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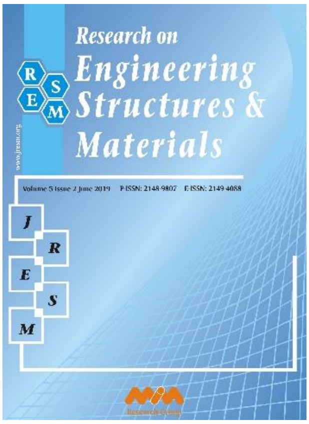

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Research Article

Constructal design applied to geometrical evaluation of rectangular plates with inclined stiffeners subjected to uniform transverse load

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Abstract

This study employs the Constructal Design method associated with the computational modeling (by the Finite Element Method) to evaluate the mechanical behavior related to deflections of stiffened rectangular steel plates submitted to uniform transverse loading. To do so, a rectangular reference plate without stiffeners was used. A material volume portion of this plate was converted into stiffeners through the parameter ϕ, which represents the ratio between the stiffeners volume and the total volume of the reference plate. By adopting $\phi = 0.3$, 50 stiffened plates were configured: 25 with orthogonal stiffeners to the plate edges and 25 with stiffeners oriented at 45°. Keeping the total volume of material constant, the influence of the orientation of the stiffeners and the effect of the degree of freedom h_s/t_s (ratio between height and thickness) variation were numerically analyzed. The results showed that variations in the geometry of the stiffeners can increase the rigidity of the plate, reducing its deflection. Further, it has been found that depending on the number of stiffeners, orienting them at 45° may be an alternative to minimize deflections in rectangular plates.

1. Introduction

Plates are straight, flat and two-dimensional structural components because their thickness is much smaller than the other dimensions [1]. In marine structures the plates are used specifically to resist the longitudinal loads advent from the ship's flexion, besides the hydrostatic pressure and the different transported items (static loading) [2].

To increase rigidity, usually, are inserted reinforcements in the longitudinal and/or transverse directions of the plates, called stiffeners. The geometrical proportions of stiffeners have an important role in the performance of these structural components [3].

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The Fig. 1 shows the cross-section of a ship, where it is possible to observe the stiffened plates that compose the structure, as well as the load that request on it.

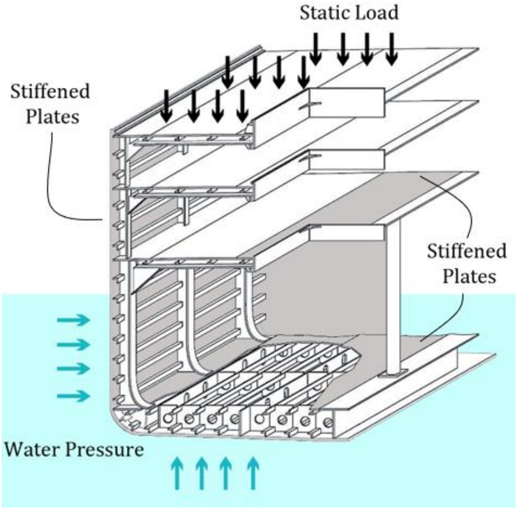

Fig. 1 Cross section of a ship (adapted from [4])

The analysis of the structural behavior of stiffened plates faces the difficulty of establishing analytical solutions due the geometric complexity. The existing proposals can be applied only to cases of simplified geometries, so that numerical methods become an alternative for solving this type of problem.

The Finite Element Method (FEM) was used by Rossow and Ibrahimkhail [5] to apply the Constraint Method in the analysis of stiffened plates with eccentric and concentric stiffeners. Bedair [6] using Quadratic Sequential Programming implemented a method of analysis for stiffened plates, where the structure was idealized considering plate and stiffeners rigidly connected. Peng et al. [7] presented a meshless method to analyze stiffened plates with eccentric and concentric stiffeners, based on the First Order Shear Deformation Theory.

Hasan [8] used the FEM through NASTRAN® software to analyze the better locations of stiffeners in plates under bending with different boundaries conditions. Hosseini and Soltani [9] investigated the structural behavior of square and rectangular plates under bending with different geometries of stiffeners, in concentric and eccentric conditions, using the Meshless Collocation Method.

Stiffened plates of isotropic material under bending were studied by Singh et al. [10]. Using the ANSYS® software, several configurations of stiffeners were analyzed considering different loads and boundary conditions, with the objective of reducing plate deflection. Finally, Troina [11] applied the Construal Design method using the FEM by ANSYS® software in the search for geometric arrangements that minimized the central deflection in plates with rectangular stiffeners.

Thus, the objective of the present study was to numerically evaluate the mechanical behavior related to the deflection of rectangular stiffened plates under uniform transverse loading, regarding the influence of the stiffeners orientation and the variation of the geometric parameter h_s/t_s (ratio between height and thickness of the stiffeners) through the application of the Constructal Design method.

2. Computational Modeling

The computational modeling of the plates analyzed in this study was performed using the ANSYS® software, based on FEM. The FEM is used in physical solution of engineering problems, which usually involve a structure or structural component subject to certain loads [12]. Zienkiewicz [13] explains that finite element analysis basically consists of four steps: creation of model geometry, mesh generation, load and boundary conditions application and problem solving.

According to Madenci and Guven [14], modeling an engineering problem for a static analysis with linear elastic behavior of the material through the FEM, requires the assembly of a global system of equations composed of the characteristic matrixes of the element and the vector of forces:

$$[K]\,.\{u\} = \{F\} \tag{1}$$

where $[K]$ is the global stiffness matrix; $\{u\}$ is the vector of unknown nodal displacements and $\{F\}$ represents the vector of external loads.

2.1. Computational Modeling Verification

To verify the computational model, it was simulated through ANSYS® the simply supported stiffened plate shown in Fig. 2. The plate was subjected a uniform transverse load of 0.006895 kN/cm^2 and its material has a Poison's ratio of 0.3 and Young's modulus of 20,684.27 kN/cm^2.

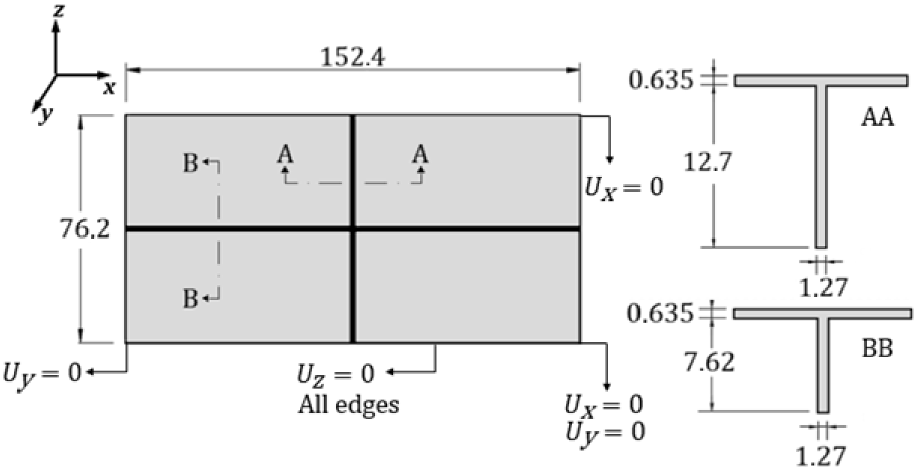

Fig. 2 - Rectangular plate with two orthogonal stiffeners (in cm).

To solve this problem, it was adopted the triangular finite element SHELL281, totalizing 30,400 elements after the mesh convergence test presented in Fig. 3. The results of central deflection U_z, as well, the results of other authors to the same plate model can be observed in Table 1.

There was a meaningful difference comparing the result found in this verification using the element SHELL281 with the results found by Rossow e Ibrahimkhail [5] and Peng et al. [7]. This can be justified considering the precision of the current model, which due to the available computational power, allows a greater refinement of mesh in the numerical model discretization.

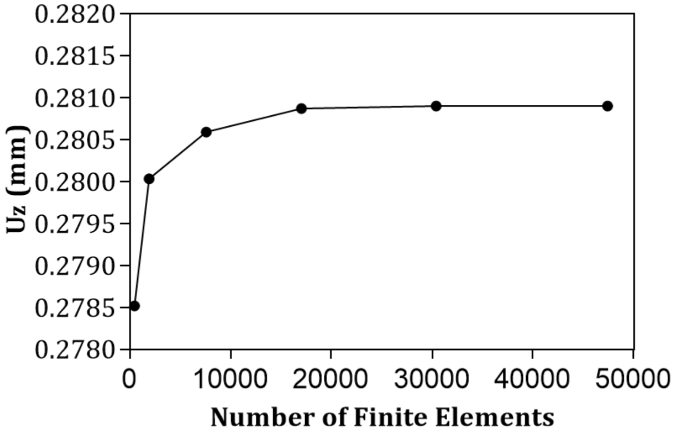

Fig. 3 - Results of mesh convergence test for computational model verification

Table 1 - Computational Model Verification

Authors	U_z (mm)	Difference (%)
Rossow e Ibrahimkhail [5]	0.2245	20.07
Peng et al. [7]	0.2185	22.20
Troina [11]	0.2781	0.99
Present Study	0.2809	--

In addition, comparing with the result found by Troina [11], which was obtained using the 3D finite element SOLID95, one can note a good agreement. This small difference can be explained because the element SOLID95 considers all components of stress and strain of the three-dimensional solid; while the two-dimensional element SHELL281, have some simplifications relative to the plane state of stress and strain. So, the proposed model can be considered as verified.

3. Constructal Design Method

According to Bejan and Lorente [16] the Constructal Law dictates that flow systems (river basins, lungs, atmospheric circulation, vascularized tissues, etc.) should evolve over time, acquiring better and better configurations to provide more access for the currents that flow through them.

The Construtal Design method is the way in which the Construtal Law is applied in practical situations, it is the generation of the flow architecture. The performance of a system is global and carries fixed global constraints, which may include the space allocated to the system, available material and components, ranges temperature limits, pressure or stress. Thus, the designer gathers all the components and optimizes the arrangement, so that can develop the flow architecture that achieves the better performance [17]. Bejan and Lorente [16] explain that in the analysis of mechanical structures the Constructal Law is applied in a similar way to any flow system, when subjected to a load the mechanical arrangements work as networks in which the stress flow through their components.

To apply the Constructal Design method, a rectangular reference plate without stiffeners was adopted. So, the volumetric fraction ϕ, presented in Eq. (2) and Eq. (3), was established for orthogonal stiffeners to the plate edges and stiffeners oriented at 45°, respectively. The parameter ϕ represents the ratio between the material volume of the stiffeners and the total material volume of the reference plate. In this study, ϕ = 0.3 was considered, i.e., 30% of the material volume of the reference plate was removed from the thickness and transformed into stiffeners.

$$\phi = \frac{V_s}{V_r} = \frac{N_{sx}(ah_s t_s) + N_{sy}[(b - N_{sx}t_s)h_s t_s]}{abt} \tag{2}$$

$$\phi = \frac{V_s}{V_r} = \frac{\sum_{d=1}^{n}[(d_1 + d_2 + d_3 \dots d_n)h_s t_s] - (N_{int}h_s {t_s}^2)}{abt} \tag{3}$$

where V_s is the volume of reference plate transformed into stiffeners; V_r is the total volume of the reference plate; a = 2000 mm, b = 1000 mm and t = 20 mm are, respectively, the length, width and thickness of the reference plate; h_s and t_s are, respectively, the height and thickness of the stiffeners, common to all configured plates. Exclusively for the geometric arrangements formed with stiffeners oriented at 45°, the parameters d_1, d_2, d_3, d_n, represents the length of the stiffeners, being n the total number of stiffeners and N_{int} the number of intersections among them. It is important to observe that all stiffeners thicknesses adopted in this study were based on commercial plates.

Just for the plates with stiffeners oriented at 45° an auxiliary coordinate system x', y' and z' was considered. Then, was adopted the format P(N_{sx}, N_{sy}) and P'$(N_{sx'}, N_{sy'})$, for orthogonal stiffeners to the plate edges and stiffeners oriented at 45°, respectively. Being $N_{sx'}$ and $N_{sy'}$, respectively, the number of stiffeners in x' and y' directions, as well, N_{sx} and N_{sy} represents, respectively, the number of stiffeners in x and y directions.

Thus, from the reference plate were configured 25 stiffened plates with orthogonal stiffeners to the plate edges (non-inclined) and 25 stiffened plates with stiffeners oriented at 45° (inclined). The Figs. 4 and 5 show the plates P(3,3) e P'(3,2), respectively.

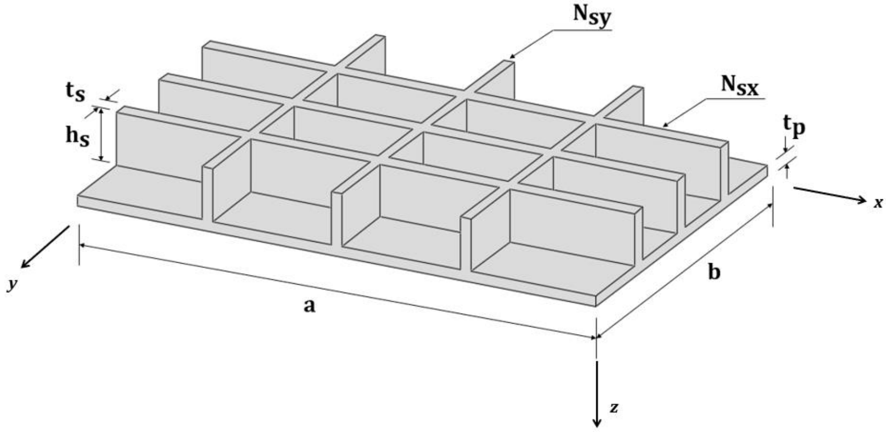

Fig. 4 - Plate P(3,3)

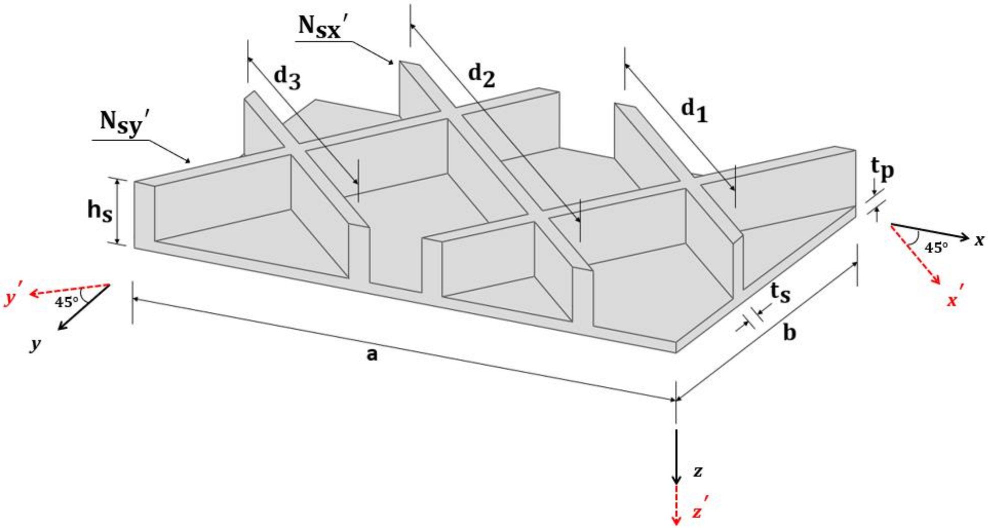

Fig. 5 - Plate P'(3,2)

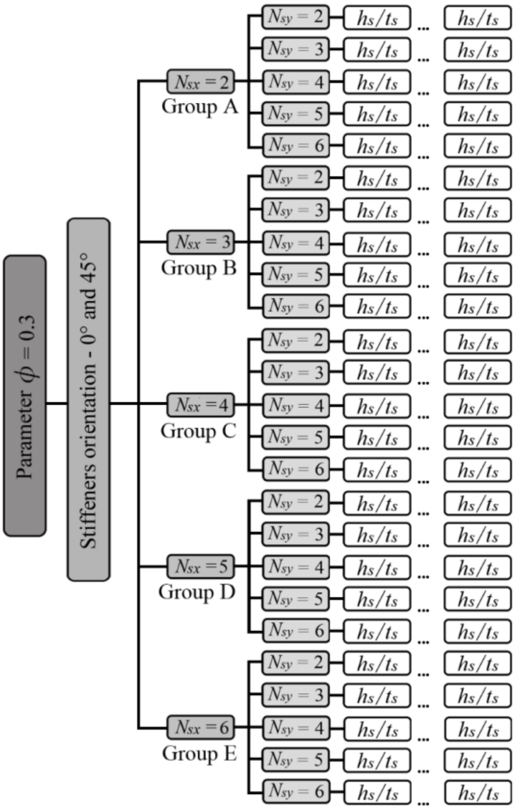

Fig. 6 - Plates arrangements analyzed

To facilitate the organization of the results presented in this study all proposed geometric configurations were divided into groups, as shown in Fig. 6. In addition, to avoid excessive disproportion between the dimensions of the plate and the dimensions of the stiffeners, the maximum stiffener height was limited to 300 mm.

In order to evaluate the influence of the degree of freedom h_s/t_s on the mechanical behavior of stiffened plates in relation to the deflection, for each new considered thickness, a new height was formed, maintaining constant the total material volume. The length of the reference plate a = 2000 mm and its width b = 1000 mm were preserved; however, its thickness t = 20 mm became t_p = 14 mm after material removal to create the stiffeners.

4. Mesh Convergence Test

A mesh convergence test was performed to define the size of the finite elements that would be used in the simulations. For this, the plate with greater geometric complexity P'(6,6) with h_s/t_s = 92.275 was chosen. The spatial discretization was done with SHELL281 finite element in its triangular shape, due to the geometric complexity of the plates with inclined stiffeners.

Six mesh configurations were analyzed, in each analysis the size of the finite elements was reduced successively as shown in Table 3. According to Fig. 7, it can be observed that from the third simulation the value of the center (U_z) and maximum (U_{zMax}) deflections of the plate were kept constant. It is important to highlight that in this geometric configuration used for the convergence mesh test $U_z = U_{zMax}$. Hence, the mesh M3 with finite element size of 25 mm was adopted, as can be viewed in Fig. 8.

Table 2 - Convergence mesh test

Meshes	Finite element size	Number of finite elements	Deflection U_z=U_{zMax} (mm)	Relative difference (%)
M1	100	1560	0.0181	2.162
M2	50	5090	0.0185	0.537
M3	25	20478	0.0186	0.000
M4	16,67	47338	0.0186	-
M5	12,5	84502	0.0186	-
M6	10	132322	0.0186	-

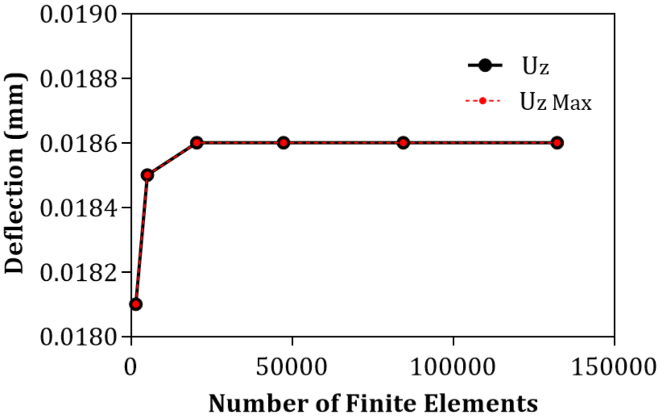

Fig. 7 - Result of mesh convergence test

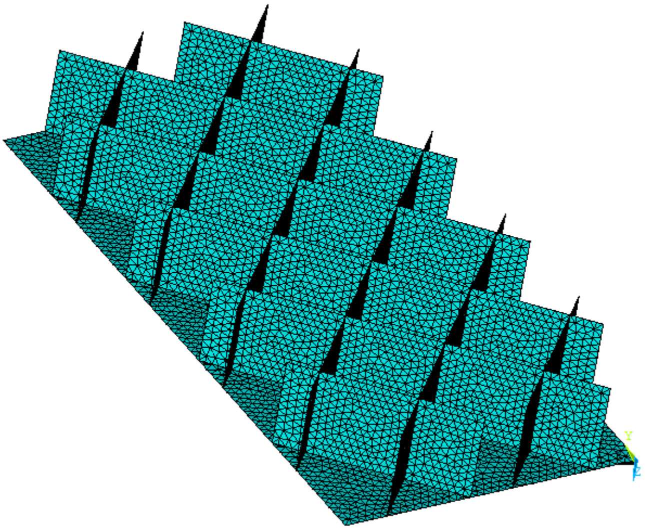

Fig. 8 - Discretization of the computational model for plate P'(6,6) with h_s/t_s = 92.275

5. Results and Discussions

For all plates simulated in this study, a uniform transverse load of 10 kN/m^2 was adopted. The material of plate is A-36 steel with modulus of elasticity E = 200 GPa and Poisson's ratio ν = 0.3.

Firstly, it was observed that transforming part of the volume of the reference plate into stiffeners always promote an improvement of the mechanical behavior concerning the deflections. The reference plate presented central and maximum displacement equal to $U_z = U_{zMax} = 0.697$ mm. Therefore, all proposed geometric configurations for the stiffened plates have central and maximum displacements lower than 0.697 mm.

Another observation was that for some geometric configurations of stiffened plates the central and maximum displacements are equal; while for other geometries different values for central and maximum deflections are obtained. This is due to the distribution of stiffeners. In the plates with orthogonal stiffeners, the distribution was carried out in function of the length and the width of the plate. However, for the plates with inclined stiffeners, the distribution was performed regarding the plate diagonal. Thus, whenever a stiffener crosses the center of the plate its central displacement is less than its maximum displacement.

For discussion purpose, only the results of groups A and D were presented, which showed distinct behaviors. That way, it was possible to analyze the influence of the stiffeners orientation, as well as the effect of its geometry variation. The other results of this study are presented in Appendix A. Moreover, for presentation purposes, the values of the center and maximum displacements were considered in the opposite direction of the z axis.

Observing the Fig. 9, it's noted that all plates of group A with orthogonal stiffeners to the edges (non-inclined) presented better performance (lower displacements) if compared to the plates with stiffeners oriented at 45°. The plates with the best results in this group were: P(2,5) with h_s/t_s = 59.409 and P'(2,5) with h_s/t_s = 62.891. The plate P(2,5) showed reductions of 34% for the central displacement and 40% for the maximum displacement, when compared with the plate P'(2,5).

However, all plates of group D with stiffeners oriented at 45° presented better results, both for the central displacement and maximum, when compared with orthogonal stiffened plates (see Fig. 10). The plates that presents better performances were: P(5,3) with h_s/t_s = 41.137 and P'(5,3) with h_s/t_s = 58.129 The plate P'(5,3) with stiffeners oriented at 45° showed a reduction of 50.2% for the central displacement and 34.2% for de maximum displacement, when compared with the plate P(5,3).

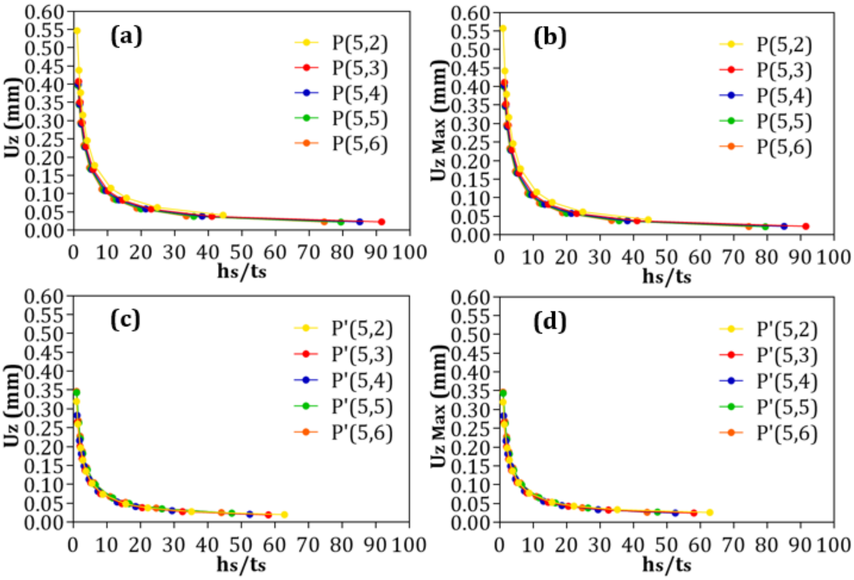

Fig. 9 - Group A - Central and maximum displacements – (a) and (b) Orthogonal stiffeners to edges; (c) and (d) stiffeners oriented at 45°

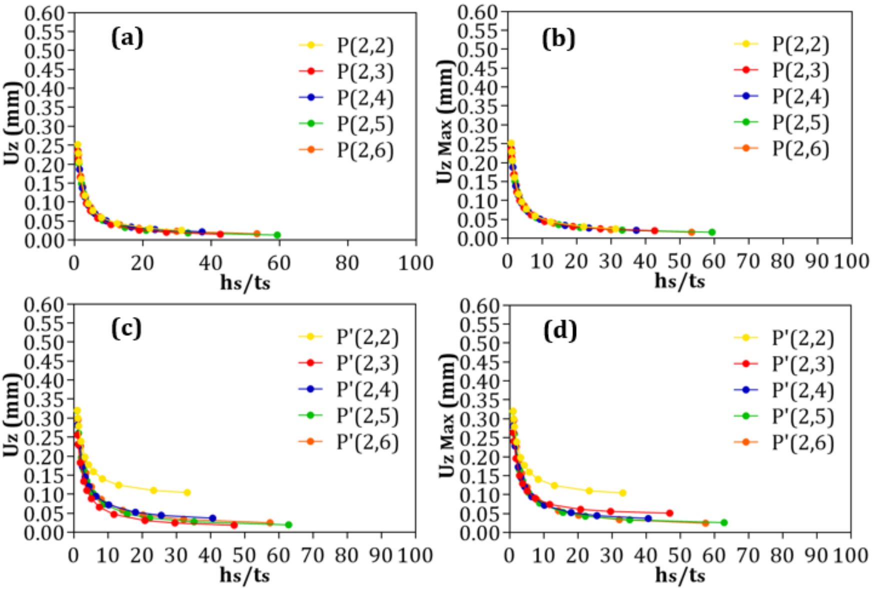

Fig. 10 - Group D - Central and maximum displacements – (a) and (b) Orthogonal stiffeners to edges; (c) and (d) stiffeners oriented at 45°

The displacements distribution of the best performing plates in Group A and D are showed in Figs. 11 and 12. It is interesting to note that the plates with orthogonal stiffeners to the edges have a uniform displacement pattern, while in the plates with stiffeners oriented at 45° the displacement is presented of irregular way.

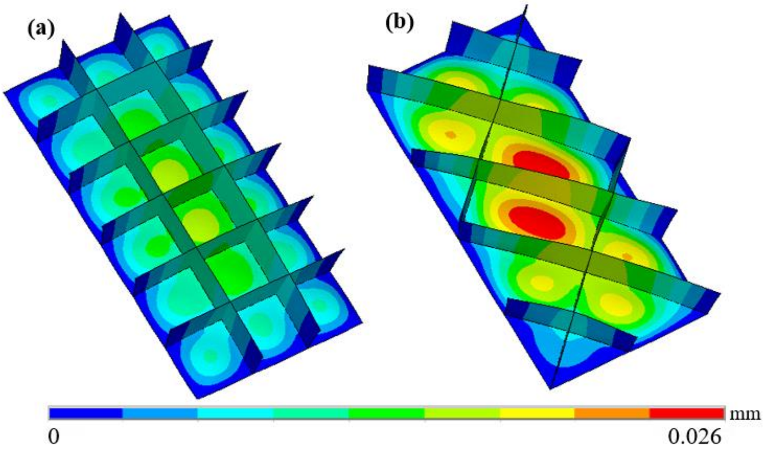

Fig. 11 - Group A - Displacements distribution: (a) P(2,5); (b) P'(2,5)

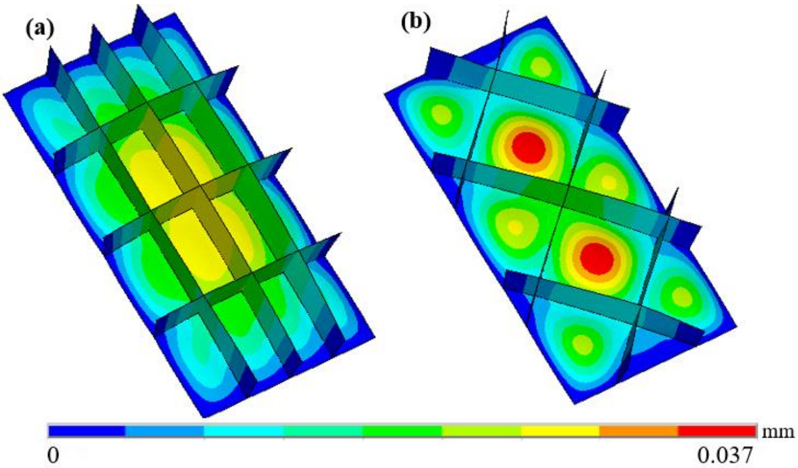

Fig. 12 - Group D - Displacements distribution: (a) P(5,3); (b) P'(5,3)

Another important observation indicates that a greater number of stiffeners does not necessarily reduce the deflections. This trend is valid for both investigated stiffeners orientations. The fact of keeping constant the total material volume of the plate, makes that in plates with a greater number of stiffeners the stiffener height is smaller, hence reducing its moment of inertia.

6. Conclusion

It was observed that transform part of the reference plate material into stiffeners always promote a reduction in the central e and maximum deflections, if compared the stiffened plate and the reference non-stiffened plate. In its turn, the increasing of the ratio h_s/t_s in stiffened plates always conduct to a reduction of central and maximum deflections. In other words, the rearrangement of material while keeping constant the volume of plate material may increase its stiffness.

Regarding the stiffeners orientation, it has been observed that orienting the stiffeners at 45° can lead to smaller displacements depending on the number of stiffeners. In addition, the results indicated that a greater number of stiffeners does not mean smaller deflections in both cases of plates, with stiffeners oriented at 45° and orthogonal to the edges of the plate.

The results also showed that for plates with two stiffeners in the x' direction (Group A), independent of the number of stiffeners in the direction y', oriented them at 45° is not advantageous in reducing the central and maximum deflections. However, for three or more stiffeners in the x' direction, independent of the number of stiffeners in the direction y', significant reductions in the central and maximum displacements can be obtained by orienting the stiffeners by 45°.

Therefore, the use of the Construtal Design method associated with computational modeling is an effective tool for the analysis of mechanical behavior of plates. In addition, modification of the orientation and geometry of the stiffeners can lead to an improvement in the mechanical behavior of hardened rectangular plates. In some cases, reaching a reduction of greater than 60% in relation to their deflections. For instance, the plates P(6,2) with h_s/t_s = 5.373 and P'(6,2) with h_s/t_s = 8.094. The plate P'(6,2) with stiffeners oriented at 45° showed a reduction of 64.8% for the central displacement and 64.8% for de maximum displacement, when compared with the plate P(6,2).

In future works, it is worth investigating stiffened plates with other orientations and amounts of stiffeners. As well as, other values of ϕ, i.e., other ratios of material transformed into stiffeners from the reference plate. It is also possible to evaluate the influence of the degrees of freedom considered in this study in the stress distribution from the uniform transverse loading.

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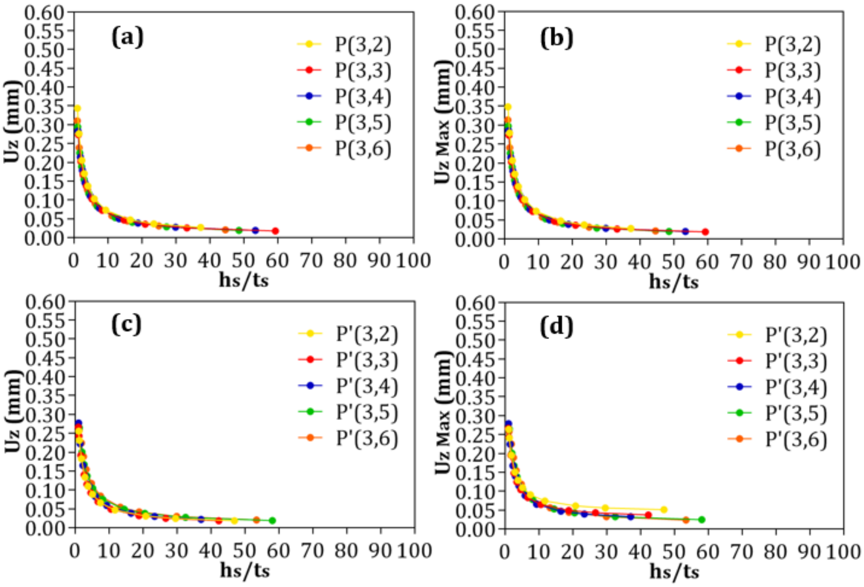
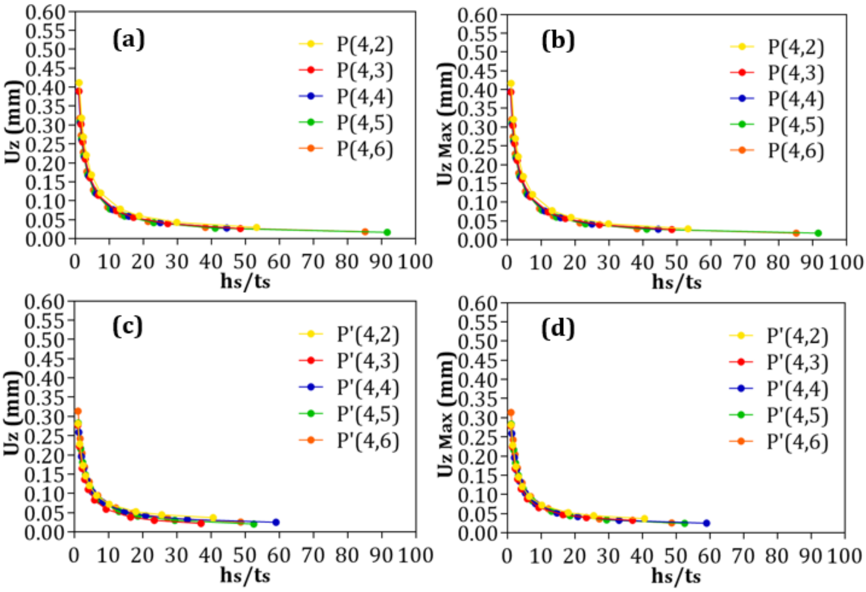
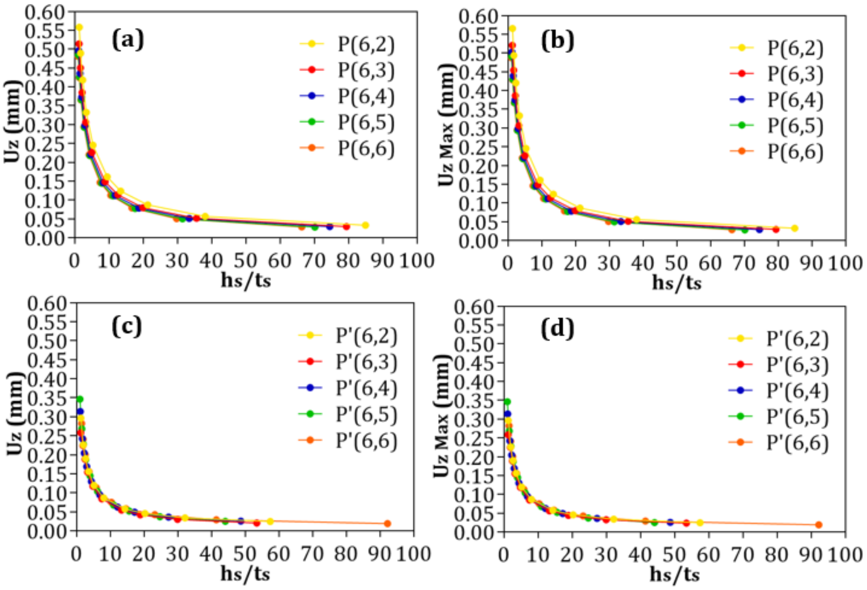
APPENDIX A



A. 1 - Group B - Central and maximum displacements – (a) e (b) Orthogonal stiffeners to edges; (c) e (d) stiffeners oriented at 45°



A. 2 - Group C - Central and maximum displacements – (a) e (b) Orthogonal stiffeners to edges; (c) e (d) stiffeners oriented at 45°



A. 3 - Group E - Central and maximum displacements – (a) e (b) Orthogonal stiffeners to edges; (c) e (d) stiffeners oriented at 45°

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