.2 academic version with the SHELL281 finite element. The study analyzed the maximum deflection and the maximum von Mises stress of the different geometric configurations of plates that were generated from the application of the Constructal Design Method (Rocha, Lorente & Bejan [8]). They were defined by varying the number of longitudinal (N_{ls}) and transversal (N_{ts}) stiffeners; and by the use of different height-to-thickness ratios of stiffeners (h_s/t_s). All the geometric configurations were then numerically simulated, allowing the identification of the one that minimized the maximum deflection and the maximum von Mises stress, characterizing a process of geometric optimization through the Exhaustive Search Technique.

2. Computational Modeling

The software ANSYS, based on FEM, was used to generate the numerical models presented in this work. According to Stolarski, Nakasone & Yoshimoto [9], the ANSYS package can be used to numerically solve a wide variety of mechanical problems involving static or dynamic structural analysis, both of which can be linear or non-linear; problems involving heat transfer or fluid transfer; acoustic problems; electromagnetic problems, among others.

These computational models adopted the SHELL281 finite element (Fig. 1). According to ANSYS [7], the element is composed by eight nodes, having in each node six degrees of freedom (three translations in directions x, y, z, and three rotations around the axes x, y, z); and its use is suitable for the analysis of shell and plate structures that are thin to moderate in thickness.
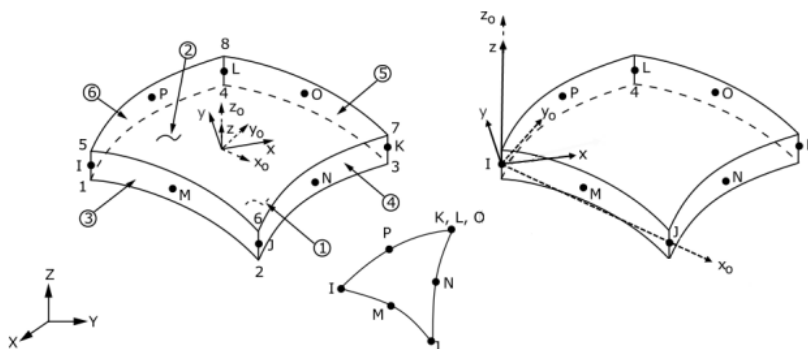

Fig. 1 Illustration of SHELL281 finite element (ANSYS [7])

In relation to FEM, its definition is given as the discretization of a mathematical model in a finite number of parts, whose geometry, boundary condition and load imposed on the model are defined according to the proposed physical problem. The accuracy of the method is directly related to the size of the elements, because the smaller the size, the greater the number of finite elements for the discretization of the computational domain and, consequently, the greater the precision of the results; but longer is the computational processing time required to develop the simulations. Also, tests that evaluate the minimum number of elements are necessary due to the low variation of result that the computational models present from a certain point, characterizing, thus, the stabilization of a mesh convergence test. Regarding the solution obtained by this method, this will only occur from the assembly of the global matrix of elements that will allow the nodal solution and the values of displacement and stress suffered by each element. In order to exemplify this explanation, for cases involving linear elastic behavior problems, the global matrix is represented by (Zienkiewicz [10], Rao [11]).

$$[K] \cdot \{u\} = \{F\} \tag{1}$$

where $[K]$ is the stiffness matrix of the structure, $\{u\}$ is the vector of unknown nodal displacements and $\{F\}$ is the external forces vector. The stiffness matrix of the system is obtained through the strain-displacement relations of the structure, from the discretization of the model and the properties of the material.

3. Constructal Design Method and Exhaustive Search Technique

The Constructal Law, the basic law of the Constructal Theory, deals with the design forecast (the flow configuration) and its evolution over time. Also, it is a way of seeing, as it teaches that the evolution of the flow system can be observed at all-time scales. That is, it is not only a principle from which the geometric form and the structure are deduced in nature, but also an engineering method (Design Construtal) used to optimize the paths of the flows through open systems of finite size. In this way, the Construtal Law indicates that the best flow architecture is one that minimizes the resistance of global flow or maximizes access to the global flow (Rocha, Lorente & Bejan [8]).

Therefore, the Constructal Design is a method that allows the obtaining of a geometry that gives the best performance of systems when submitted to some kind of flow. For this, the flow must be ajustable and the geometry must be deduced in order to maximize overall performance. In addition, global constraints and variations of certain degrees of freedom must be subjected to geometry (Bejan [12]). About the Exhaustive Search Technique, according to Khoury [13], refers to any search algorithm that analyzes a range of solutions until it finds the most adequate solution or reaches the maximum number of pre-established attempts.

The use of the Constructal Design Method in this article has the purpose to set the search space by changing the geometric parameters, so that the volume of the material does not change. For this purpose, a non-stiffened plate of length a, width b and thickness t was adopted as reference. After that, a fraction of its volume ϕ is transformed into stiffeners by reducing its thickness, maintaining the length and width with constant values (Fig. 2). Consequently, all the analyzed geometries have the same amount of material, allowing a comparative evaluation of the structural performance of these plates. The improvement of the mechanical behavior will be considered in the definition of the optimized geometric configuration. In addition, the volume fraction of the material is a constraint parameter of the Constructal Design Method and can be defined by:

$$\phi = \frac{V_s}{V_r} = \frac{N_{ls}\,(a\,h_s\,t_s) + N_{ts} \cdot [(b - N_{ts}\,t_s)\,h_s\,t_s]}{abt} \tag{2}$$

where V_s is the volume of the plate transformed into stiffener and V_r is the volume of the plate used as a reference, N_{ls} is the number of longitudinal stiffeners, N_{ts} is the number of transversal stiffeners, h_s is the height of the stiffeners and t_s is the thickness of the stiffeners. As for degrees of freedom, according to [6], for problems involving stiffened plates, it is possible to consider: height-to-thickness ratios of stiffeners h_s/t_s, the number of longitudinal stiffeners N_{ls} and the number of transverse stiffeners N_{ts}.

The analysis involved plates formed by 10 combinations of longitudinal and transverse stiffeners, obeying the P(N_{ls},N_{ts}) format, varying the following degrees of freedom: $N_{ls} = 2, 3$ and $N_{ts} = 2, 3, 4, 5, 6$. Therefore, in this study the following plates were analyzed: (2,2), P(2,3), P(2,4), P(2,5), P(2,6), P(3,2), P(3,3), P(3,4), P(3,5) and P(3,6). On the stiffeners, they have a rectangular cross section and commercial values of thicknesses of steel plates were adopted. In addition, two geometrical constraints are imposed: the height of the stiffeners cannot be greater than 0.3 m, in order to avoid geometric disproportions between the height of the stiffener and the lateral dimensions of the plate; and the ratio h_s/t_s must be greater than 1 to avoid that the thickness of the stiffener be greater than its height, a reason that would de-characterize the stiffener that should have a height greater than its thickness.
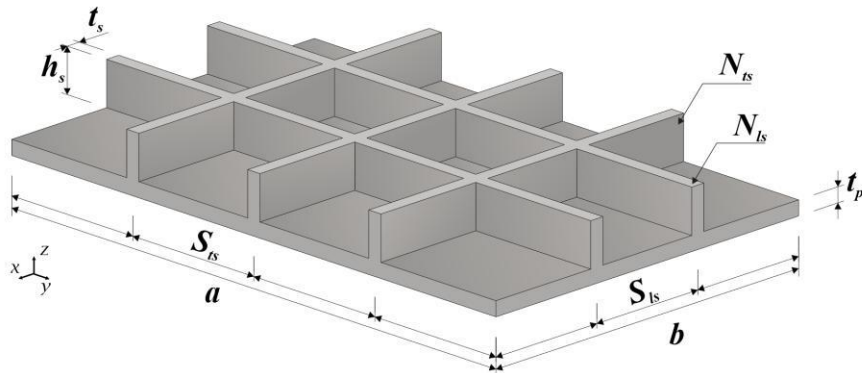
Fig. 2 Stiffened plate with 'its' variables

The adopted reference plate has a length of 2.00 m, width of 1.00 m, thickness of 0.02 m, and is of structural steel A-36 with modulus of elasticity of 200 GPa and Poisson coefficient of 0.3. The ratio of the volume of material transformed in stiffeners to the volume of the reference plate is φ = 0.3, that is, 30% of the volume was used in the stiffeners. The reference plate and the stiffened plates were considered to be clamped in the four edges, being subjected to a uniformly distributed transverse loading of 10 kPa.

4. Results and Discussions

Initially, two verifications were performed. The first one, through a comparison of the results of the central deflection of the stiffened plates P(2,5), and P(2,6), with simply support and subject to uniform transverse loading, with numerical results presented by Troina [6]. The second verification was a comparison between the deflection results and the von Mises stress of a stiffened plate (whole, without considering the condition of symmetry) in relation to a quarter of the plate considering the symmetry boundary condition, both with simply supported on its edges.

Figure 3 shows the first verification, so that the central deflection results obtained with SHELL281 finite element (used in this study) were compared with the SOLID95 finite element (used by [6]). The verification showed coherent results, since the largest difference between the displacements was 5.54%, found on the plate P(2,6) with h_s/t_s equal to 1.2337. While the smallest difference was 0.97% in the plate P(2,5) with h_s/t_s equal to 59.4087. The variation of the accuracy of the results may be due to the SOLID95 being an element that considers all the components of the stress and strain state as a three-dimensional system of a solid body, whereas the shell element SHELL281 has some simplifications in the components of its plane of stress and deformation.
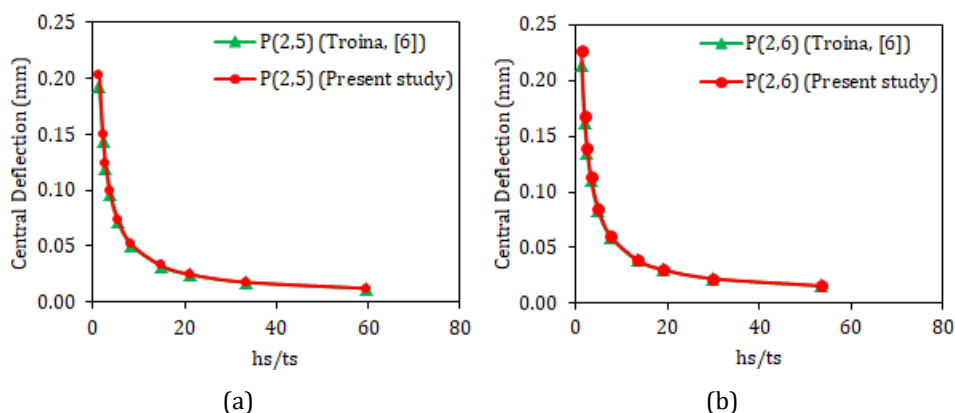
(a)　　　　　　　　　　　　　　　　　(b)

Fig. 3 Comparison of the numerical results of the central deflection of the stiffened plate due to the variation of degree of freedom h_s/t_s

The second verification was exposed in a mesh convergence test, shown in Tabs. 1 and 2. The objective here was to compare the stabilization of the displacement and von Mises stress results for the plate with greater geometric complexity among all simulated cases, that is, the plate P(3,6).

Table 1 Mesh convergence test for entire stiffened plate P(3,6)

Element size (mm)	Number of elements	w (mm)	σ_{vM} (MPa)
352	73	0.007	3.363
176	292	0.009	4.317
88	804	0.009	4.813
44	2631	0.009	5.552
22	9978	0.009	6.598

Table 2 Mesh convergence test for stiffened plate P (3,6) with symmetry

Element size (mm)	Number of elements	w (mm)	σ_{vM} (MPa)
376	34	0.008	3.416
188	80	0.009	4.317
94	222	0. 009	4.813
47	796	0.009	5.553
23.50	2718	0.009	6.568
11.75	9970	0.009	7.692

One can observe that the symmetry plate model (Tab. 2) presented similar results to the simulations that treated the entire plate (Tab. 1), which verifies the proposed computational model. In addition, it can be seen that for a deflection analysis it was possible to determine the independent mesh solution, as seen in Tabs. 1 and 2. In its turn, for the von Mises stress the results indicated the need for continuity of the mesh convergence test, i.e., the use of more refined meshes. However, the academic version of the software ANSYS 18.2 limits the simulations to a maximum use of 32000 nodes. Meshes with element sizes smaller than 11.75 mm in the symmetric model (see Tab. 2) exceed this node limit.

A solution for this problem on the mesh convergence test of the von Mises stress would be if it was used an elasto-plastic analysis rather than an elastic linear analysis. This new analysis could solve this problem because, in this case, the yielding of the material would occur at this point. Then, there would be a limit to the value of the von Mises stress at this stress concentration point. Thus, the maximum value of the von Mises stress will be used as a means of comparison, and not as an absolute value.

Then, the case study was performed using the computational model with the symmetry boundary condition. To do so, based on the results of Tab 2, the regular meshes were generated with quadrilateral SHELL281 finite elements with size of 11.75 mm. Figure 3 shows the numerical results obtained for the maximum deflection and in Fig. 4 for the maximum von Mises stress of the stiffened plates with symmetry subjected to the distributed uniform transverse loading. Besides, the reference plate was also numerically simulated. A maximum displacement of 0.1739 mm and a maximum von Mises stress of 11.047 MPa were obtained, being included in Figs. 4 and 5, respectively.
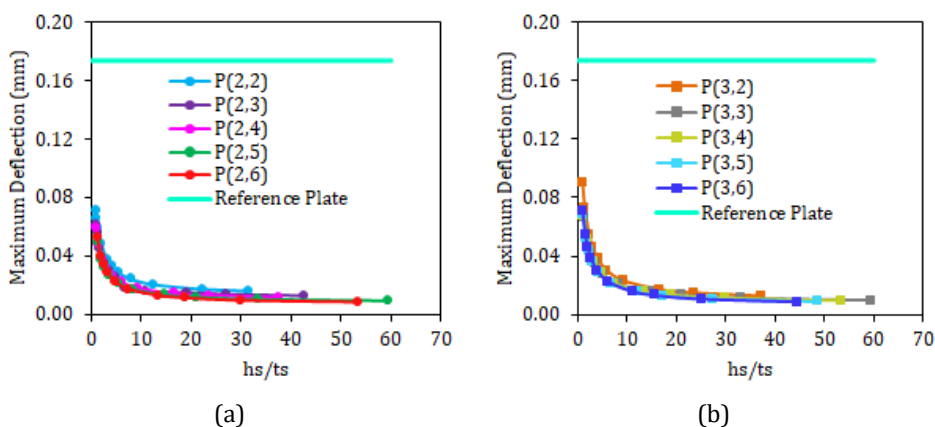

(a)   (b)

Fig. 4 Variations of the maximum deflection in relation to h_s/t_s for: (a) N_{ls} = 2; and (b) N_{ls} = 3

The results of Fig. 4 indicate that the transformation of 30% of reference plate material into stiffeners always leads to an improvement over deflection, i.e., all the stiffened plates have a maximum deflection smaller than that of the reference plate. However, for the maximum von Mises stress, it can be noted that geometric configurations with values of h_s/t_s greater than approximately 5 (Fig. 5 (a)) and 10 (Fig. 5 (b)) show mechanical behavior superior to the plate and reach lower stresses; while for smaller values of h_s/t_s there was an increase in the maximum stress and, consequently, a worsening in the mechanical behavior.

Another aspect observed is that as the value of h_s/t_s increases, the deflection (Fig. 4) and the von Mises stress (Fig. 5) of the stiffened plates decrease. Table 3 shows the best geometric configuration, that is, the one with the lowest deflection and lowest von Mises stress for each combination of longitudinal and transverse stiffeners.
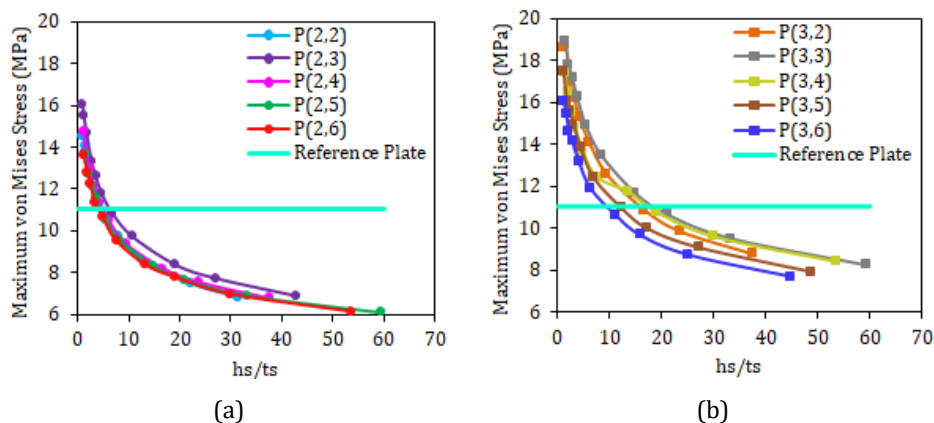
Fig. 5 Variations of the maximum von Mises stress in relation to h_s/t_s for: (a) N_{ls} = 2; and (b) N_{ls} = 3

Table 3 Better geometric configurations that minimize displacements and stresses

Plate	h_s/t_s	w (mm)	σ_{vM} (MPa)
P(2,2)	31.4176	0.0158	6.5167
P(2,3)	42.7470	0.0126	6.6846
P(2,4)	37.4378	0.0115	6.5674
P(2,5)	59.4087	0.0091	6.0334
P(2,6)	53.4905	0.0083	6.0244
P(3,2)	37.3781	0.0128	8.5811
P(3,3)	59.3771	0.0098	8.2325
P(3,4)	53.4905	0.0096	7.7104
P(3,5)	48.6658	0.0089	7.8466
P(3,6)	44.6394	0.0087	7.6923

The stiffened plate that presented the smallest displacement was P(2,6) with h_s/t_s = 53.4905 (see Tab. 3). Its maximum deflection is 95.23% smaller than the deflection obtained by the reference plate without stiffeners and 4.82% smaller than the plate P (3,6) with h_s/t_s = 44.6394 (lower displacement reached by the degree of freedom N_{ls} = 3). Also, it has been noted that as the degree of freedom N_{ts} increases, overall the maximum deflection of the plates decreases, as can be seen in Fig. 6 (a).

In relation to the stresses (see Table 3), the geometric configurations generated by the degree of freedom N_{ls} = 2 presented a greater reduction of von Mises stress than the geometries generated by the degree of freedom N_{ls} = 3, as can be seen in Fig. 6 (b). Thus, among the studied cases, the geometric configuration that minimized the maximum stress was the plate P(2,6) with h_s/t_s = 53.4905, whose stress presented a value 45.46% lower than the reference plate and 27.69% lower than the plate P (3,6) with h_s/t_s = 44.6394 (lower stress reached by degree of freedom N_{ls} = 3).
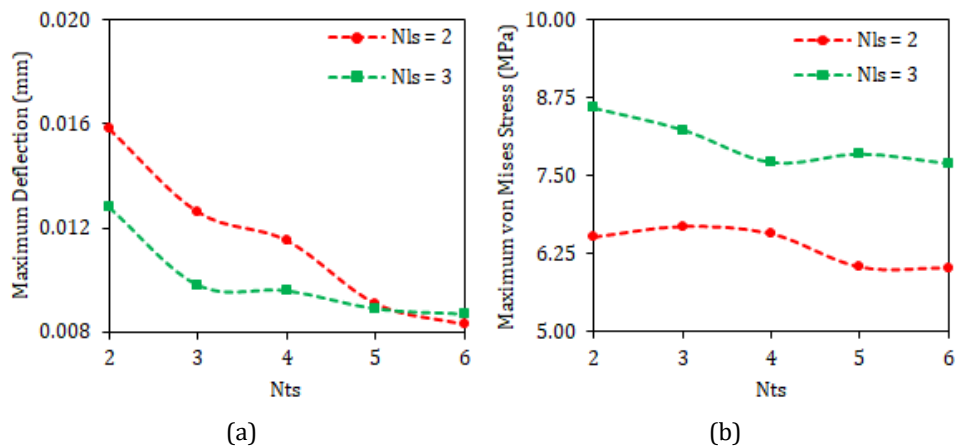
Fig. 6 Variation of (a) maximum deflection; and (b) Maximum von Mises stress in relation to Nts

5. Conclusion

Different geometric configurations of plates with stiffeners, generated from the application of the Constructal Design Method, were numerically simulated. The obtained results for the maximum deflection and for the maximum stress were compared with each other, in order to determine the optimized geometric configuration, by means the Exhaustive Search Technique. It is understood as optimal geometry the one that minimizes maximum deflection and minimizes maximum stress.

The study allowed determining a reduction in the computational processing time of the simulations that use the symmetry boundary condition in a quarter of the plate in the analysis of the deflection, because they reduce the computational domain of the problem and, as a consequence, they need less finite elements for their spatial discretization. For example, for the plate with 3 longitudinal stiffeners, 6 transverse and h_s/t_s = 44.6394 with element size of 22 mm, it was observed a reduction of 71.22% in the computational processing time to calculate the results in the simulation that employs the symmetry boundary condition (2876 elements) when compared with the simulation that uses the computational domain as entire (9978 elements). Also, it was noted that the symmetry boundary condition enables the use of more refined meshes with respect to simulations that consider the entire plate in the ANSYS software academic version. This is an important aspect, since the academic version of ANSYS has a limitation on mesh refinement.

About the results, it has been observed that the transformation of part of the volume of a non-stiffened plate into longitudinal and transverse stiffeners generally results in an improvement of structural mechanical performance.

The optimum geometric configuration for the cases studied was the plate with 2 longitudinal stiffeners, 6 transverse and h_s/t_s = 53.4905 ratio that provided reductions in maximum deflection and maximum von Mises stress of respectively 95.23% and 45.46% relative to the reference plate.

For the plates with N_{ls} = 3, the maximum stress reduction was 30.36% in relation to the reference plate. However, in the use of stiffeners with h_s/t_s of less than 5, a stress increase of 71.73% was found for the plate with 3 longitudinal stiffeners, 3 transversal stiffeners and h_s/t_s = 1.3618 ratio, that is, a worsening relative to the reference plate. Thus, it is clear

that not always the inclusion of stiffeners leads to an improvement in the mechanical behavior of the plates, evidencing the importance of the geometric evaluation in this type of structural component.

Acknowledgment

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