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Particular Matrix in the Study of the Index Hour Mathematical Model

The three phase transformer clock hour figure mathematical model can be conceived in his regular form as a 3X3 square matrix, called matrix code, or as a matrix equation, called code equation and is conceived through the elementary matrices: M_{a} , M_{b} , M_{c} or by defining matrices: M_{100} , M_{10} , M_1 . The code equation expression is dependent on the definition function: the sgn function or the trivalent variable function. Interrelated with the two possibilities are shown, defined and explained the following particular matrix: transfer matrix T. Finally are presented the interrelation between these particular matrixes and highlighted the possibilities of exploitation.

Keywords: clock hour figure, mathematical model, particular matrix

1. General considerations on the clock hour figure mathematical model

The clock hour figure mathematical model is represented through a square matrix with three lines and three columns [4, 6, 9]:

$$G_{i} = \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}$$
(1)

The Gi matrix is called code matrix. At the suggestion of academician prof. Ph.D. Emanuel DIACONESCU, to emphasize the leading role of the Electrotechnics Department of USV, in matrix discovery, the authors propose the acceptance of matrix CEUS name. The matrix code _{ij} element can be defined through the sgn function or through the trivalent element algebra [6], [9], [10].

In the first case:

$$_{ij} = \operatorname{sgn} a$$
 (2)

$$\operatorname{sgn} a = \frac{a}{|a|}$$
(3)

sgn a =
$$\begin{cases} 1 & when \quad a < 0 \\ -1 & when \quad a < 0 \\ 0 & when \quad a = 0 \end{cases}$$
(4)

All the possible code matrix obtained through this representation are presented in Table 1.

For the defining through the trivalent element algebra is valid the relation:

$$_{ij} = k; \text{ where: } k \in (2,1,0)$$
 (5)

$$k = \begin{cases} 2 & when \quad a < 0 \\ 1 & when \quad a < 0 \\ 0 & when \quad a = 0 \end{cases}$$
(6)

When defining through trivalent element algebra the matrix code configuration is the one presented in the Table 2.

The studies, highlight the interdependence between matrix code configuration (the lines and columns configuration) and the configuration on the terminal connection fixed on the transformer cover.

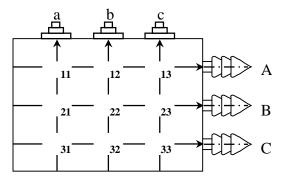


Figure 1. The interdependence between matrix code configuration and the configuration on the terminal connection [6], [9]

Table 1	Table 2
$\begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & 1 \end{pmatrix}$	$(2 \ 1 \ 0)$ $(2 \ 1 \ 2)$
$\mathbf{G_1} = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 1 \end{pmatrix} \qquad \mathbf{G_2} = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{pmatrix}$	$G_1 = \begin{bmatrix} 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} G_2 = \end{bmatrix} \begin{bmatrix} 2 & 2 & 1 \end{bmatrix}$
$\begin{pmatrix} -1 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & 1 & 1 \end{pmatrix}$	$\mathbf{G_1} = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{pmatrix} \qquad \mathbf{G_2} = \begin{pmatrix} 2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & 2 & 2 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	(0 1 2) (1 1 2)
$G_3 = \begin{pmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{pmatrix} \qquad G_4 = \begin{pmatrix} -1 & -1 & 1 \\ 1 & -1 & -1 \\ -1 & 1 & -1 \end{pmatrix}$	$\mathbf{G}_{3} = \begin{pmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \\ 1 & 2 & 0 \end{pmatrix} \qquad \mathbf{G}_{4} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$
(-1 1 0) (-1 1 -1)	$\begin{pmatrix} 1 & 2 & 0 \end{pmatrix}$ $\begin{pmatrix} 1 & 2 & 1 \end{pmatrix}$
$(-1 \ 0 \ 1) \ (-1 \ 1 \ 1)$	$(1 \ 0 \ 2)$ $(1 \ 2 \ 2)$
$G_5 = \begin{pmatrix} -1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix} \qquad G_6 = \begin{pmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$	$G_5 = 2 1 0 G_6 = 2 1 2$
(0 1 -1) (1 1 -1)	$\mathbf{G_5} = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 1 & 0 \\ 0 & 2 & 1 \end{pmatrix} \qquad \mathbf{G_6} = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{pmatrix}$
$(-1 \ 1 \ 0) (-1 \ 1 \ -1)$	(1 2 0) (1 2 1)
$G_7 = \begin{bmatrix} 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} G_8 = \\ -1 & -1 & 1 \end{bmatrix}$	$\mathbf{G}_7 = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \\ 2 & 0 & 1 \end{pmatrix} \qquad \mathbf{G}_8 = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{pmatrix}$
$G_7 = \begin{pmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{pmatrix} \qquad G_8 = \begin{pmatrix} -1 & 1 & -1 \\ -1 & -1 & 1 \\ 1 & -1 & -1$	(201) (211)
$\begin{pmatrix} 0 & 1 & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & 1 & -1 \end{pmatrix}$	(0 2 1) (2 2 1)
$G_{g} = \begin{pmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{pmatrix} G_{10} = \begin{pmatrix} 1 & 1 & -1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{pmatrix}$	$\mathbf{G_9} = \begin{pmatrix} 0 & 2 & 1 \\ 1 & 0 & 2 \\ 2 & 1 & 0 \end{pmatrix} \qquad \mathbf{G_{10}} = \begin{pmatrix} 2 & 2 & 1 \\ 1 & 2 & 2 \\ 2 & 1 & 2 \end{pmatrix}$
(1 - 1 0) (1 - 1 1)	(2 1 0) (2 1 2)
$\begin{pmatrix} 1 & 0 & -1 \end{pmatrix}$ $\begin{pmatrix} 1 & -1 & -1 \end{pmatrix}$	$\begin{pmatrix} 2 & 0 & 1 \end{pmatrix}$ $\begin{pmatrix} 2 & 1 & 1 \end{pmatrix}$
$G_{11} = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \qquad G_{12} = \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}$	$\mathbf{G_{11}} = \begin{pmatrix} 2 & 0 & 1 \\ 1 & 2 & 0 \\ 0 & 1 & 2 \end{pmatrix} \mathbf{G_{12}} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{pmatrix}$
$\begin{pmatrix} 0 & -1 & 1 \end{pmatrix}$ $\begin{pmatrix} -1 & -1 & 1 \end{pmatrix}$	

The lines position reflect the modifications in the high voltage windings terminal and the columns position reflect the modification in the low voltage windings terminal [2], [3], [4], [5], [6], [9].

These conclusion represent the most remarcable and interesting aspects. The connection between the matrix code and the configuration on the terminal connection, is expressed in a suggestive manner in figure 1.

The code matrix reflects faithfully the following changes in the transformer connection diagram [6], [9]:

- circular permutation of terminal connection;

- circular permutation of the transformer terminal notation;

- the convesion between them of two terminal connection at the primary winding and secondary winding;

- the inversion of the transformer supply power(from the high voltage to the low voltage or vice versa);

- the connection variation from the N in Z at the delta connection and zig-zag connection;

- reverse direction for the phase winding wrapping;

- reversal the beginning with the end of a phase winding;

- the terminal notation reversal for a winding phase.

Concerning to the mentioned modification are recommended and used the following notations and symbols [6], [9]:

 $A\downarrow$ - for the matrix obtained through the lines permutation in direct orientation (arrow orientation):

$$\mathbf{A} = \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right) \rightleftharpoons \begin{pmatrix} a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix} = \mathbf{A} \checkmark$$
(7)

A \uparrow - for the matrix obtained through the lines reverse permutation (arrow orientation):

$$\mathbf{A} = \left(\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \right) \Rightarrow \begin{pmatrix} a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \\ a_{11} & a_{12} & a_{13} \end{pmatrix} = \mathbf{A}^{\uparrow}$$
(8)

 \overrightarrow{A} - for the matrix obtained through the column permutation in direct orientation (arrow orientation)

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} a_{13} & a_{11} & a_{12} \\ a_{23} & a_{21} & a_{22} \\ a_{33} & a_{31} & a_{32} \end{pmatrix} = \vec{A}$$
(9)

 \overleftarrow{A} - for the matrix obtained through the column permutation in reverse orientation (arrow orientation)

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} a_{12} & a_{13} & a_{11} \\ a_{22} & a_{23} & a_{21} \\ a_{32} & a_{33} & a_{31} \end{pmatrix} = \mathbf{A}$$
(10)

 $A_{\text{invL12C23}}$ - for the matrix obtained through the inversion of the first line with the second line and through the inversion of the second column with the third column.

$$\mathbf{A} = \begin{pmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} a_{21} & a_{22} & a_{23} \\ a_{11} & a_{12} & a_{13} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} a_{21} & a_{23} & a_{22} \\ a_{11} & a_{13} & a_{12} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Rightarrow \begin{pmatrix} a_{21} & a_{23} & a_{22} \\ a_{11} & a_{13} & a_{12} \\ a_{31} & a_{33} & a_{32} \end{pmatrix} = \mathbf{A}_{invL^{12}C^{23}}$$

 A^{T} – for the transpose matrix (by changing lines with columns):

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \Longrightarrow \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix} = \mathbf{A}^{\mathsf{T}}$$
(12)

2. Particular matrix identified within the study of the clock hour figure mathematical model

Based on the matrix code decomposition facility in three elementary matrices $M_{a,}\,M_{b,}\,M_{c,}\,we$ obtain the general [6], [9]:

$$G_i = M_a + M_b + M_c \tag{13}$$

where:

$$M_{a} = \begin{pmatrix} y_{11} & 0 & 0 \\ 0 & y_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix}; M_{b} = \begin{pmatrix} 0 & y_{12} & 0 \\ 0 & 0 & y_{23} \\ y_{31} & 0 & 0 \end{pmatrix};$$
(14)
$$M_{c} = \begin{pmatrix} 0 & 0 & y_{13} \\ y_{21} & 0 & 0 \\ 0 & y_{32} & 0 \end{pmatrix}$$

In connection with elementary matrix are identified and defined three defining matrices $M_{100,}\,M_{10}\,and\,M_{1,};$

$$M_{100} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}; \quad M_{10} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}; \quad M_{1} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$
(15)

This form is valid if the mathematical model is defined through the signum function or trough the trivalent element algebra.

The clock hour figure mathematical model can be configured in mathematical terms through several equations code. The equations general form depends on the defining manner (through the signum function or through the trivalent element algebra).

When defining through the sgn function the code equation general form has the form [6]:

$$G_{i} = \left(\mathbf{sgnT}_{\mathbf{a}}, \ \mathbf{sgnT}_{\mathbf{b}}, \ \mathbf{sgnT}_{\mathbf{c}} \right) \cdot \begin{pmatrix} \boldsymbol{M}_{100} \\ \boldsymbol{M}_{10} \\ \boldsymbol{M}_{1} \end{pmatrix}$$
(16)

where $sgnT_{a}$, $sgnT_{b}$, $sgnT_{c}$ coefficient represents the first line of the code matrix defined trough the sgn function [9]:

sgn $T_a = {}_{11}$; sgn $T_b = {}_{12}$; sgn $T_c = {}_{13}$; where ${}_{ij}$ (-1, 1, 0).

When defining through the trivalent element algebra the code equation general form has the form [9]:

$$G_{i} = \begin{pmatrix} \mathbf{K}_{\mathbf{T}_{a}}, \ \mathbf{K}_{\mathbf{T}_{b}}, \ \mathbf{K}_{\mathbf{T}_{c}} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{M}_{100} \\ \mathbf{M}_{10} \\ \mathbf{M}_{1} \end{pmatrix}$$
(17)

where $k_{Ta,} k_{Tb,} k_{Tc}$ coefficients represent trivalent constant defined through the relation:

 $k_{Ta} = {}_{11}; \quad k_{Tb} = {}_{12}; \quad k_{Tc} = {}_{13}; \text{ unde } {}_{ij} \quad (0, 1, 2)_{..}$ The code equation expression for the sgn function defining are presented in table 3 and for the trivalent element algebra are presented in table 4.

The authors propose a unified form for the code equation expression.

$$G_{i} = \begin{pmatrix} \mathbf{q}_{a}, \mathbf{q}_{b}, \mathbf{q}_{c} \end{pmatrix} \cdot \begin{pmatrix} \boldsymbol{M}_{100} \\ \boldsymbol{M}_{10} \\ \boldsymbol{M}_{1} \end{pmatrix}$$
(18)

where: $q_a = {}_{11}$; $q_b = {}_{12}$; $q_c = {}_{13; 11, 12, 13}$ being defined, by case, through the sgn function or through the trivalent element algebra.

Given the connection between the defining matrices $M_{1,}$ $M_{10,}$ M_{100} and the transfer matrix T and that the connection is analytically expressed through the relations:

$$T^{1} = M_{1}; \quad T^{2} = M_{10}; \quad T^{3} = M_{100}$$
 (19)

result the general expression of the code equation that has the form: $(\tau 3)$

$$G_{i} = \left(\mathbf{q}_{a}, \mathbf{q}_{b}, \mathbf{q}_{c} \right) \cdot \begin{pmatrix} \mathbf{7}^{2} \\ \mathbf{7}^{2} \\ \mathbf{7}^{1} \end{pmatrix}$$
(20)

The unitary form of the code equation creates the conditions of expression, in the same embodiment connected to the sgn function (figure 2) and other connected to the trivalent element algebra (figure 3).

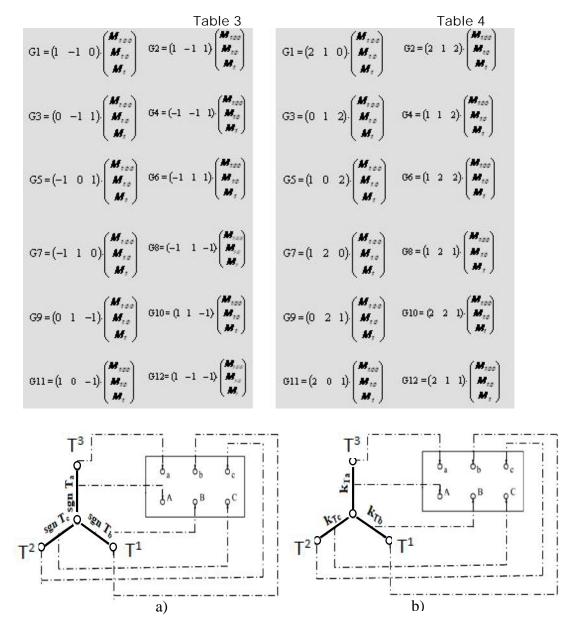


Figure 2. The connection between the graph code configuration and the three – phase transformer terminal connection configuration a) through sgn function; b) through trivalent element algebra [6,9]

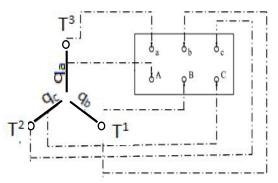


Figure 3. The connection between the graph code configuration and the threephase transformer terminal connection configuration q_a , q_b , q_c coefficients are defined in the same form as in the code equation.

3. Conclusions

1. One of the most important discovery of the USV's Research Centre EMAD is the clock hour figure mathematical model represented in the first phase through a square matrix with three lines and three columns. The lines position reflect the modification in the high voltage windings terminal and the columns position reflect the modification in the low voltage winding terminal.

2. Starting from the equation code, the research has marked out the possibility of clock hour figure mathematical modelling, through a code equation, respectively through a code graph.

3. Modelling the clock hour figure in the three forms of expression above mentioned, is realised through the sgn function either through trivalent element algebra, obtaining distinct mathematical expression for either manner of defining.

4. The unification possibility of mathematical model expression form was confirmed by the fact that the elementary matrices $M_{a,} M_{b,} M_{c,}$ defining matrices $M_{1,} M_{10,} M_{100,}$ lagrange matrix L and the transfer matrix T have the same form and in the case of defining through sgn function either trough trivalent element algebra.

5. At the end of the paper are presented uniform expression of the mathematical model through the code equation and code graph that have the same form through sgn function either trough trivalent element algebra.

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