MOBILITY & VEHICLE MECHANICS

DOI: 10.24874/mvm.2018.44.03.03
UDC: 004.94:656.084

A COMPUTATIONAL MODEL FOR THE RECONSTRUCTION OF VEHICLE COLLISIONS

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Received in September 2018 Accepted in October 2018

RESEARCH ARTICLE

ABSTRACT: The reconstruction of a vehicle collision consists of the processes of investigation, analysis and conclusions about the causes and the events during the traffic accident. In this perspective, a vehicle collision is considered in three distinct phases, the pre-collision, the collision and the post-collision phase. For the analysis of the collision phase two main approaches exist in the literature, the energy based and the momentum based one. The latter has been described in details by Brach et al. and can find a solution to a given set of parameters to reconstruct a collision. These parameters can be known or assumed using monitoring systems of the vehicle or physical evidence. In the present paper a computational model implementing the Planar Impact Mechanics (PIM) collision model has been set up in Matlab ® and its coupling with the least squares method has been investigated. As test cases, the RICSAC database, which consists of twelve staged collisions, has been used. Special attention has been given in the number of parameters which have been considered known (or assumed). The results indicate the importance of each parameter.

KEY WORDS: traffic accident reconstruction, Planar Impact Mechanics, RICSAC, least squares method, computational model

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PRORAČUNSKI MODEL REKONSTRUKCIJE SUDARA VOZILA

REZIME: Rekonstrukcija sudara vozila se sastoji od procesa: istraživanja, analize i zaključaka o uzrocima i događajima tokom saobraćajne nezgode. Na ovaj način, sudar vozila se razmatra u tri različite faze, pre sudara, tokom sudara i posle sudara. Za analizu faze sudara postoje dva glavna pristupa u literaturi, zasnovani na održanju energije i održanja impulsa. Zakon održanju impulsa je opisao Brach et all i našli su rešenja za parametre kojima se rekonstruiše sudar. Ovi parametri mogu biti poznati ili se pretpostavljajući koristeći sisteme za praćenje vozila ili fizičke dokaze. U ovom radu je u MATLAB-u formiran model mehanike ravanskog sudara (PIM) zasnovan na metodi najmanjih kvadrata. Baza RICSAC, koja se sastoji od dvanaest realizovanih sudara je korišćena za testiranje modela. Posebna pažnja je posvećena broju parametara koji su usvojeni kao poznati (ili pretpostavljeni). Rezultati ukazuju na važnost svakog parametra.

KLJUČNE REČI: rekonstrukcija saobraćajnih nezgoda, mehanika ravanskog sudara, RICSAC, metoda najmanjih kvadrata, računski model

A COMPUTATIONAL MODEL FOR THE RECONSTRUCTION OF VEHICLE COLLISIONS

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1. INTRODUCTION

Traffic accident reconstruction involves the qualitative and quantitative estimation of the way such an accident occurred. This process includes the use of engineering, scientific and mathematical laws and it is based on data and physical evidence collected from the accident scene.

A typical traffic accident involves the collision of two vehicles and it can be considered in three distinct time phases, the pre-collision, the instantaneous collision and the post-collision phase. In the literature, two broad approaches are used for the simulation of the collision phase, the one based on the conservation of linear and/or angular momentum and the one based on the conservation of energy. The momentum based models are these defined by Brach (Brach, 1987) and Ishikawa (Ishikawa, 1993) while the model considering the conservation of energy is this defined by McHenry (McHenry, 1981).

The momentum based collision simulation model presented by Brach is based on the second law of Newton and the principle of impulse – momentum. This collision model is referenced in the literature as Planar Impact Mechanics model and it consists of six algebraic equations. Since the movement of a vehicle is quantified with three velocity components, the normal, the tangential and the rotational velocity, twelve velocity components, six initial (three per vehicle) and six final ones are involved in this model. Furthermore, this model incorporates three impact coefficients quantifying physical constraints of the collision, namely, the restitution coefficient, the equivalent coefficient of friction and the momentum coefficient of restitution. All fifteen parameters, velocity components and impact coefficients, need to be known, assumed or calculated in order for a traffic accident reconstruction to be accomplished.

In the present paper the Planar Impact Mechanics collision model, which is presented in details in the following section, has been set up in the programming environment of Matlab ®. Planar Impact Mechanics collision model has been coupled with the least squares method in order to calculate the unknown velocity components and/or the impact coefficients. A minimization problem has been set up, using the least squares method equation as a cost function. Using the deterministic optimization method of SQP, implemented in Matlab through the fmincon function, the values of the impact coefficients where determined.

As test cases, the RICSAC database, which consists of twelve staged collisions, has been used. Special attention has been given in the number of velocity components which have been considered known (or assumed) exploring the behaviour of three different cost functions. For the evaluation of the performance of the optimization procedure and the efficiency of each cost function the measured velocity values for each RICSAC test have been used. The results consider the performance of the optimization set up, the performance of each cost function and the importance of the impact parameters.

2. MATERIALS & METHODS

In this section the equations of the Planar Impact Mechanics collision model are going to be introduced. Moreover, the test collisions, belonging in the RICSAC database, are going to be presented. Finally, the least squares method along with the optimization procedure are going to be described providing information on the design variables, the evaluated cost functions and the applicable constraints.

2.1 Planar Impact Mechanics collision model

In 1977 Brach (Brach, 1977) presented a set of six linear algebraic equations which simulate the planar collision of two vehicles. These equations are based on the Newton's second law and the impulse - momentum principles and achieve the calculation of the velocity changes of two particles due to impact (Brach, 1983). This set of equations can be applicable to the simulation of a collision of two vehicles provided that the following assumptions are satisfied (Brach, 1984):

1. The resultant intervehicular impulse is much larger than the impulses of other forces such as friction with the ground, drive train drag and aerodynamic drag which are neglected
2. The resultant impulse vector of the intervehicular force acts at a single point, assumed to be know, called the centre of impact
3. Changes in the position of the centre of mass and in angular orientation in every vehicle are small over the time interval of contact
4. A hypothetical, fixed contact surface is presumed in such a way that motion normal to this surface is due to deformation while motion parallel to this surface has the nature of relative motion corresponding to frictional sliding
5. The time duration of contact is small.

Vehicular collisions typically have contact times less than 0.2 s. Time intervals of this magnitude, coupled with the assumption of large forces cause large accelerations, finite velocity changes and small displacements. All of these considered together, usually, cause the above assumptions to be satisfied for the study of vehicle collisions.

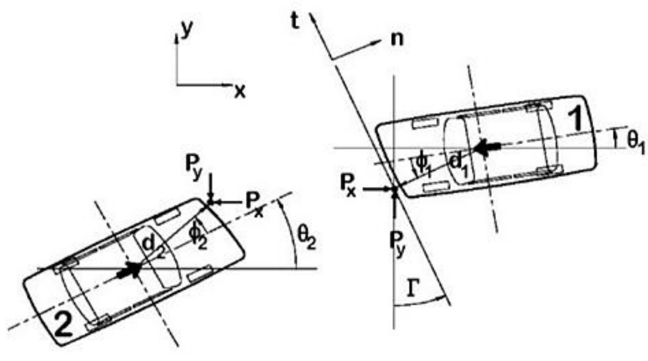


Figure 1. Free body diagram of the collision of two vehicles (Brach, 1983)

In Figure 1, the free body diagram of two vehicles (Vehicle 1 and Vehicle 2) in the collision phase is presented. These vehicles are in contact along the crush surface (line forming angle Γ with y-axis of the global Cartesian Coordinate System (CCS)). Each vehicle has an a priori known mass and inertia, m_i,I_i, respectively. Since the momentum is conserved in each axis of the global CCS Eq. 1 and Eq. 2 are constructed.

$$m_1 \cdot \left(V_{1fx} - V_{1ix}\right) + m_2 \cdot \left(V_{2fx} - V_{2ix}\right) = 0 \tag{1}$$

$$m_1 \cdot \left(V_{1fy} - V_{1iy}\right) + m_2 \cdot \left(V_{2fy} - V_{2iy}\right) = 0 \tag{2}$$

he indices 1 and 2 represent each vehicle while the indices i and f stand for the initial (i.e. start of collision phase) and the final (end of collision phase) values, respectively. These indices are used in all following equations.

In Figure 1 the distances d_1 and d_2 correspond to the distance between the centre of mass of each vehicle and the centre of impact. Furthermore, φ_1 and φ_2 represent the angle between d_1 and d_2 and the longitudinal axis of each vehicle while with θ_1 and θ_2 the angle between the the longitudinal axis of each vehicle and the x-axis of the global CCS is denoted. Using these geometrical quantities and the principle of conservation of angular momentum Eq. 3 is derived.

$$I_1 \cdot \left(\Omega_{1f} - \Omega_{1i}\right) + I_2 \cdot \left(\Omega_{2f} - \Omega_{2i}\right) + m_1 \cdot (d_b + d_d) \cdot \left(V_{1fy} - V_{1iy}\right) + m_2 \cdot (d_a + d_c) \cdot \left(V_{2fx} - V_{2ix}\right) = 0 \tag{3}$$

In Eq. 3 $d_a, d_{b,} d_c, d_d$ are correlated to d_1 and d_2 with the following trigonometrical functions.

$$d_a = d_2 \cdot sin(\theta_2 + \varphi_2) \tag{3a}$$

$$d_b = d_2 \cdot cos(\theta_2 + \varphi_2) \tag{3b}$$

$$d_c = d_1 \cdot sin(\theta_1 + \varphi_1) \tag{3c}$$

$$d_d = d_1 \cdot sin(\theta_1 + \varphi_1) \tag{3d}$$

The following three equations (Eq.4 – 6) are provided considering the impact coefficients. The coefficient of restitution, e, is used to model energy loss due to material deformation in a mode normal or perpendicular to the crush surface.

$$\left(V_{1fy} - d_d \cdot \Omega_{1f} - V_{2fy} - d_b \cdot \Omega_{2f}\right) \cdot sin\Gamma + \left(V_{1fx} - d_c \cdot \Omega_{1f} - V_{2fx} - d_a \cdot \Omega_{2f}\right) \cdot cos\Gamma = e \cdot \left[\left(\left(V_{1iy} - d_d \cdot \Omega_{1i} - V_{2iy} - d_b \cdot \Omega_{2i}\right) \cdot sin\Gamma\right) + \left(V_{1ix} - d_c \cdot \Omega_{1i} - V_{2ix} - d_a \cdot \Omega_{2i}\right) \cdot cos\Gamma\right] \tag{4}$$

The equivalent friction coefficient, μ, corresponds to the ratio of the tangential to normal impulse components which develop between the vehicles. The tangential impulse is typically attributed to and referred to as friction, though shear deformation is probably equally significant (Brach 1987).

$$m_1 \cdot (V_{1fy} - V_{1iy}) \cdot (cos\Gamma - \mu \cdot sin\Gamma) + m_2 \cdot (V_{2fx} - V_{2ix}) \cdot (sin\Gamma + \mu \cdot cos\Gamma) = 0 \quad (5)$$

Finally, the third coefficient, the moment coefficient of restitution, e_m, governs the rotational effects. A value of this coefficient of unity (1) implies that no moment impulse is developed between the vehicles during collision and that the centre of impact is known. Otherwise any value in the range of [-1, 0] implies that the centre of impact is not known (Barch 1987).

$$(\Omega_{1f} - \Omega_{2f}) \cdot (1 - e_m) = -e_m \cdot \left[\left((\Omega_{1f} - \Omega_{1i}) - m_1 \cdot d_c \cdot \frac{(V_{1fx} - V_{1ix})}{I_1} + m_1 \cdot d_d \cdot \frac{(V_{1fy} - V_{1iy})}{I_1} - (\Omega_{2f} - \Omega_{2i}) - m_2 \cdot d_a \cdot \frac{(V_{2fx} - V_{2ix})}{I_2} + m_2 \cdot d_b \cdot \frac{(V_{2fy} - V_{2iy})}{I_2} \right) \right] \quad (6)$$

he Planar Impact Mechanics collision model through the use of these coefficients models the energy loss which is present in all real collisions, since there is loss of kinetic energy, mostly through deformation, friction and vibrational energy. According to the literature (Brach, 1987), typical values of energy loss due to collision range from 25% to 95%.

It is noteworthy that Eq. 1 – 6 correlate (a) six initial velocity components (three for each vehicle – V_{ix}, V_{iy}, Ω_i), (b) six final velocity components (three for each vehicle – V_{fx}, V_{fy}, Ω_f), (c) vehicle inertial properties (I, m) and (d) the collision geometry (d_a, d_b, d_c, d_d, Γ). The aforementioned equations written in a matrix form (Eq. 7) constitute the mathematical collision model (Brach, 1983).

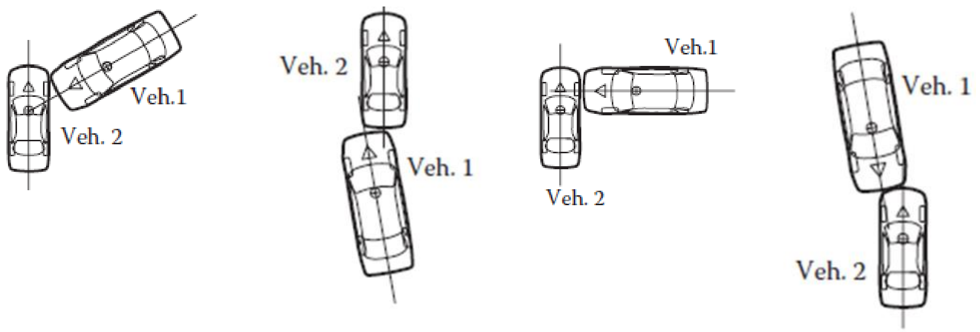
$$A \cdot V_f - C \cdot V_i = 0 \quad (7)$$

2.2 RICSAC database

In the 1970's, while the first computational methods for the simulation of traffic accidents appeared in the literature, the need for a database with fully defined vehicle collisions arose. Within this context a research project named the "Research Input for Computer Simulation of Automobile Collisions" (RICSAC) and funded by the National Highway Traffic Safety Administration (NHTSA), provided the researches with a test matrix of 12 full – scale crash tests. Within this project cameras and accelerometers were attached to the vehicles and a set of staged collision scenarios has been performed resulting into a test matrix of 12 crashes. For each RICSAC test, at least 13 accelerometers were mounted on each vehicle in order to monitor the acceleration components. At three locations, triaxial (XYZ) packages were installed to provide coverage between the front and the rear of the vehicle. The front steer angles were measured on each vehicle by a linear stroke potentiometer attached to the vehicle steering linkage. The time history of the change in vehicle yaw, pitch and roll angles and yaw rate were recorded by two degrees of freedom, free gyroscopes and rate gyro (McHenry, 1987). The final test reports include, also, objective information on the impact speeds, vehicle weights, vehicle dimensions, weight distributions, spin-out trajectories and positions of rest.

RICSAC test database contains vehicle collisions engaging six different vehicles included in four categories of vehicle sizes. The different vehicles used in RICSAC tests are namely (V1) Chevrolet Chevelle, (V2) Ford Pinto, (V3) Ford Torino, (V4) Honda Civic, (V5) VW Rabbit and (V6) Chevrolet Vega. The tests can be classified into four impact configurations (IC1 – IC4) (Figure 2) according to the relative orientation of the vehicles at the time of collision. In the IC1 belong the Tests no. 1, 2, 6 and 7, in the IC2 belong the Tests no. 3, 4 and 5, in IC3 belong the Tests no. 8, 9 and 10 and in IC4 belong the Tests no. 11 and 12. Each crash test involved vehicles of different size categories, except for Test no. 8, which involved two intermediate vehicles. In the front-to-rear collisions (Figure 2b) the car struck in the rear was stopped while in all other tests, both cars were moving.

In the model of structure-borne noise, the engine emission, the properties of its design are integrated into an equivalent cylindrical shell, for which the oscillatory characteristics are known. Equivalence conditions are: equality of mass, length and area of the outer surface of the engine and of such a shell [3].



(a) (b) (c) (d)

Figure 2. (a) IC1 - Front corner to corner at 60 o, (b) IC2 - Rear offset oblique at 10 o (c) IC3 - Side perpendicular offset and (d) IC 4 - Frontal offset oblique at 10o (Struble, 2013)

In Table 1, that follows, the measured velocity components of both vehicles for each RICSAC test are presented. This and the following tables are organized per IC and the data of RICSAC test no. 2 are not included due to loss of experimental measurements.

Table 1. Linear and angular velocity for each vehicle for all RICSAC tests

		Impact Configuration (IC)										
		1			2			3			4	
RISAC	Units	1	6	7	8	9	10	3	4	5	11	12
Vi1x	m/s	-8.95	-9.61	-13.01	-9.3	-9.48	-14.89	-9.48	-17.30	-17.75	-9.12	-14.8
Vi1y	m/s	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-4.28
Ωi1	rad/s	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Vi2x	m/s	4.43	4.66	6.50	0.00	0.00	0.00	0.00	0.00	0.00	8.98	13.87
Vi2y	m/s	7.67	8.32	11.27	9.3	9.48	14.89	0.00	0.00	0.00	-1.58	2.44
Ωi2	rad/s	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Vf1x	m/s	-3.76	-5.69	-7.74	-3.12	-0.86	-1.55	-5.34	-8.94	-10.46	1.77	4.28
Vf1y	m/s	2.41	1.26	1.48	3.27	4.52	8.59	-0.32	-0.44	0.17	0.62	-0.49
Ωf1	rad/s	-1.57	-0.52	-0.52	-1.99	-3.14	-5.24	-0.42	-0.65	-0.21	0.52	1.57
Vf2x	m/s	-2.07	-1.28	-2.22	-3.66	-3.02	-4.44	-6.73	-9.92	-11.32	1.96	1.93
Vf2y	m/s	5.17	5.49	8.64	6.01	7.38	11.44	1.14	0.42	0.84	-1.26	-2.94
Ωf2	rad/s	0.00	-3.14	-3.35	-0.31	0.79	1.26	-0.42	-0.52	-1.22	0.00	1.05

In Table 2 the geometrical properties of each collision are presented, i.e. the crush angle (Γ) for each IC along with the initial heading angle (θi) and the angle to center of collision (φi). Both angles θi and φi are presented for each vehicle while angle φi is also presented for each RICSAC.

Table 2. Crush angle per RICSAC, initial heading angle and angle to center of collision per vehicle and RICSAC

		Impact Configuration (IC)										
		1			2			3			4	
RISAC	Units	1	6	7	8	9	10	3	4	5	11	12
Γ	deg	-30.0			0.0			-10.0			0.0	
θ1		0.0			0.0			0.0			-10.0	
θ2		60.0			90.0			170.0			0.0	
φ1		-19.8	-17.9	17.9	0.0	6.0	0.0	-17.0	-18.2	-20.7	9.4	9.6
φ2		-38.7	-90.0	-90.0	-68.8	-29.7	-29.2	171.4	171.7	-168.0	11.3	10.3

2.3 Least squares method and minimization problem

As mentioned above, the least squares method is utilized as a means of retrieving a combination of unknown parameters in a way that the equations of the Planar Impact Mechanics model are satisfied and the specified velocity components are closely matched to the estimated ones. The assumed values of the velocity components may result from monitoring devices mounted on the vehicle and/or physical evidence. Such a monitoring device installed in, more vehicles as time progresses, is the Event Data Recorder (EDR). Thus, in the least squares method the velocity components have been included, since an estimate of their value might be available.

In order to utilize the least squares method the vector of all the velocity components included in Eq. 7 is renamed to the vector U in the following way:

$$\{U_1 \quad U_2 \quad U_3 \quad U_4 \quad U_5 \quad U_6 \quad U_7 \quad U_8 \quad U_9 \quad U_{10} \quad U_{11} \quad U_{12}\}^T = \{V_{i1x} \quad V_{i1y} \quad \Omega_{i1} \quad V_{i2x} \quad V_{i2y} \quad \Omega_{i2} \quad V_{f1x} \quad V_{f1y} \quad \Omega_{f1} \quad V_{f2x} \quad V_{f2y} \quad \Omega_{f2}\}^T \tag{8}$$

In order to evaluate the effect of the number of the known and/or estimated velocity components, Eq. 9 – 11 have been set up.

$$Q_1 = \sum_{k=1}^{6} w_k \cdot (U_k - U_k^{est})^2 \tag{9}$$

$$Q_2 = \sum_{k=7}^{12} w_k \cdot (U_k - U_k^{est})^2 \tag{10}$$

$$Q_3 = \sum_{k=1}^{12} w_k \cdot (U_k - U_k^{est})^2 \tag{11}$$

In all three equations w_k, are weighting factors in the range of (0,1] allowing for the definition of different confidence levels for each estimate. In the present paper all w_k have been considered equal to unity stating that all the estimates have the same level of confidence. The experimental estimates U_k^{est} are considered having the corresponding values available in Table 1, i.e. the measured values available in the RICSAC database.

In Q1 (Eq. 9) all the final velocity components have been considered exactly know and their values, equal to those appearing in Table 1, are used in Eq.7. As far as the initial velocity components are concerned, estimates have been considered available, and they were treated as unknowns in Eq.7. On the contrary, in Q2 (Eq. 10), all the initial velocity components have been considered exactly know (Table 1), while experimental estimates have been considered available for the final velocity components. Finally, in Q3 (Eq.11), no velocity component has been considered a priori known, but estimates are considered to exist for all of them. For all three equations, the geometrical properties of the collisions are considered exactly known and having the values appearing in Table 2 while for the impact coefficients (e, μ and em) no experimental values have been considered available.

In order to retrieve the impact coefficients (e, μ and em) these three equations (Eq. 9 – 11) have been coupled with an SQP based optimization subroutine (fmincon) available in Matlab ® in order to achieve the minimization of each Q, forming an equal number of optimization problems.

$$FC_i = \min(Q_i)\,, i = 1 - 3 \tag{12}$$

Each optimization procedure (Eq. 12) provides the values of the design variables, being the three impact coefficients (e, μ and e_m), which minimizes Q_i with U_k values calculated via the least squares method. The Planar Impact Mechanics collision model are included in the optimization procedure in the form of non-linear constraints. Other active constraints of the optimization procedure are the boundary values of the velocity components and these of the design variables. In more details, the linear velocities have been considered in the range of [-20, 20] m/s and the angular velocities have been considered in the range of [-5, 5] rad/s. Respectively, the ranges of the design variables have been considered as: $e \in [0,0.2]$,

$e_m \in [-1,0]$ and $\mu \in [0,1.2]$. The constraints are imported in the optimization procedure through the augmented Lagrange function (L).

3. RESULTS

In the results section the outcome of all three optimization procedures is presented in terms of minimum values of cost function (FCi).

In order to investigate the feasibility of the solution of each minimization procedure along with the value of the cost function, also the values of the derivative of the augmented Lagrange function (∇L) and the vector of the linear constraints (C) are monitored. Moreover, the optimized values of the impact coefficients are introduced and the results regarding the velocity component values are presented in terms of absolute error with respect to the measured, corresponding values. All results are organized per IC and cost function.

In Figure 3 the results of the optimization procedure, for all RICSAC tests, is presented. Each bar represents a different cost function FC_i . It is obvious that FC_2 , which considers the initial velocity components as known quantities provides considerably higher values of cost function in tests 1, 6, 9, 10 and 11. For the rest of RICSAC tests the minimum value of all FC_i is almost the same regardless cost function.

In Table 3, for tests 1, 6, 7, 3, 5 and 11, can be observed that the design variable vector which minimizes FC2 leads to the violation of the constraints. This violation is quantified through the values of the derivative of the augmented Lagrange function (∇L) which has a value significantly greater than 0 in the tests 1, 6, and 11. The value of C is significantly greater than 0 in tests 1, 6, 7, 3 and 5.

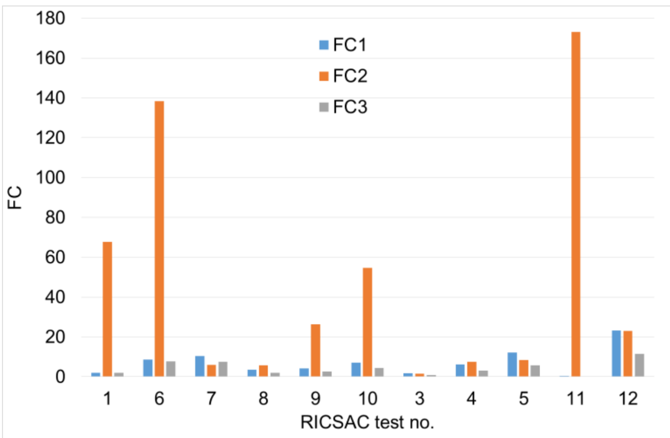


Figure 3. Value of cost function for all cases at the end of the optimization procedure

Table 3. Constraint satisfaction for all optimization problems per RICSAC test

	IC1						**IC2**					
	RICSAC1		**RICSAC6**		**RICSAC7**		**RICSAC8**		**RICSAC9**		**RICSAC10**	
	∇L	**C**	∇L	**C**	∇L	**C**	∇L	**C**	∇L	**C**	∇L	**C**
F1	0.001	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.002	0.002
F2	102.570	0.732	0.122	2.620	0.000	2.830	0.000	0.000	0.000	0.000	0.000	0.003
F3	0.001	0.000	0.002	0.000	0.000	0.000	0.000	0.000	0.001	0.000	0.002	0.000

	IC3						**IC4**			
	RICSAC3		**RICSAC4**		**RICSAC5**		**RICSAC11**		**RICSAC12**	
	∇L	**C**	∇L	**C**	∇L	**C**	∇L	**C**	∇L	**C**
F1	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000
F2	0.005	0.260	0.003	0.000	0.000	1.010	23.060	0.003	0.004	0.001
F3	0.001	0.000	0.000	0.000	0.001	0.000	0.000	0.000	0.001	0.000

In the following Tables 4, 5 and 6 the absolute error values for each velocity component and every RICSAC test for all three cost functions (FC_1 – FC_3) are presented.

Table 4. Absolute error for all velocity components in m/s for each vehicle for all RICSAC tests for the cost function FC_1

		Impact Configuration (IC)										
		1			**2**			**3**			**4**	
	RISAC	**1**	**6**	**7**	**8**	**9**	**10**	**3**	**4**	**5**	**11**	**12**
Absolute error	**Vf1x**	0.25	-0.61	-1.18	1.07	1.01	1.50	0.08	1.63	1.13	0.10	-0.18
	Vf1y	-1.23	-0.47	-0.85	-0.22	0.76	0.33	0.40	0.52	0.92	-0.07	0.99
	Ωf1	0.35	1.52	2.17	0.13	-1.44	-1.13	0.36	0.86	1.01	0.17	-0.97
	Vf2x	0.94	1.48	0.38	1.20	0.49	1.35	-0.29	0.57	-0.12	0.13	-0.68
	Vf2y	0.71	1.53	1.75	0.39	-0.33	-0.61	0.00	1.07	1.13	0.67	-2.44
	Ωf2	0.84	-1.10	-0.66	0.90	-0.41	-1.09	0.36	0.99	0.00	0.17	0.21

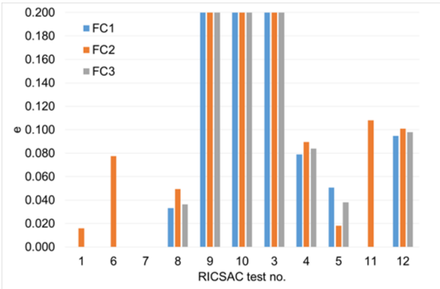
Table 5. Absolute error for all velocity components in m/s for each vehicle for all RICSAC tests for the cost function FC_2

		Impact Configuration (IC)										
		1			**2**			**3**			**4**	
	RISAC	**1**	**6**	**7**	**8**	**9**	**10**	**3**	**4**	**5**	**11**	**12**
Absolute error	**Vi1x**	-4.16	-5.56	0.34	-1.00	2.43	4.71	0.10	-1.68	-1.07	-7.25	-0.16
	Vi1y	2.07	1.79	0.48	0.03	2.25	1.86	0.84	0.68	1.21	0.62	-0.99
	Ωi1	2.67	1.68	1.79	0.42	3.15	2.78	0.79	0.30	2.06	1.11	1.98
	Vi2x	-3.92	-0.68	-0.81	1.27	2.07	4.38	0.08	0.49	0.01	-7.09	0.69
	Vi2y	-2.00	-3.69	-1.21	-0.15	1.01	1.36	0.08	1.32	1.06	-0.32	-1.44
	Ωi2	3.24	3.61	0.65	1.73	0.46	0.49	0.79	1.49	0.61	0.06	2.41

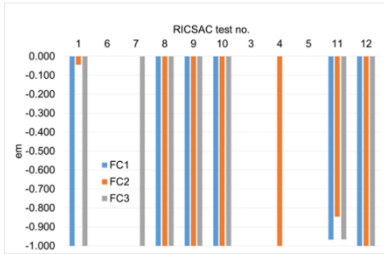
Table 6. Absolute error for all velocity components in m/s for each vehicle for all RICSAC tests for the cost function FC_3

		Impact Configuration (IC)										
		1			**2**			**3**			**4**	
	RISAC	**1**	**6**	**7**	**8**	**9**	**10**	**3**	**4**	**5**	**11**	**12**
Absolute error	**Vi1x**	-0.34	-0.33	0.13	-0.53	-0.25	-0.36	0.03	-0.81	-0.56	0.03	-0.09
	Vi1y	0.47	0.14	0.14	0.09	0.45	0.24	0.41	0.25	0.63	0.03	-0.49
	Ωi1	0.44	0.39	1.07	0.15	0.46	0.35	0.45	0.60	1.30	0.08	0.89
	Vi2x	0.23	0.09	-0.29	0.61	0.39	0.95	0.11	0.29	0.04	-0.08	0.34
	Vi2y	-0.20	-0.37	-0.57	-0.18	0.20	0.34	0.02	0.53	0.57	-0.33	-1.71
	Ωi2	0.47	0.19	0.20	0.39	0.22	0.01	0.23	0.30	0.38	0.08	1.32
	Vf1x	-0.02	-0.86	-1.00	0.53	0.38	0.69	0.00	0.81	0.56	0.06	-0.09
	Vf1y	-0.68	-0.54	-0.64	-0.09	0.45	0.24	0.00	0.25	0.29	-0.03	0.49
	Ωf1	-0.05	1.23	1.39	-0.08	-1.10	-0.92	0.00	0.38	-0.08	0.09	-1.11
	Vf2x	0.60	1.27	0.58	0.59	0.27	0.62	0.00	0.29	-0.04	0.06	-0.34
	Vf2y	0.41	1.06	1.08	0.18	-0.20	-1.06	0.00	0.53	0.57	0.33	-1.71
	Ωf2	0.45	-1.39	-1.06	0.61	-0.61	-1.07	0.00	0.51	-1.09	0.08	-1.00

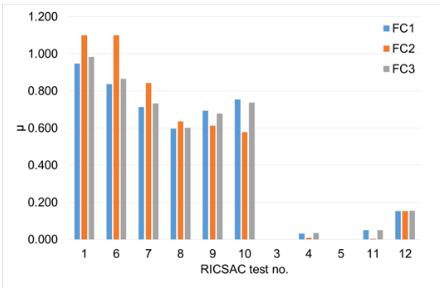
The maximum absolute error for the use of cost function FC_1 is -2.44 m/s and it appears in RICSAC test 12 in the velocity component Vf2y. If FC_2 is used then the maximum absolute error is -7.25 m/s and it appears in RICSAC 11 in the Vi1x. Finally, the maximum absolute error using FC_3 is -1.71 m/s and it appears again in RICSAC 11 in the velocity component Vi2y. In Figure 4 the values of the impact coefficients are presented in the form of bar diagrams. For all impact coefficients, the vertical axis represent the applicable range of values while the horizontal axis is organized per RICSAC test. The different color in bars indicates use of different cost function, FC_i. Lack of a bar indicates that using the corresponding cost function FC_i the optimization procedure converges for the minimum value of this impact coefficient. In RICSAC tests 9, 10 and 3 the coefficient of restitution (Figure 4a) reached its maximum allowable value regardless cost function. In the rest of the tests every cost function provides different value for the coefficient of restitution. In all RICSAC tests, except RICSAC 5 FC_2 provides the lowest value of coefficient of restitution. Furthermore, FC_3 provides higher value for the coefficient of restitution compared to FC_1 with the exception of RICSAC 5. The value of the moment coefficient of restitution (Figure 4b) is more uniform. In all RICSAC tests except 1, 7, 4 and 11 all cost functions provide the same value for this coefficient which is either its lower (6, 3, 5) or its upper bound (8, 9, 10, 12). As far as the values of the equivalent coefficient of friction (Figure 4c) is concerned they show significant non-uniformity. Nevertheless in RICSAC tests belonging to IC1, FC_2 seems to provide the highest values, followed by FC3. In RICSAC tests 9 and 10 the opposite seems to happen, while in tests 3, 5 and 12 all cost functions provide the same value of friction coefficient.



(a)



(b)



(c)

Figure 4. Optimized value of the (a) restitution coefficient e, (b) moment coefficient of restitution em and (c) coefficient of friction μ for all RICSAC and all cost functions

4. DISCUSSION

In Figure 3 is obvious that FC_3 provides the overall minimum value for all RICSAC tests, while FC_2 provides the highest value in most RICSAC tests. The fact that FC_3 provides results of better quality is also obvious in Table 6 where the absolute error for all velocity components are presented. In Table 3 where the values of ∇L and C are presented, is obvious that FC_2 leads to violation of the constraints in tests 1, 6, 7, 3, 5 and 11, meaning that Eq. 7 is not satisfied, i.e. these solution vectors cannot be taken under consideration.

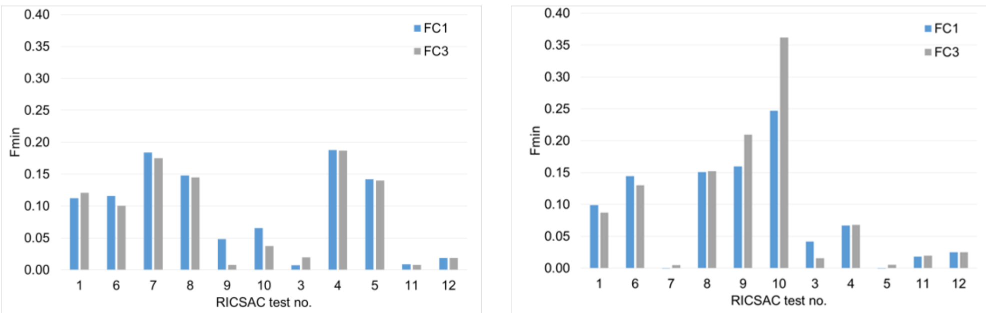
In order to quantify, in more details, the quality of each solution the error of the change of velocity in every vehicle is also considered. The components of the vehicle's change of velocity (ΔV_i) were computed by subtracting the initial velocity at impact from the velocity at the time of separation. The procedure was performed for the X and Y velocity component separately. The measured change in velocity ΔV is provided in the literature (Brach, 1982) as an overall measure of the severity of a collision. In Table 7 the measured velocity changes are presented per IC and RICASC test.

In Figure 5 the relative computed velocity changes are presented for Vehicle 1 (Figure 5a) and Vehicle 2 (Figure 5b) for the cost functions FC_1 and FC_3. The relative computed velocity change was calculated as the difference between the measured and the computed velocity change divided by the measured one. FC2 was omitted for Figure 5 since, as it was discussed earlier, it leads to solution vectors that may violate both the linear and the non-linear constraints.

The accuracy of both cost functions as far as the velocity change is concerned is fairly good. The relative computed velocity change ranges from 0 to 25% for FC_1 and from 0 to 36% for FC_3. The maximum value of relative computed velocity change appears for both cost functions in RICSAC test 10 and Vehicle 2. In general, the values of the relative computed velocity change are larger in Vehicle 2 where 36% is met, than in Vehicle 1. The highest value Vehicle 1 is 19%.

Table 7. Absolute measured ΔV for each vehicle and RICSAC test

IC1	**RICSAC**	**1**		**6**		**7**	
	Vehicle	**1**	**2**	**1**	**2**	**1**	**2**
	Measured ΔV (m/s)	5.72	6.96	4.12	6.58	5.47	9.11
IC2	**RICSAC**	**8**		**9**		**10**	
	Vehicle	**1**	**2**	**1**	**2**	**1**	**2**
	Measured ΔV (m/s)	6.99	4.92	9.73	3.68	15.87	5.81
IC3	**RICSAC**	**3**		**4**		**5**	
	Vehicle	**1**	**2**	**1**	**2**	**1**	**2**
	Measured ΔV (m/s)	4.15	6.83	8.37	9.93	7.29	11.35
IC4	**RICSAC**	**11**		**12**			
	Vehicle	**1**	**2**	**1**	**2**		
	Measured ΔV (m/s)	10.91	7.03	18.75	13.10		



(a) (b)

Figure 5. Relative velocity change for cost functions 1 and 3 for vehicle (a) 1 and (b) 2

As far as the impact coefficients are concerned, in Figure 4, is obvious that only equivalent friction coefficient depends on the cost function used. Both the restitution coefficient, and the moment coefficient of restitution demonstrate a dependence on the test conditions. Furthermore, the moment coefficient of restitution in most tests obtains one of its boundary values.

5. CONCLUSIONS

In the present paper the Planar Impact Mechanics collision model has been implemented in Matlab® and coupled with the least squares method. Three different cost functions based on least squares method have been minimized with the deterministic optimization method of SQP in order to calculate the velocity components of two vehicles in collision and provide the impact coefficients which verify the PIM collision model. The presented methodology is a way to utilize EDR data (Brach, 2011). Comparing three scenarios through different cost functions it was indicated that the most reliable results where produced when estimates for all velocity components were available (FC3).

The values of absolute error of the velocity components are comparable for all RICSAC tests and all cost functions, but are quite high. It is worth mentioning that a concern has risen in the past, with respect to the accuracy of the velocity components documented in RICSAC reports. In 1997 a re-evaluation of the provided data has been proposed, since it was realized that the measuring devices were not placed on the centre of mass of each vehicle (McHenry and McHenry, 1997). Furthermore, in the reports of the RICSAC tests (Jones, 1978) it is acknowledged that the value of the separation velocity in all tests was contaminated by the effects of rotation of the vehicles between impact and separation. The abovementioned facts influence the performed analysis within this study, in terms of relative values, since the re-evaluated values have not been taken under consideration. Finally, as far as the impact coefficients are concerned, they were successfully computed regardless cost function, with slight differences in most RICSAC tests for both the moment coefficient of restitution and the coefficient of restitution. On the other hand each cost function leads to a different value for the friction coefficient, leading to the conclusion that this is the more decisive impact coefficient.

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