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Weighted Composition of m-Quasi k-Paranormal Operators

D. Senthilkumar[1] and P. Tamil Selvi[1,*]

1 Department of Mathematics, Government Arts College (Autonomous), Coimbatore, Tamil Nadu, India.

Abstract: In this paper we discuss the conditions for a composition operator and a weighted composition operator to be m-quasi k-paranormal operator and also the characterization of m-quasi k-paranormal operator on weighted Hardy space.

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1. Introduction

Let H be an infinite dimensional complex Hilbert and $B(H)$ denote the algebra of all bounded linear operators acting on H. Recall that an operator $T \in B(H)$ is positive, $T \geq 0$, if $\langle Tx, x\rangle \geq 0$ for all $x \in H$. An operator $T \in B(H)$ is said to be hyponormal if $T^*T \geq TT^*$. Hyponormal operators have been studied by many authors and its known that hyponormal operators have many interesting properties similar to those of normal operators. An operator T is said to be p-hyponormal if $(T^*T)^p \geq (TT^*)^p$ for $p \in (0,1]$ and an operator T is said to be log-hyponormal if T is invertible and $log \mid T \mid \geq log \mid T^* \mid$. p-hyponormal and log-hyponormal opertaors are defined as extension of hyponormal operator.

An operator T is called paranormal if $\|Tx\|^2 \leq \|T^2x\| \|x\|$ for all $x \in H$.An operator T is called quasi-paranormal if $\|T^2x\|^2 \leq \|T^3x\| \|Tx\|$ for all $x \in H$. An operator T is called k-quasi-paranormal if $\|T^{k+1}x\|^2 \leq \|T^{k+2}x\| \|T^kx\|$ for all $x \in H$. An operator $T \in B(H)$ is said to be m-quasi k-paranormal for some positive integer m and k if for all $x \in H$, $\|T^{m+k+1}x\| \|T^mx\|^k \geq \|T^{m+1}x\|^{k+1}$.

1.1. Preliminaries

Let $L^2 = L^2(\Omega, A, \mu)$ denote the space of all complex-valued measurable function for which $\int_\Omega |f|^2 d\mu < \infty$. A composition operator on L^2, induced by a non-singular measurable transformation T, is denoted by C_T and is given by $C_Tf = f \circ T$ for each $f \in L^2$. Then for $f \in L^2$ and for any positive integer k, $C_T^kf = f \circ T^k$, $C_T^*f = h.E(f) \circ T^{-1}$ and $C_T^{*k}f = h_k.E(f) \circ T^{-k}$, where $h_k = d\mu T^{-k}/d\mu$. Let $W = W_{(u,T)}$, denote the weighted composition operator on L^2 given by $(f \mapsto u.f \circ T)$ induced by a complex-valued measurable mapping u on Ω and the measurable transformation $T : \Omega \longmapsto \Omega$. The adjoint W^* of the weighted composition operator W is given by

$$W^*f = h.E(u.f) \circ T^{-1}$$

* E-mail: tamilselviirugur@gmail.com

$$W^k f = u_k.f \circ T^k$$
$$W^{*k} f = h_k.E(u_k.f) \circ T^{-k}$$
$$W^* W f = h.E(u^2) \circ T^{-1}$$
$$W W^* f = u(h \circ T).E(u.f) \text{ for each } f \in L^2.$$

The operator C_T are not necessarily defined on all of $H^2(\beta)$. They are ever where defined in some special cases in the classical Hardy space H^2. Let w be a point on the disk. Define $k^{\beta}_{w(z)} = \sum_{n=0}^{\infty} \frac{z^n \bar{w}^n}{\beta_n^2}$. Then the function k^{β}_w is a point evaluation for $H^2(\beta)$. Then k^{β}_w is in $H^2(\beta)$ and $\left\|k^{\beta}_w\right\|^2 = \sum_{n=0}^{\infty} \frac{|\, w\,|^{2n}}{\beta_n^2}$. Thus, $\|k_w\|$ is an increasing function of $|\, w\,|$. If $f(z) = \sum_{n=0}^{\infty} a_n z^n$ then $\left\langle f, k^{\beta}_w \right\rangle_{\beta} = f(w)$ for all f and k^{β}_w is known as the point evalution kernal at w. It can be easily shown that $C_T^* k^{\beta}_w = k^{\beta}_{T(w)}$ and $k^{\beta}_0 = 1$.

2. m-Quasi k-Paranormal Composition Operators

In this chapter, some properties of m-quasi k-paranormal composition operators are discussed.

Theorem 2.1. *Let $C_T \in B(L^2)$. Then the following are equivalent*

(a). C_T is m-quasi k-paranormal.

(b). $(k+1)^{\frac{1}{2}}\, \mu^{\frac{1}{2}} \left\| \sqrt{h_m E(h) \circ T^{-m}} f \right\| - k^{\frac{1}{2}} \mu^{\frac{k+1}{2}} \left\| \sqrt{h_m} f \right\| \leq \left\| \sqrt{h_{m+k} E(h) \circ T^{-(m+k)}} f \right\|$ *for each* $f \in L^2$.

(c). $(k+1)\, \mu^k h_m E(h) \circ T^{-m} - k\mu^{k+1} h_m \leq h_{m+k} E(h) \circ T^{-(m+k)}$ *where* $h_m = d\mu T^{-m}/d\mu$.

(d). $(k+1)\, \mu^k h_{m-1} h \circ T^{-(m-1)} E(h) \circ T^{-m} - k\mu^{k+1} h_{m-1} h \circ T^{-(m-1)} \leq h_{m+k} E(h) \circ T^{-(m+k)}$.

(e). $(k+1)\, \mu^k h h_{m-1} \circ T^{-1} E(h) \circ T^{-m} - k\mu^{k+1} h h_{m-1} \circ T^{-1} \leq h_{m+k} E(h) \circ T^{-(m+k)}$.

Proof.

To prove (a) ≡ (b)

Let C_T is m-quasi k-paranormal.

$$(k+1)\mu^k \left\langle \left(C_T^{*m+1} C_T^{m+1}\right) f, f \right\rangle - k\mu^{k+1} \left\langle \left(C_T^{*m} C_T^{m}\right) f, f \right\rangle \leq \left\langle \left(C_T^{*m+k+1} C_T^{m+k+1}\right) f, f \right\rangle \tag{1}$$

Consider

$$\left(C_T^{*m+1} C_T^{m+1}\right) f = h_m E(h) \circ T^{-m}.f \tag{2}$$
$$C_T^{*m} C_T^{m} f = h_m f \tag{3}$$
$$C_T^{*m+k+1} C_T^{m+k+1} f = h_{m+k} E(h) \circ T^{-(m+k)}.f \tag{4}$$

Sub (2), (3) and (4) in (1)

$$(k+1)\mu^k \left\langle h_m E(h) \circ T^{-m} f, f \right\rangle - k\mu^{k+1} \left\langle h_m f, f \right\rangle \leq \left\langle h_{m+k} E(h) \circ T^{-(m+k)} f, f \right\rangle$$
$$(k+1)\mu^k h_m E(h) \circ T^{-m} f - k\mu^{k+1} h_m f \leq h_{m+k} E(h) \circ T^{-(m+k)} f$$
$$(k+1)^{\frac{1}{2}}\, \mu^{\frac{1}{2}} \left\| \sqrt{h_m E(h) \circ T^{-m}} f \right\| - k^{\frac{1}{2}} \mu^{\frac{k+1}{2}} \left\| \sqrt{h_m} f \right\| \leq \left\| \sqrt{h_{m+k} E(h) \circ T^{-(m+k)}} f \right\|$$

To prove (b) $\equiv$ (c)

Consider

$$(k+1)^{\frac{1}{2}}\,\mu^{\frac{1}{2}}\left\|\sqrt{h_m E(h)\circ T^{-m}}f\right\| - k^{\frac{1}{2}}\mu^{\frac{k+1}{2}}\left\|\sqrt{h_m}f\right\| \leq \left\|\sqrt{h_{m+k}E(h)\circ T^{-(m+k)}}f\right\|$$
$$(k+1)\mu^k h_m E(h)\circ T^{-m} - k\mu^{k+1}h_m \leq h_{m+k}E(h)\circ T^{-(m+k)}$$

To prove (c) $\equiv$ (d)

Consider

$$(k+1)\mu^k h_m E(h)\circ T^{-m} - k\mu^{k+1}h_m \leq h_{m+k}E(h)\circ T^{-(m+k)} \tag{5}$$
$$h_m = \mu T^{-m}(B) \tag{6}$$

We have,

$$\mu T^{-m}(B) = \mu^{-1}\left(T^{-(m-1)}(B)\right)$$
$$= \int_{T^{-(m-1)}(B)} m d\mu$$
$$\mu T^{-m}(B) = \int_B h_{m-1}h\circ T^{-(m-1)}d\mu \tag{7}$$

Sub (7) in (6)

$$h_m = h_{m-1}h\circ T^{-(m-1)} \tag{8}$$

Sub (8) in (5)

$$(k+1)\,\mu^k h_{m-1}h\circ T^{-(m-1)}E(h)\circ T^{-m} - k\mu^{k+1}h_{m-1}h\circ T^{-(m-1)} \leq h_{m+k}E(h)\circ T^{-(m+k)}$$

To prove (c) $\equiv$ (e)

Consider

$$(k+1)\mu^k h_m E(h)\circ T^{-m} - k\mu^{k+1}h_m \leq h_{m+k}E(h)\circ T^{-(m+k)} \tag{9}$$
$$h_m = \mu T^{-m}(B) \tag{10}$$

We have,

$$\mu T^{-m}(B) = \mu T^{-(m-1)}\left(T^{-1}(B)\right)$$
$$= \int_{T^{-(B)}} h_{m-1}d\mu$$
$$\mu T^{-m}(B) = \int_B h.h_{m-1}\circ T^{-1}d\mu \tag{11}$$

Sub (11) in (10)

$$h_m = hh_{m-1}\circ T^{-1} \tag{12}$$

Again sub (12) in (9)

$$(k+1)\,\mu^k h.h_{m-1}\circ T^{-1}E(h)\circ T^{-m} - k\mu^{k+1}h.h_{m-1}\circ T^{-1} \leq h_{m+k}E(h)\circ T^{-(m+k)}$$

Hence the proof. □

Corollary 2.2. *If $T^{-1}(A) = A$ then C_T is m - quasi k-paranormal if and only if* $(k+1)\,\mu^k h_m.h \circ T^{-m} - k\mu^{k+1} h_m \le h_{m+k}.h \circ T^{-(m+k)}$.

Theorem 2.3. *If C_T^* is m - quasi k-paranormal then* $(k+1)\,\mu^k \left\langle h_{m+1} \circ T^{m+1}E(f), f\right\rangle - k\mu^{k+1} \left\langle h_m \circ T^m E(f), f\right\rangle \le \left\langle h_{m+k+1} \circ T^{m+k+1}E(f), f\right\rangle$.

Proof.

Case (i): Let C_T^* is m-quasi k-paranormal.

$$(k+1)\mu^k C_T^{m+1} C_T^{*m+1} - k\mu^{k+1} C_T^m C_T^{*m} \le C_T^{m+k+1} C_T^{*m+k+1}$$

$$(k+1)\mu^k \left\langle \left(C_T^{m+1} C_T^{*m+1}\right) f, f\right\rangle - k\mu^{k+1} \left\langle \left(C_T^m C_T^{*m}\right) f, f\right\rangle \le \left\langle \left(C_T^{m+k+1} C_T^{*m+k+1}\right) f, f\right\rangle \tag{13}$$

Consider

$$\left(C_T^{m+1} C_T^{*m+1}\right) f = h_{m+1} \circ T^{m+1}.E(f) \tag{14}$$

$$C_T^m C_T^{*m} f = h_m \circ T^m E(f) \tag{15}$$

$$C_T^{m+k+1} C_T^{*m+k+1} f = h_{m+k+1} \circ T^{m+k+1}.E(f) \tag{16}$$

Sub (14), (15) and (16) in (13)

$$(k+1)\mu^k \left\langle h_{m+1} \circ T^{m+1}.E(f), f\right\rangle - k\mu^{k+1} \left\langle h_m \circ T^m E(f), f\right\rangle \le \left\langle h_{m+k+1} \circ T^{m+k+1}.E(f), f\right\rangle$$

Case (ii): Let $(k+1)\mu^k \left\langle h_{m+1} \circ T^{m+1}.E(f), f\right\rangle - k\mu^{k+1} \left\langle h_m \circ T^m E(f), f\right\rangle \le \left\langle h_{m+k+1} \circ T^{m+k+1}.E(f), f\right\rangle$.

$$(k+1)\mu^k h_{m+1} \circ T^{m+1}.E(f) - k\mu^{k+1} h_m \circ T^m E(f) \le h_{m+k+1} \circ T^{m+k+1}.E(f) \tag{17}$$

Consider

$$h_{m+1} \circ T^{m+1}.E(f) = C_T^{m+1} C_T^{*m+1} f \tag{18}$$

$$h_m \circ T^m E(f) = C_T^m C_T^{*m} f \tag{19}$$

$$h_{m+k+1} \circ T^{m+k+1}.E(f) = C_T^{m+k+1} C_T^{*m+k+1} f \tag{20}$$

Sub (18), (19) and (20) in (17)

$$(k+1)\mu^k C_T^{m+1} C_T^{*m+1} f - k\mu^{k+1} C_T^m C_T^{*m} f \le C_T^{m+k+1} C_T^{*m+k+1} f$$

$$(k+1)\mu^k C_T^{m+1} C_T^{*m+1} - k\mu^{k+1} C_T^m C_T^{*m} \le C_T^{m+k+1} C_T^{*m+k+1}$$

Hence C_T^* is m - quasi k-paranormal. Hence the proof. □

Corollary 2.4. *If $T^{-1}(A) = A$ then C_T^* is m - quasi k-paranormal if and only if* $(k+1)^{\frac{1}{2}}\,\mu^{\frac{k}{2}} \left\| \sqrt{h_{m+1} \circ T^{m+1} f} \right\| - k^{\frac{1}{2}} \mu^{\frac{k+1}{2}} \left\| \sqrt{h_m \circ T^m f} \right\| \le \left\| \sqrt{h_{m+k+1} \circ T^{m+k+1} f} \right\|$ *for each $f \in L^2$.*

3. m-Quasi k-Paranormal Weighted Composition Operators

In this Chapter,m-quasi k-paranormal weighted composition operators on a Hilbert space are characterized.

Theorem 3.1. *Let W is m-quasi k-paranormal if and only if* $(k+1)\,\mu^k\,\left\|u_{m+1}f\circ T^{m+1}\right\|^2 - k\mu^{k+1}\left\|u_m f\circ T^m\right\|^2 \leq \left\|u_{m+k+1}f\circ T^{m+k+1}\right\|^2$ *for each* $f\in L^2$.

Proof.

Case (i): Let W is m-quasi k-paranormal.

$$(k+1)\mu^k W^{*m+1}W^{m+1} - k\mu^{k+1}W^{*m}W^m \leq W^{*m+k+1}W^{m+k+1}$$

$$(k+1)\mu^k\left\langle W^{m+1}f, W^{m+1}f\right\rangle - k\mu^{k+1}\left\langle W^m f, W^m f\right\rangle \leq \left\langle W^{m+k+1}f, W^{m+k+1}f\right\rangle$$

$$(k+1)\,\mu^k\left\|W^{m+1}f\right\|^2 - k\mu^{k+1}\left\|W^m f\right\|^2 \leq \left\|W^{m+k+1}f\right\|^2 \tag{21}$$

Consider

$$W^{m+1}f = u_{m+1}.f\circ T^{m+1} \tag{22}$$

$$W^m f = u_m.f\circ T^m \tag{23}$$

$$W^{m+k+1}f = u_{m+k+1}.f\circ T^{m+k+1} \tag{24}$$

Sub (22), (23) and (24) in (21)

$$(k+1)\,\mu^k\left\|u_{m+1}f\circ T^{m+1}\right\|^2 - k\mu^{k+1}\left\|u_m f\circ T^m\right\|^2 \leq \left\|u_{m+k+1}f\circ T^{m+k+1}\right\|^2$$

Case (ii): We have

$$(k+1)\,\mu^k\left\|u_{m+1}f\circ T^{m+1}\right\|^2 - k\mu^{k+1}\left\|u_m f\circ T^m\right\|^2 \leq \left\|u_{m+k+1}f\circ T^{m+k+1}\right\|^2 \tag{25}$$

Consider

$$W^{m+1}f = u_{m+1}.f\circ T^{m+1} \tag{26}$$

$$W^m f = u_m.f\circ T^m \tag{27}$$

$$W^{m+k+1}f = u_{m+k+1}.f\circ T^{m+k+1} \tag{28}$$

Sub (26), (27) and (28) in (25)

$$(k+1)\,\mu^k\left\|W^{m+1}f\right\|^2 - k\mu^{k+1}\left\|W^m f\right\|^2 \leq \left\|W^{m+k+1}f\right\|^2$$

$$(k+1)\mu^k W^{*m+1}W^{m+1}f - k\mu^{k+1}W^{*m}W^m f \leq W^{*m+k+1}W^{m+k+1}f$$

$$(k+1)\mu^k W^{*m+1}W^{m+1} - k\mu^{k+1}W^{*m}W^m \leq W^{*m+k+1}W^{m+k+1}$$

Hence W is m - quasi k-paranormal. Hence the proof. □

Corollary 3.2. *If $T^{-1}(A) = A$ then W is m - quasi k-paranormal if and only if* $(k+1)\,\mu^k h_{m+1}.u_{m+1}^2\circ T^{-(m+1)} - k\mu^{k+1}h_m u_m^2\circ T^{-m} \leq h_{m+k+1}u_{m+k+1}^2\circ T^{-(m+k+1)}$.

Theorem 3.3. *Let W^* is m-quasi k-paranormal if and only if* $(k+1)\,\mu^k \left\|h_{m+1}E(u_{m+1}f)\circ T^{-(m+1)}\right\|^2 - k\mu^{k+1}\left\|h_m E(u_m f)\circ T^{-m}\right\|^2 \leq \left\|h_{m+k+1}E(u_{m+k+1}f)\circ T^{-(m+k+1)}\right\|^2$ *for each* $f \in L^2$.

Proof.

Case (i): Let W^* is m-quasi k-paranormal.

$$(k+1)\mu^k W^{*m+1}W^{m+1} - k\mu^{k+1}W^{*m}W^m \leq W^{*m+k+1}W^{m+k+1}$$

$$(k+1)\mu^k \left\langle \left(W^{*m+1}W^{m+1}\right) f, f\right\rangle - k\mu^{k+1}\left\langle \left(W^{*m}W^m\right) f, f\right\rangle \leq \left\langle \left(W^{*m+k+1}W^{m+k+1}\right) f, f\right\rangle \tag{29}$$

consider

$$W^{*m+1}f = h_{m+1}E(u_{m+1}.f)\circ T^{-(m+1)} \tag{30}$$

$$W^m f = h_m E(u_m.f)\circ T^{-m} \tag{31}$$

$$W^{m+k+1}f = h_{m+k+1}E(u_{m+k+1}.f)\circ T^{-(m+k+1)} \tag{32}$$

Sub (30), (31) and (32) in (29)

$$(k+1)\,\mu^k \left\|h_{m+1}E(u_{m+1}.f)\circ T^{-(m+1)}\right\|^2 - k\mu^{k+1}\left\|h_m E(u_m.f)\circ T^{-m}\right\|^2 \leq \left\|h_{m+k+1}E(u_{m+k+1}.f)\circ T^{-(m+k+1)}\right\|^2$$

Case (ii): We have

$$(k+1)\,\mu^k \left\|h_{m+1}E(u_{m+1}.f)\circ T^{-(m+1)}\right\|^2 - k\mu^{k+1}\left\|h_m E(u_m.f)\circ T^{-m}\right\|^2 \leq \left\|h_{m+k+1}E(u_{m+k+1}.f)\circ T^{-(m+k+1)}\right\|^2 \tag{33}$$

Sub (30), (31) and (32) in (33)

$$(k+1)\,\mu^k \left\|W^{*m+1}f\right\|^2 - k\mu^{k+1}\left\|W^{*m}f\right\|^2 \leq \left\|W^{*m+k+1}f\right\|^2$$

$$(k+1)\mu^k \left\langle W^{*m+1}f, W^{*m+1}f\right\rangle - k\mu^{k+1}\left\langle W^{*m}f, W^{*m}f\right\rangle \leq \left\langle W^{*m+k+1}f, W^{*m+k+1}f\right\rangle$$

$$(k+1)\mu^k \left\langle \left(W^{m+1}W^{*m+1}\right) f, f\right\rangle - k\mu^{k+1}\left\langle \left(W^{m}W^{*m}\right) f, f\right\rangle \leq \left\langle \left(W^{m+k+1}W^{*m+k+1}\right) f, f\right\rangle$$

$$(k+1)\mu^k W^{m+1}W^{*m+1}f - k\mu^{k+1}W^mW^{*m}f \leq W^{m+k+1}W^{*m+k+1}f$$

$$(k+1)\mu^k W^{m+1}W^{*m+1} - k\mu^{k+1}W^mW^{*m} \leq W^{m+k+1}W^{*m+k+1}$$

Hence W^* is m - quasi k-paranormal. Hence the proof. ☐

Corollary 3.4. *If* $T^{-1}(A) = A$ *then* W^* *is m - quasi k-paranormal if and only if* $(k+1)\,\mu^k u_{m+1}h_{m+1}\circ T^{m+1}.u_{m+1}\circ T^{m+1} - k\mu^{k+1}u_m h_m \circ T^m.u_m\circ T^m \leq u_{m+k+1}h_{m+k+1}\circ T^{m+k+1}.u_{m+k+1}\circ T^{m+k+1}$.

4. m-Quasi k-Paranormal Composition Operators On Weighted Hardy Space

In this Chapter,m-quasi k-paranormal composition operators on weighted Hardy space are characterized.

Theorem 4.1. *If C_T is m-quasi k-paranormal operator on $H^2(\beta)$ then $\mu = 1$.*

Proof. Assume that C_T is m-quasi k-paranormal operator on $H^2(\beta)$.

To prove: Let C_T is m-quasi k-paranormal operator.

$$C_T^{*m+k+1}C_T^{m+k+1} - (k+1)\mu^k C_T^{*m+1}C_T^{m+1} + k\mu^{k+1}C_T^{*m}C_T^m \geq 0$$
$$\left\|C_T^{m+k+1}f\right\|^2 - (k+1)\mu^k \left\|C_T^{m+1}f\right\|^2 + k\mu^{k+1}\left\|C_T^m f\right\|^2 \geq 0$$

Let $f = k_0^\beta$ then

$$\left\|C_T^{m+k+1}k_0^\beta\right\|^2 - (k+1)\mu^k \left\|C_T^{m+1}k_0^\beta\right\|^2 + k\mu^{k+1}\left\|C_T^m k_0^\beta\right\|^2 \geq 0$$
$$\left\|C_T^{k+1}\left(C_T^m k_0^\beta\right)\right\|_\beta^2 - (k+1)\mu^k \left\|C_T\left(C_T^m k_0^\beta\right)\right\|_\beta^2 + k\mu^{k+1}\left\|C_T^m k_0^\beta\right\|_\beta^2 \geq 0$$
$$\left\|C_T^{k+1}k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|C_T k_0^\beta\right\|_\beta^2 + k\mu^{k+1}\left\|k_0^\beta\right\|_\beta^2 \geq 0$$
$$\left\|C_T^k C_T k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|C_T k_0^\beta\right\|_\beta^2 + k\mu^{k+1}(1) \geq 0$$
$$\left\|C_T^k k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|k_0^\beta\right\|_\beta^2 + k\mu^{k+1} \geq 0$$
$$\left\|C_T^k k_0^\beta\right\|_\beta^2 - (k+1)\mu^k(1) + k\mu^{k+1} \geq 0$$
$$\left\|k_0^\beta\right\|_\beta^2 - (k+1)\mu^k(1) + k\mu^{k+1} \geq 0$$
$$1 - (k+1)\mu^k(1) + k\mu^{k+1} \geq 0$$

If $k = 1$,

$$1 - 2\mu + \mu^2 \geq 0$$

By elementary properties of real quadratic form, we get $\mu = 1$. □

Theorem 4.2. *If C_T^* is m-quasi k-paranormal operator on $H^2(\beta)$ then $\mu = 1$.*

Proof. Assume that C_T^* is m-quasi k-paranormal operator on $H^2(\beta)$.

To prove: Let C_T^* is m-quasi k-paranormal operator.

$$C_T^{m+k+1}C_T^{*m+k+1} - (k+1)\mu^k C_T^{m+1}C_T^{*m+1} + k\mu^{k+1}C_T^m C_T^{*m} \geq 0$$
$$\left\|C_T^{*m+k+1}f\right\|^2 - (k+1)\mu^k \left\|C_T^{*m+1}f\right\|^2 + k\mu^{k+1}\left\|C_T^{*m} f\right\|^2 \geq 0$$

Let $f = k_0^\beta$ then

$$\left\|C_T^{*m+k+1}k_0^\beta\right\|^2 - (k+1)\mu^k \left\|C_T^{*m+1}k_0^\beta\right\|^2 + k\mu^{k+1}\left\|C_T^{*m} k_0^\beta\right\|^2 \geq 0$$
$$\left\|C_T^{*k+1}C_T^{*m}k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|C_T^* C_T^{*m} k_0^\beta\right\|_\beta^2 + k\mu^{k+1}\left\|C_T^{*m} k_0^\beta\right\|_\beta^2 \geq 0$$
$$\left\|C_T^{*k+1}k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|C_T^* k_0^\beta\right\|_\beta^2 + k\mu^{k+1}\left\|k_0^\beta\right\|_\beta^2 \geq 0$$
$$\left\|C_T^{*k} C_T^* k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|C_T^* k_0^\beta\right\|_\beta^2 + k\mu^{k+1}(1) \geq 0$$
$$\left\|C_T^{*k} k_0^\beta\right\|_\beta^2 - (k+1)\mu^k \left\|k_0^\beta\right\|_\beta^2 + k\mu^{k+1} \geq 0$$
$$\left\|k_0^\beta\right\|_\beta^2 - (k+1)\mu^k(1) + k\mu^{k+1} \geq 0$$

$$(1) - (k+1)\mu^k + k\mu^{k+1} \geq 0$$
$$1 - (k+1)\mu^k + k\mu^{k+1} \geq 0$$

If $k = 1$,

$$1 - 2\mu + \mu^2 \geq 0$$

By elementary properties of real quadratic form, we get $\mu = 1$. □

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