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DYNAMIC STABILITY OF A MODEL OF A TRACTOR-LORRY-TRAILER COMBINATION

Summary. In this paper, we consider the mobility and stability of a model of a tractor-lorry-trailer combination, consisting of a two-axle tractor and a single-axle semi-trailer, and its possible stationary states with fixed steering. A feature of this research is the study of a non-linear mathematical model of a two-link tractor-lorry-trailer combination. The set of steady-state conditions for the movement of the tractor-lorry-trailer combination model is determined on the basis of the developed mathematical model; it provides the necessary mobility for the passage of the circular overall traffic lane. The range of stable steady-state conditions of the road train is limited and the character of the loss of stability in the direct motion of the road train (divergent, flutter) is checked. The phase portraits of the system are constructed at different speeds, which allow us to estimate the range of attraction for direct motion. Stability issues are also considered, namely, the influence of the control parameters (θ, v) on stability or instability.

Keywords: dynamic stability; mobility of tractor-lorry-trailer combination; steady-state condition

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1. INTRODUCTION

Fast and economical delivery of indivisible wide loads in many industries is becoming increasingly important [1]. Road trains play a major role in solving this problem [2-5]. Their operation can be complicated by design features, primarily the limited manoeuvrability of long road trains in restricted urban conditions [6-8].

In this case, a double (two-trailer) road train is considered, which consists of a leading link (tractor) and a driven link (semitrailer).

The mathematical model of the canonical road train is the object of research for many authors, where the results of analysis of linearized models are mainly presented [9-13].

The specificity of this paper is the study of the non-linear mathematical model of a double road train, whose aim is to find possible rearrangements of the road train configuration in different initial disturbances of phase variables. This requires non-standard analysis methods (phase portrait construction and evaluation of the domain of attraction for stable steady-state regimes).

The purpose of the paper is as follows:

a) Investigate the manoeuvrability and stability of the model of a double road train

b) Determine a set of stationary traffic conditions, which provides the necessary manoeuvrability by passing a circular overall lane

c) Estimate domains of attraction for a stable steady-state regime

d) Check the type of stability loss in direct motion (divergent, flutter)

2. CONSTRUCTION OF A MATHEMATICAL MODEL FOR SEMI-TRAILER TRUCK MOTION

For the most complete description of and research into possible stationary states of a semi-trailer truck (Fig. 1) with rigid steering, it is necessary to choose a suitable mathematical model and applicable state variables (Fig. 2).

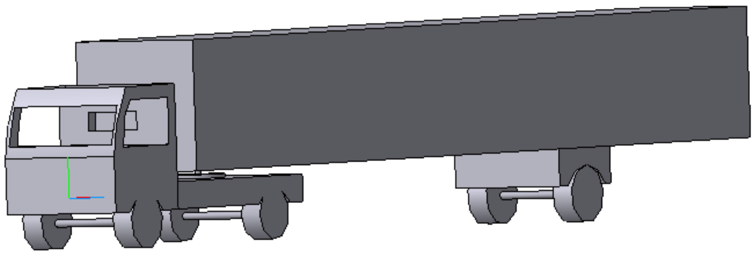

Fig. 1. Model of semi-trailer truck

The front axle of the tractor can be turned by an angle θ. The connection between the links is carried out by a cylindrical hinge, which enables the free relative rotation of the links in the plane of motion.

The configuration of each link is described by coordinates x_i and y_i, its centre-of-mass C_i and course angle ψ_i (it is enclosed between the longitudinal axis of the corresponding link and the X-axis of the fixed coordinate system).

The system parameters are as follows:

v - longitudinal velocity component of the centre of mass of the tractor

a; b - distance from the centre of mass of the tractor to the attachment point of the front and back axles of the tractor
c - distance from the centre of mass of the tractor to the hitch point with the back link
d_1 - distance from the centre of mass of the back link to the hitch point
2K - overall width of the road train
k_f - friction coefficient
k_1, k_2, k_3 - the factors influencing lateral skid on the axes
χ_1, χ_2, χ_3 - adhesion factors in determining the force of lateral skid
θ - assignable wheels' turning angle for the subordinate module
Y_1, Y_2, Y_3 - lateral reaction of the highway area

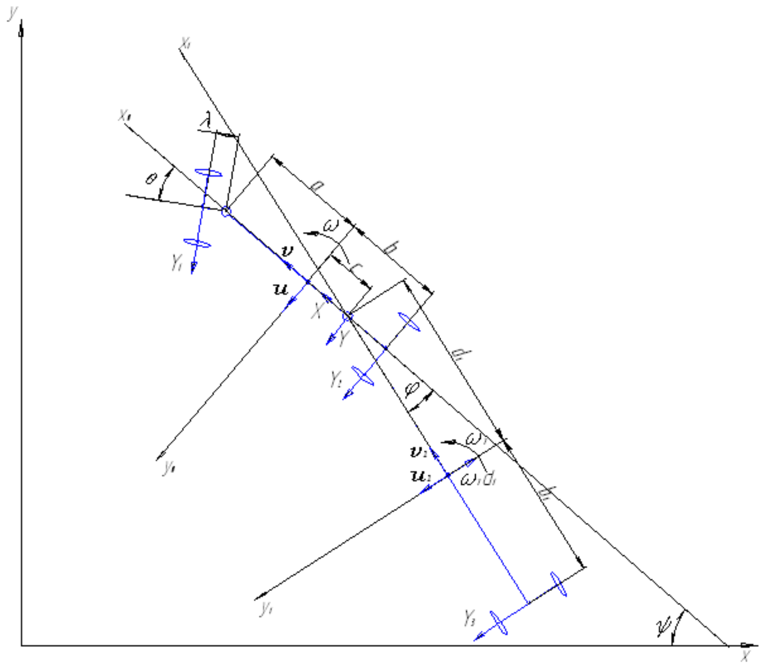

Fig. 2. Traffic plan of a semi-trailer truck

If we assume that C, C_1 are the mass centres of the tractor and semi-trailer), m, m_1 are the masses of the tractor and semi-trailer, I, I_1 are the central moments of inertia about the vertical axes, $\omega=\psi'$, $\omega_1=\psi_1'$ are absolute angular velocities of the driving and driven links, and φ is the angle of folding (it is enclosed between the longitudinal axes of the tractor and semitrailer), then $\omega_1 = \omega - \dot{\varphi}$.

We set the absolute velocities of points C, C_1 by their resolution along the axes of the corresponding bases:

$$\begin{aligned} &\mathrm{v}_C = i \cdot v + j_0 u & &\mathrm{v}_{C1} = i_1 \cdot v_1 + j_1 u_1 \\ &\mathrm{v} = \dot{x} \cdot \cos\psi + \dot{y} \cdot \sin\psi & &\mathrm{u} = -\dot{x} \cdot \sin\psi + \dot{y} \cdot \cos\psi \\ &\mathrm{v}_1 = v \cdot \cos\varphi - (u - \omega \cdot c) \cdot \sin\varphi & &\mathrm{u}_1 = v \cdot \sin\varphi + (u - \omega \cdot c) \cdot \cos\varphi \end{aligned} \tag{1}$$

The differential equation system of motion for the semi-trailer truck describes the variation in phase variables (u, ω ,φ, Φ), where: u - cross speed of the centre of mass of the tractor (quasi-velocity); U - its derivative in the moving coordinates; ω - angular acceleration relative to the vertical axis; and Φ - velocity of jack-knifing angle φ.

Among the different theories for the rolling of elastically deformable wheels, the field of axiomatics has become the most widespread, according to which the lateral reaction Y_i of the highway area is applied in tooth bearing centre of the rolling elastic wheel, which is a function of slip angle δ_i.

The reduced angles of lateral skid of the wheel axles are given by the following expressions:

$$\delta_1 = \theta - arctg\frac{(u + a\omega)}{v};$$
$$\delta_2 = arctg\frac{-u + b\omega}{v}; \tag{2}$$
$$\delta_3 = arctg\frac{-u_1 + b_1\omega_1}{v_1}.$$

The dependencies of the forces of lateral skid are of empirical origin [2] and can be approximated by expressions (the strictly increasing function is the rate of the curve of saturation):

$$Y_i = k_i\delta_i / \sqrt{(1 + (k_i\delta_i / \chi_i Z_i)^2}, \tag{3}$$

where Z_i is the reaction of the bearing area on the axes.

We neglect the redistribution of normal reactions between the lateral wheels and instead consider the lateral wheels of each axis that is replaced by one reduced wheel with a centre in the middle of the axis:

Then:

$$Z_1 = \frac{1}{l}[mgb - m_2 g\frac{b_1}{L_1}(c - b);$$
$$Z_2 = \frac{1}{l}[mga + m_2 g\frac{b_1}{L_1}(c + a)]; \tag{4}$$
$$Z_3 = m_2 g\frac{d_1}{L_1};\quad l = a + b;\ L_1 = d_1 + b_1 .$$

2.1. The derivation of the system of equations in the normal Cauchy form

The derivation of the differential equations for the plane-parallel motion of a semi-trailer truck is performed by the cut set method [4].

Using this method, we obtain the following equations for the plane-parallel motion, which in axial projections are invariably associated with links for the tractor and semi-trailer, respectively:

a) The motion equations of the tractor are:

$$\mathrm{m}\cdot(\mathrm{V} - \mathrm{u}\cdot\omega) - X + Y_1\cdot\sin\theta = 0;$$
$$\mathrm{m}\cdot(\mathrm{U} + \mathrm{v}\cdot\omega) - Y - Y_1\cdot\cos\theta - Y_2 = 0; \tag{5}$$
$$\mathrm{J}\cdot\Omega + Y\cdot c - Y_1\cdot a\cdot\cos\theta + Y_2\cdot b = 0.$$

b) The motion equations of the trailer are:

$$\begin{aligned}
&m_1 \cdot (V_1 - u_1 \cdot \omega_1) + X \cdot \cos\varphi - Y \cdot \sin\varphi = 0;\\
&m_1 \cdot (U_1 + v_1 \cdot \omega_1) + X \sin\varphi + Y \cdot \cos\theta - Y_3 = 0;\\
&J_1 \cdot \Omega_1 + X \cdot d_1 \cdot \sin\varphi + Y \cdot d_1 \cdot \cos\varphi + Y_3 \cdot b_1 = 0.
\end{aligned} \tag{6}$$

We eliminate the internal forces X, Y of the interaction of subsystems from Eqs. (5) and (6), and we obtain a system of non-linear differential equations in (7):

a) With variable quantity v:

$$m \cdot (V - u \cdot \omega) - Y_3 \cdot \sin\varphi + m_1 \cdot d_1 \cdot PP \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Omega \cdot \sin\varphi + m_1 \cdot d_1 \cdot \Phi^2 \cdot \cos\varphi -$$
$$- 2 \cdot m_1 \cdot d_1 \cdot \Phi \cdot \omega \cdot \cos\varphi + m_1 \cdot d_1 \cdot \omega^2 \cdot \cos\varphi + \omega \cdot u \cdot m_1 + V \cdot m_1 + \omega^2 \cdot m_1 \cdot c + Y_1 \cdot \sin\theta = 0;$$

b) With variable quantity u:

$$\begin{aligned}
&m \cdot (U + v \cdot \omega) + 2 \cdot m_1 \cdot d_1 \cdot \Phi \cdot \omega \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Phi^2 \cdot \sin\varphi - m_1 \cdot d_1 \cdot \omega^2 \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Omega \cdot \cos\varphi +\\
&+ m_1 \cdot d_1 \cdot PP \cdot \cos\varphi + U \cdot m_1 - Y_3 \cdot \cos\varphi - \Omega \cdot m_1 \cdot c + \omega \cdot v \cdot m_1 - Y_1 \cdot \cos\theta - Y_2 = 0;
\end{aligned} \tag{7}$$

c) With variable quantity ω:

$$J \cdot \Omega - (2 \cdot m_1 \cdot d_1 \cdot \Phi \cdot \omega \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Phi^2 \cdot \sin\varphi - m_1 \cdot d_1 \cdot \omega^2 \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Omega \cdot \cos\varphi) \cdot c +$$
$$+ (m_1 \cdot d_1 \cdot PP \cdot \cos\varphi + U \cdot m_1 - Y_3 \cdot \cos\varphi - \Omega \cdot m_1 \cdot c + \omega \cdot v \cdot m_1) \cdot c - Y_1 \cdot a \cdot \cos\theta + Y_2 \cdot b = 0;$$

d) With a jack-knifing angle:

$$J_1 \cdot (\Omega - PP) + (Y_3 \cdot \sin\varphi - m_1 \cdot d_1 \cdot PP \cdot \sin\varphi + m_1 \cdot d_1 \cdot \Omega \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Phi^2 \cdot \cos\varphi) \cdot d_1 \cdot \sin\varphi +$$
$$+ (2 \cdot m_1 \cdot d_1 \cdot \Phi \cdot \omega \cdot \cos\varphi - m_1 \cdot d_1 \cdot \omega^2 \cdot \cos\varphi + \omega \cdot u \cdot m_1 - V \cdot m_1 - \omega^2 \cdot m_1 \cdot c) \cdot d_1 \cdot \sin\varphi -$$
$$- (2 \cdot m_1 \cdot d_1 \cdot \Phi \cdot \omega \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Phi^2 \cdot \sin\varphi - m_1 \cdot d_1 \cdot \omega^2 \cdot \sin\varphi - m_1 \cdot d_1 \cdot \Omega \cdot \cos\varphi) \cdot d_1 \cdot \cos\varphi +$$
$$+ (m_1 \cdot d_1 \cdot PP \cdot \cos\varphi + U \cdot m_1 - Y_3 \cdot \cos\varphi - \Omega \cdot m_1 \cdot c + \omega \cdot v \cdot m_1) \cdot d_1 \cdot \cos\varphi + Y_3 \cdot b_1 = 0.$$

Consider a uniform motion, then v=const; therefore, V=0. We substitute into the system of equations in (7) the value V=0 and solve it in relation to the higher derivative (U, PP, Ω), where PP is the angular acceleration of the driven link relative to the vertical axis.

We get a system of equations in the normal Cauchy form (8):

$$\begin{aligned}
&U = U(u, \omega, \varphi, \Phi);\\
&\Omega = \Omega(u, \omega, \varphi, \Phi);\\
&PP = PP(u, \omega, \varphi, \Phi).
\end{aligned} \tag{8}$$

3. NUMERICAL ANALYSIS RESULTS OF THE MATHEMATICAL MODEL OF THE ROAD TRAIN

Applying the numerical methods of integration in the Maple package, we get the following numerical analysis results for the mathematical model of a road train:

1. Locating a road train under circular steady-state conditions of motion

Circular trajectories of all points of a double road train on a road plane meet the stationary solutions (that is, the equilibrium state, singular points and rest points), in which ω=const, u=const, φ=const of the system with v=const, and θ=const.

Setting the system parameters:

m=6,500 kg; m_2=36,500 kg; a=0.4 m; b=3.2 m; c=2.7 m; b_1=2.8 m; d_1=5.4 m;

v=4.5 m/s; θ=0.38; **k_1=160,000 H; k_2=326,000 H; k_3=365,000 H;**

$J=0.35\cdot m\cdot a\cdot b$ **($kg\cdot m^2$);** $J_2=0.8\cdot m_1\cdot d_1\cdot b_1$ **($kg\cdot m^2$);** χ_i=0.8; K=1.5 m

The given parameters and values of the control parameters v and θ are substituted in the systems of equations in (8), which we then solve to obtain the following results.

Thus, the circular steady-state conditions correspond to the values of v=4.8 m/s and θ=0.42 rad; the trajectory of the tractor's centre-of-gravity motion in the plane of the road and the position of the tractor are shown in Fig. 3.

We obtain a circular stationary regime with v=5 m/s and θ=0.36 rad, as shown in Fig. 4.

The attitude of the road train is shown when moving along a circular corridor in Figs. 3 and 4, limiting the dimensions of which correspond to EU standards. It can be seen that, for the given control parameters, in the first case, the semi-trailer and, in the second case, the tractor and semi-trailer go beyond the dimensions of the corridor.

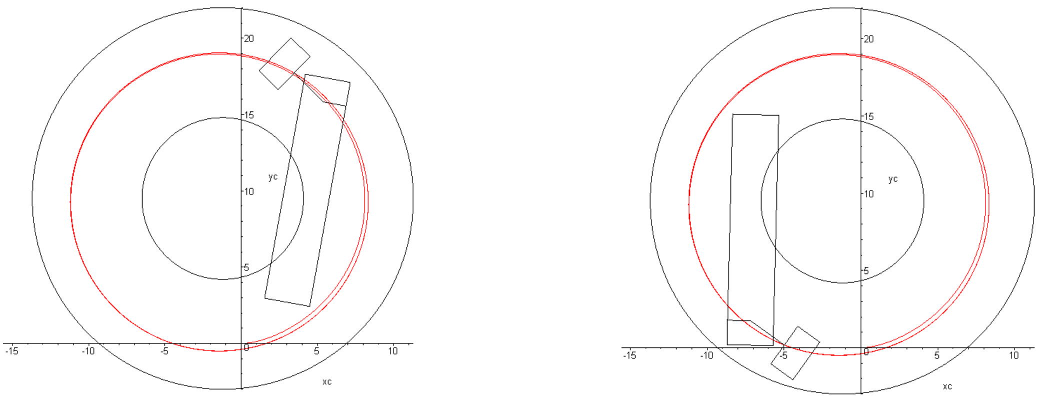

Fig. 3. Trajectory of the tractor's centre-of-gravity motion in the plane of the road (v=4.8 m/s and θ=0.42 rad)

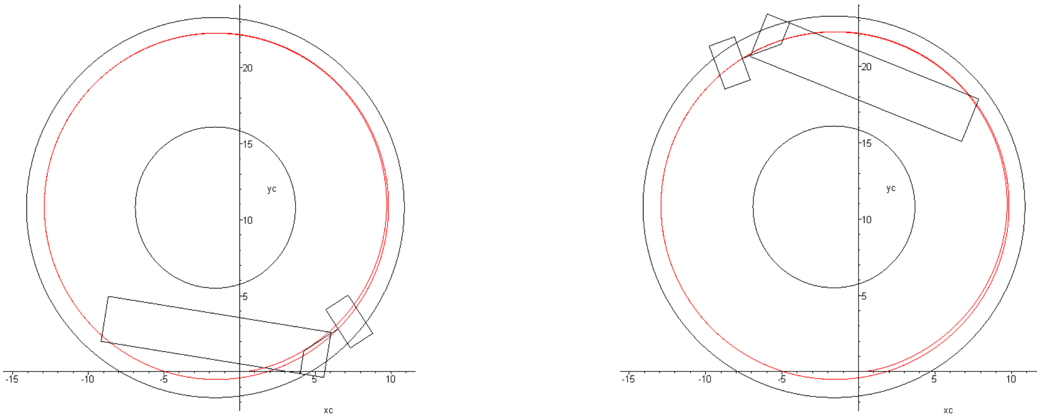

Fig. 4. Trajectory of the tractor's centre-of-gravity motion in the plane of the road (v=5 m/s and θ=0.36 rad)

Thus, there are values of v and θ at which the road train will pass a circular corridor, fitting its dimensions.

Control parameters were selected by the method of progressive approximation v=5 m/s and θ=0.36 rad, which corresponds to a circular steady-state condition with a trajectory. This is shown in Fig. 5.

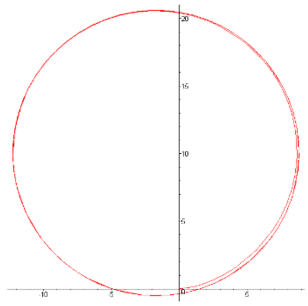

Fig. 5. Trajectory of the tractor's centre-of-gravity motion in the plane of the road (obtained by numerical integration in the Maple package)

This trajectory corresponds to the passage by a road train of a circular corridor, the dimensions of which correspond to EU standards, as shown in Fig. 6.

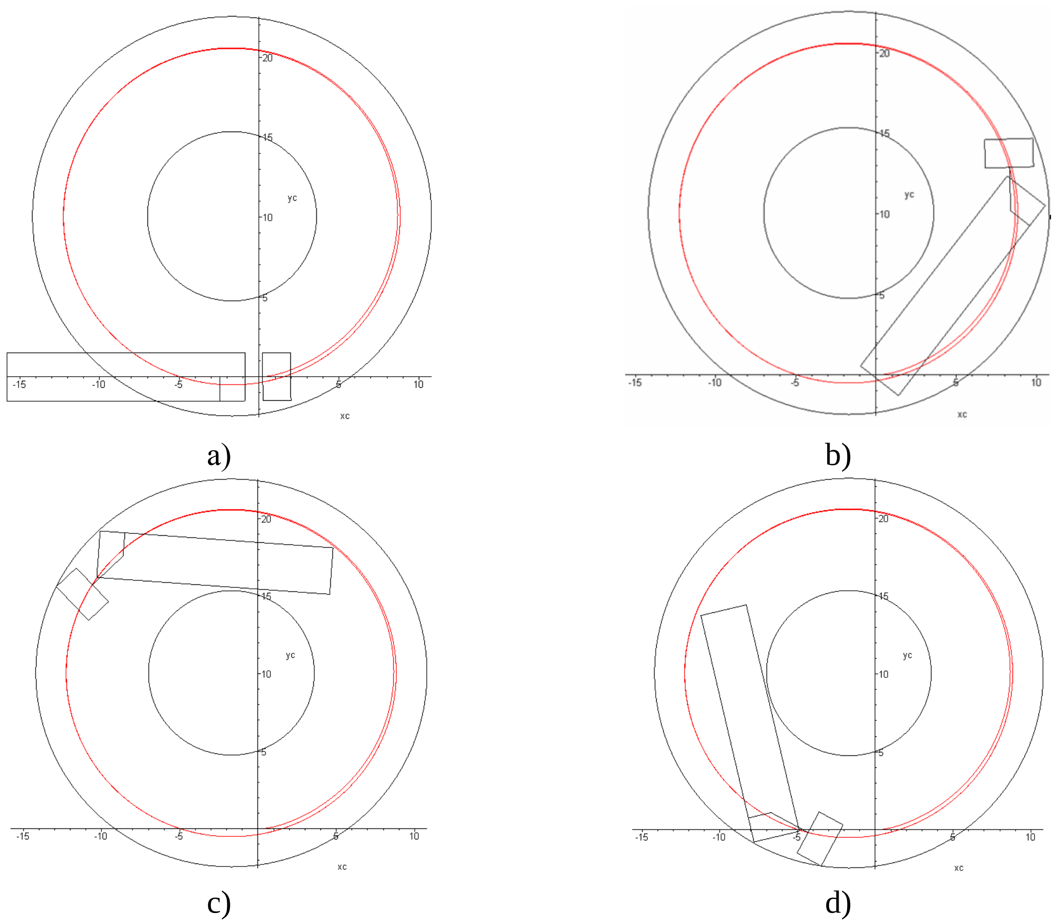

Fig. 6. A double road train passes a circular corridor (corridor dimensions comply with EU standards): a) entering the corridor; b-d) passing of the corridor

2. Determination of the stability range of the rectilinear regime in the parameter space (analytical and numerical definition of the critical speed of rectilinear motion)

The linear approximation of the initial system is used for the numerical determination of critical velocity. The eigenvalue spectrum is determined for the different parameter value *v*. This approach makes it possible to establish the existence of stability (instability) for a pattern of design factor. The method of interval bisection facilitates the determination of the moment of the loss of stability (v_{kr}) [3].

Take, for example, the following set of parameters: m=6,500 kg; m_2=36,500 kg; a=0.4 m; b=3.2 m; c=2.7 m; b_1=2.8 m; d_1=5.4 m; k_1=160,000 H; k_2=226,000 H; k_3=270,000 H; $J=0.35\cdot m\cdot a\cdot b$ (kg·m²); $J_2=0.8\cdot m_1\cdot d_1\cdot b_1$ (kg·m²); $\chi_i=0.8$; $\theta=0$; K=1.5 m

This corresponds to the eigenvalue spectrum at the value v=20 m/s:

$$eigv := -0.6241640318 + 1.342794302\,I,\ -0.4253230590,\ -1.932232332,\ -0.6241640318 - 1.342794302\,I$$

The eigenvalue spectrum of the system (8) at v=20 m/s is shown in Fig. 7.

Since the roots of the performance equation of a system experiencing variations are negative real parts, according to the Lyapunov theorem, the linear traffic condition is stable.

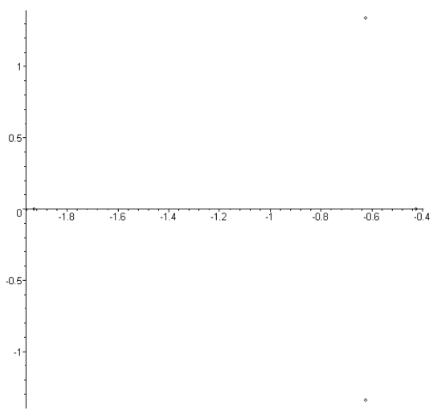

Fig. 7. The eigenvalue spectrum of the system (8) at v=20 m/s.

We have at v=35 m/s:

$$eigv := 0.08371808044,\ -0.3860627656 + 1.512924892\,I,\ -1.372097383,\ -0.3860627656 - 1.512924892\,I$$

If one real root is positive, then the regime is unstable.

The eigenvalue spectrum of the system (8) at v=35 m/s is shown in Fig. 8.

Consequently, there is a loss of stability in the linear motion in the speed range 20 m/s<v<35 m/s. The zero eigenvalue corresponds to the velocity value $v_{кр}$ (the so-called critical case of one zero root involves a divergent loss of stability). In this case, the initial perturbations of the phase variables are grown aperiodically. The case of a couple of complex eigenvalues with zero real parts corresponds to a periodic increase in the initial perturbations of the phase variables, leading to flutter instability.

We have at v=31 m/s:

$$eigv := 0.0007759302256,\ -0.4319365867 + 1.490179879\,I,\ -1.463279177,\ -0.4319365867 - 1.490179879\,I$$

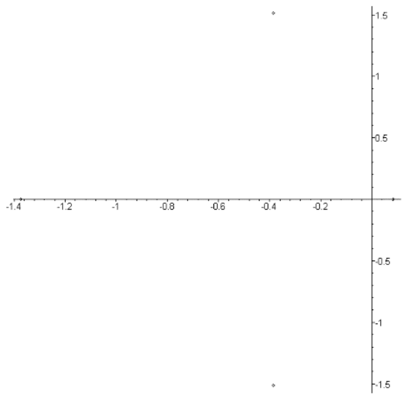

Fig. 8. The eigenvalue spectrum of the system (8) at v=35 m/s

The eigenvalue spectrum of the system (8) at v=31 m/s is shown in Fig. 9.

One of the real roots with some degree of accuracy is equal to zero, that is, a divergent loss of stability is going to occur at the velocity value v_{kr}=31 m/s.

The analytic expression for determining the critical velocity is given by:

$$v_{kr} = \sqrt{\frac{k_1 \cdot k_2 \cdot L_1 \cdot l^2}{(m \cdot L_1 + m_1 \cdot b_1)(k_1 \cdot a - k_2 \cdot b) + c \cdot m_1 \cdot b_1(k_1 + k_2)}}\,. \tag{9}$$

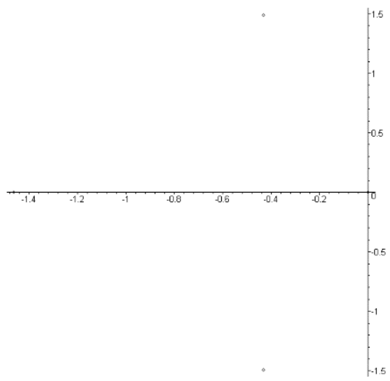

Fig. 9. The eigenvalue spectrum of the system (8) at v=31 m/s

The numerical value of the critical velocity for the selected parameters of the system is v_{kr}=30.97 m/s. This result confirms the results of the trial-and-error method.

It follows from (9) that v_{kr} depends on a certain design factor. We analyse how the value v_{kr} changes with the variation in the parameters L_1 and m_1.

If we only change the mass of the semi-trailer in the design factors of the system, as presented in Fig. 10, we obtain the dependence of critical velocity on the mass of the semitrailer $v_{kr}=f(m_1)$.

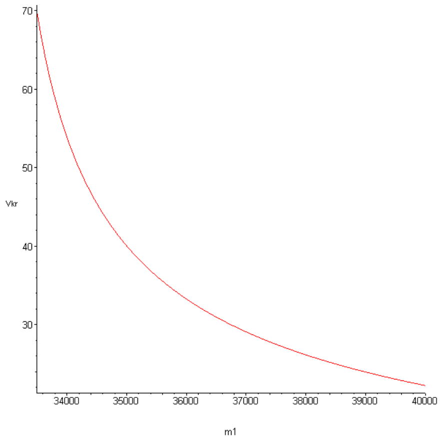

Fig. 10. Graph of the dependence of critical velocity
on the mass of the semitrailer $v_{kr}=f(m_1)$

If we assume m_1=33,000 kg, we can determine that the divergent loss of stability comes at a value of v_{kr}=123 m/s. The flutter loss of stability under certain conditions could happen earlier than at the divergent stage. As such, it is necessary to check what happens with a flutter loss of stability at lower values v_{kr}, as well as determine the eigenvalue spectrum of the system under v_{kr}=120 m/s:

$$eigv := -0.1835379687 + 1.848346011\,I,\ -0.1439906210 + 0.5802589752\,I,$$
$$-0.1439906210 - 0.5802589752\,I,\ -0.1835379687 - 1.848346011\,I$$

Since the real parts of the roots are negative, the flutter loss of stability does not set in, that is, the regime is stable.

Changing the position of the centre of gravity of the semi-trailer, that is, varying the ratio d_1/b_1, we get the following dependence of the critical velocity:

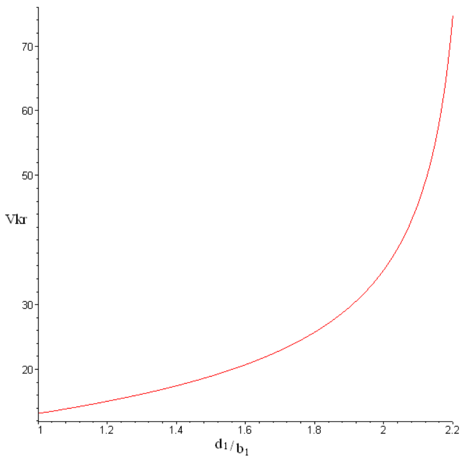

Fig. 11. Graph of the dependence of the critical velocity on the ratio d_1/b_1 ($v_{kr}=f\,(d_1/b_1)$)

From the graph, it follows that critical speed value will decrease when the semi-trailer's centre of gravity in relation to the hitch point is approximated.

4. MODELLING OF PHASE PORTRAITS OF THE MODEL (ANALYSIS OF THE STABILITY REGION OF THE RECTILINEAR REGIME)

The system (7) is allowed an obvious solution {v=const; u=0; ω=0; θ=0; φ=0; Φ=0} with balanced longitudinal forces (X_1=0; X_2=0; X_3=0). This corresponds to the uniform rectilinear motion of the road train (stationary rectilinear mode). The set of steady-state conditions is determined by the system (7) in which we substitute the following values: U=0; ω=0; Φ=0; PP=0.

We take the control parameter of the system v=20 m/s and θ=0 and construct a phase portrait in the space of variables (u, ω). The system comprises three steady-state conditions. These regimes correspond to three singular points on the phase plane: at the origin of coordinates of the stable node (it corresponds to a rectilinear regime) and two saddle points that are symmetrically located (they correspond to unstable circular regimes). Saddle special points are approximated to the origin of coordinates according to an increase in the parameter *v*. The stability of the rectilinear regime is wrecked at $v=v_{kr}$. The domain of stability of the rectilinear regime limits the incoming separatrix of the saddle points [6], as shown in Fig. 12.

The coordinates of the saddle points were numerically determined using the Maple package as a solution to the system of non-linear equations (7):

(u=-5.279; φ=-0.0653; ω=0.2198); (u=5.279; φ=0.0653; ω=-0.2198)

The phase portrait of the system at $v>v_{kr}$ is discussed in Fig. 13.

The system is one unstable rectilinear traffic condition, which corresponds to a saddle singular point at the origin of coordinates. The initial perturbations grow aperiodically, which should correspond to the phenomenon of skidding. The phase variables in this case are close to the stable separatrices of the saddle. The coordinates of the saddle singular point are as follows: (u=0; φ=0; ω=0).

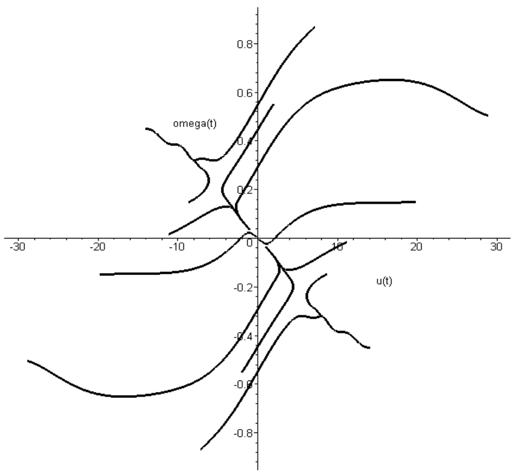

Fig. 12. Phase portrait of the system at subcritical velocity

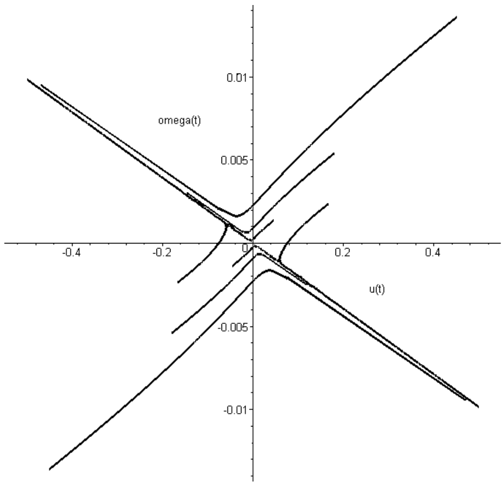

Fig. 13. Phase portrait of the system at supercritical speed

5. CONCLUSION

Stability problems, namely, the influence of motion parameters on stability (instability), are considered. The graphs of the dependence of the critical velocity on the mass of the semi-trailer and its geometric parameters are constructed. These dependencies make it possible to determine the design factors of the system. This corresponds to a divergent loss of stability. Flutter loss of stability under the considered parameters is not found.

Phase portraits of the system are constructed at different speeds, which makes it possible to estimate the domain of attraction of linear motion. The domain of attraction of the rectilinear regime is limited by separatrices. The initial values of the phase variables can be estimated in phase portraits, leading to the conclusion that the system exists in the stability domain. The implementation of these initial disturbances can result from external influences (crosswind, impact with the edge of the roadway etc.).

The values of the speed (v) and the steering angle (θ) are determined for the selected design factors of the model, which ensures the passage of the road train along the circular overall corridor.

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