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Dynamics in a Series RLC Circuit with Damping

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Abstract: This paper considers a two dimensional model of Series RLC circuit. Existence of equilibrium points is established and local stability conditions are discussed. The phase portraits are obtained for different sets of parameter values. Numerical simulations are performed.

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Keywords: RLC circuit, Differential Equation, Numerical Simulation, Stability.

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1. Introduction

An Electric Circuit corresponds to a Mass spring system. In Mass springs there is cyclical exchange between potential energy of stretched spring and kinetic energy of moving mass whereas in circuits the exchange is between the electric potential energy and magnetic field energy. The Decay of these oscillations are increased by introducing Resistor to the circuit. There are different ways in which these resistor, inductor and capacitor can be combined [4, 7]. But the most common and straight forward way of analyzing the circuit is by connecting them in series and parallel. Recently, in [5] the authors established the existence of the approximate solution of the second order differential equation of the RLC electric circuit in the sense of Hyers-Ulam and Hyers-Ulam-Rassias. Also, they obtained some interesting results for both homogeneous and non-homogeneous cases. Here we consider a series RLC circuit (Figure 1).

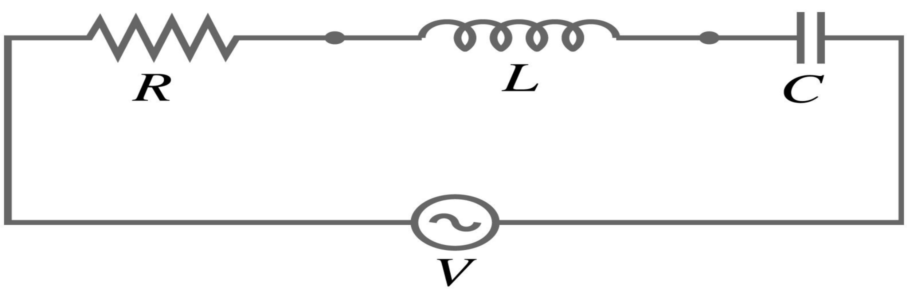

Figure 1. Series RLC circuit

The Second order Differential equation in terms of charge q for a series RLC circuit is given by

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q(t) = F(t), \tag{1}$$

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where R is the effective Resistance, L is the Inductance of the Inductor component, C is the Capacitance of the capacitor components and $F(t)$ is the Forcing function. Equation (1) takes the form

$$\frac{d^2q}{dt^2} + 2\alpha\frac{dq}{dt} + \omega_0^2 q(t) = G, \tag{2}$$

where the damping coefficient $\dfrac{R}{2L}$ is denoted by α, the resonant frequency $\dfrac{1}{\sqrt{LC}}$ is denoted by ω_0 and the forcing term $\dfrac{F(t)}{L}$ by G.

2. Discussion of the Model

2.1. System of Differential Equations

Equation (2) in the absence of the forcing term takes the form

$$\begin{aligned}\frac{dq}{dt} &= x(t)\\ \frac{dx}{dt} &= -2\alpha x(t) - \omega_0^2 q(t),\end{aligned} \tag{3}$$

where $q(t)$ is the charge across the capacitor. The system (3) has the trivial equilibrium point $(0, 0)$. The Jacobian matrix for (3) is

$$J(q, x) = \begin{bmatrix} 0 & 1 \\ -\omega_0^2 & -2\alpha \end{bmatrix}. \tag{4}$$

The eigenvalues are $\lambda_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$. The following subsections consider different damping conditions.

Under Damping

When $\alpha < \omega_0$, the system (3) is under damped with complex, conjugate eigenvalues. Let $\alpha = 1.5$ and $\omega_0 = 6$ then the eigenvalues are $\lambda_{1,2} = -1.5 \pm i\ 5.8094$ .Figure 2 explains the under damped motion of the system (3). The oscillations decay as time increases, thus the equilibrium position is attained.

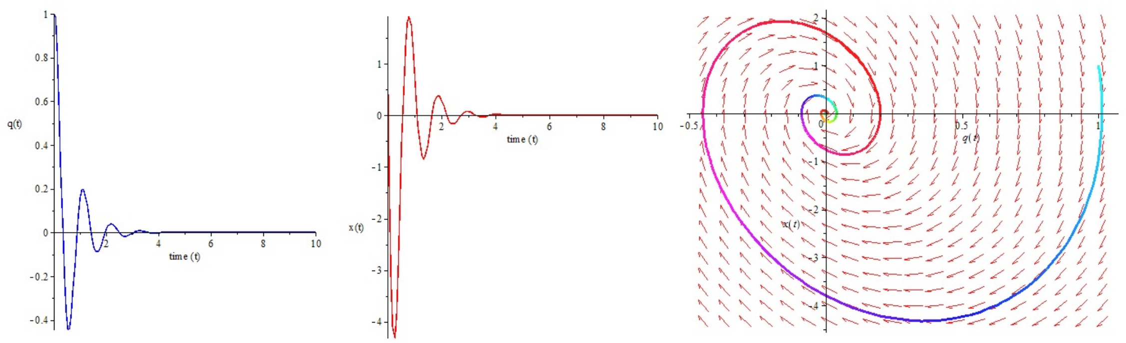

Figure 2. **Under Damping**

Over Damping

When $\alpha > \omega_0$, the system (3) is over damped with real and distinct eigenvalues. For $\alpha = 8$ and $\omega_0 = 4$ then the eigenvalues are $\lambda_1 = -1.0718$ and $\lambda_2 = -14.928$. Over damped motion of the system (3) approaching the equilibrium position as time increases is shown in Figure 3.

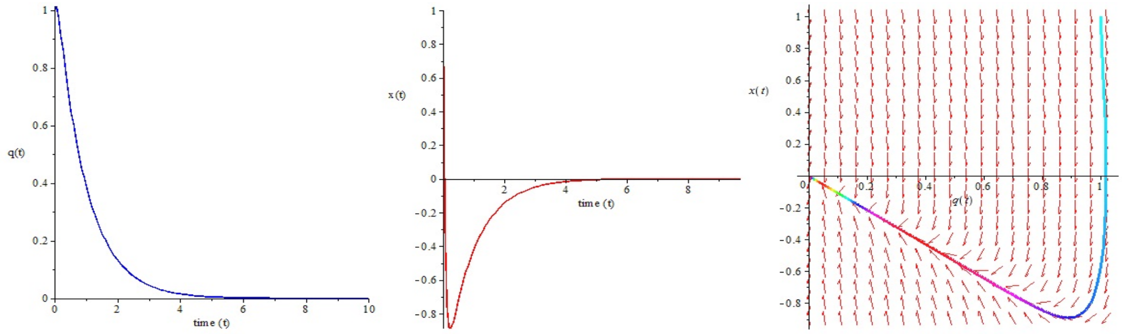

Figure 3. **Over Damping**

Critical Damping

When $\alpha = \omega_0$, the system (3) becomes Critically damped with real and equal eigenvalues. For $\alpha = 4$ and $\omega_0 = 4$ then the eigenvalues are $\lambda_{1,2} = -4$. The system (3) exhibits similar motion as that of over damped approaching the equilibrium position as shown in Figure 4.

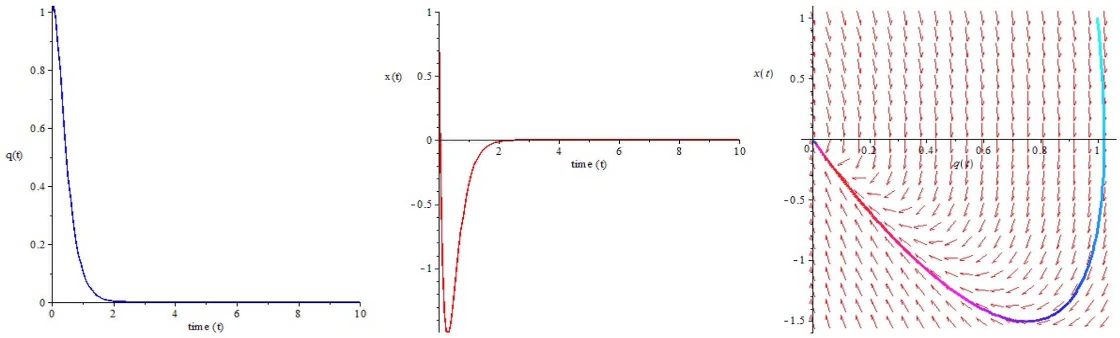

Figure 4. **Critical Damping**

2.2. Discrete Fractional Order System

Fractional order form of (3) is

$$\begin{aligned} \frac{d^\nu q}{dt^\nu} &= x(t) \\ \frac{d^\nu x}{dt^\nu} &= -2\alpha x(t) - \omega_0^2 q(t), \end{aligned} \tag{5}$$

where ν is the Fractional order. Equation (5), on discretization becomes

$$\begin{aligned} q(t+1) &= q(t) + S\ x(t) \\ x(t+1) &= x(t) - S\left(2\alpha x(t) + \omega_0^2 q(t)\right). \end{aligned} \tag{6}$$

where $S = \dfrac{h^\nu}{\Gamma(1+\nu)}$. The only equilibrium point of (6) is the trivial equilibrium point $(0,0)$. The Jacobian matrix for (6) is

$$J(q,x) = \begin{bmatrix} 1 & S \\ -S\ \omega_0^2 & 1-2S\ \alpha \end{bmatrix}. \tag{7}$$

The eigenvalues are $\lambda_{1,2} = 1 - S\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$, which in turn results in three different motions of the system.

Under Damping

If $\alpha < \omega_0$, then the Eigenvalues are complex, conjugate with motion of the system (6) being under damped. Let $\alpha = 0.43$, $\omega_0 = 2.449$, $S = 0.001$ then the eigen values are $\lambda_{1,2} = -0.9995 \pm i\ 0.0024$. From Figure 5 it is clear that the under damped motion reaches the equilibrium position.

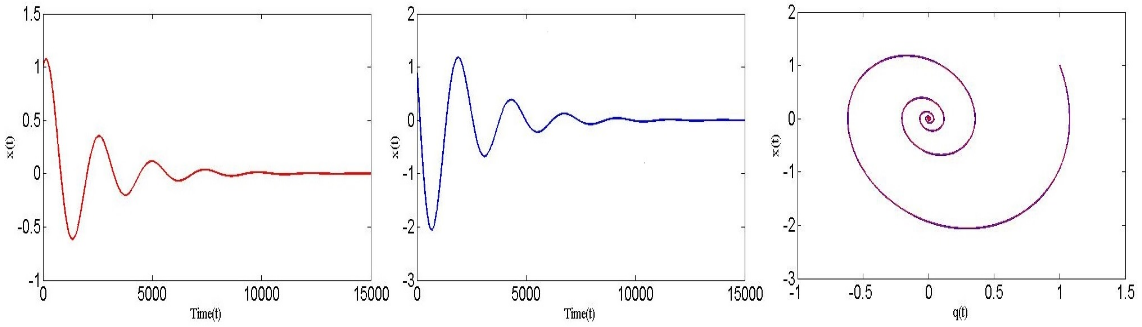

Figure 5. **Under Damping**

Over Damping

If $\alpha > \omega_0$, then the eigenvalues are real and distinct with over damped motion of the system (6). Let $\alpha = 5$, $\omega_0 = 2.449$, $S = 0.001$ then the eigen values are $\lambda_1 = -0.99064$ and $\lambda_2 = -0.99935$. The Over damped motion of the system (6) shown in Figure 6 approaches the equilibrium position as time increases.

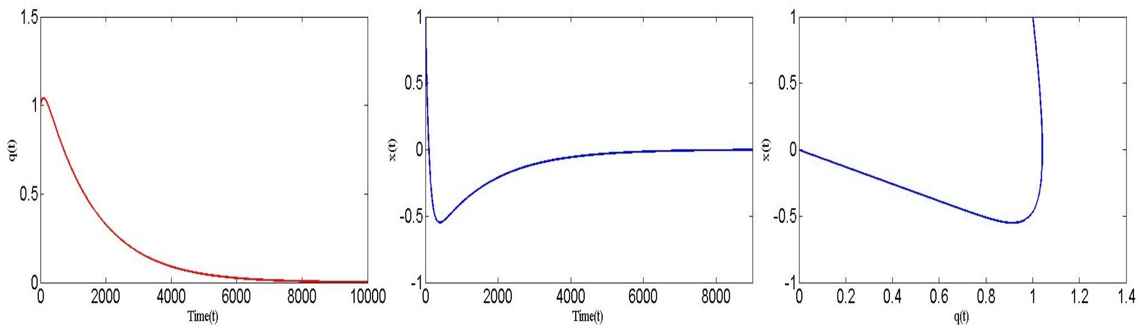

Figure 6. **Over Damping**

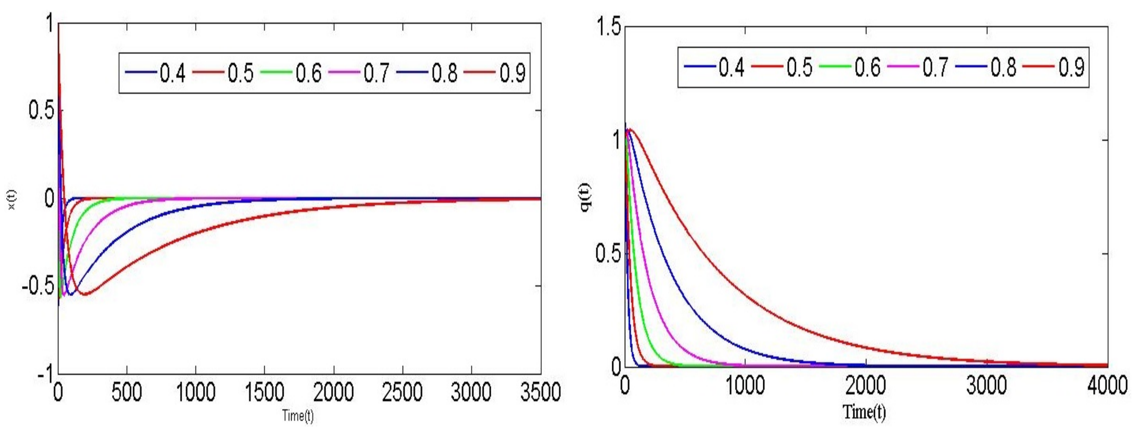

Figure 7. **Over damped motions of (6) for different values of ν**

Critical Damping

If $\alpha = \omega_0$, then the eigenvalues are real and equal. Let $\alpha = 2.449$, $\omega_0 = 2.449$, $S = 0.001$ then the eigen values are $\lambda_{1,2} = -0.9950$. The damped motion of the system (6) shown in Figure 8 similar to over damped motion approaches the equilibrium position with increase in time.

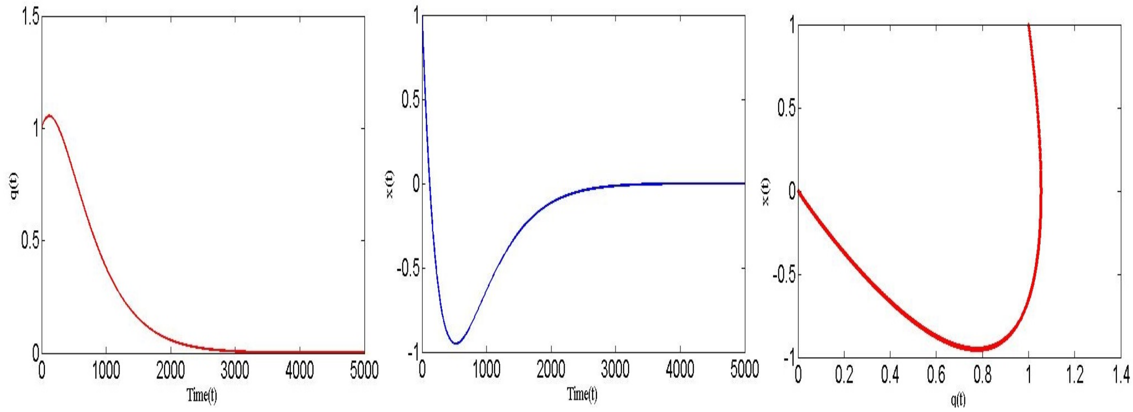

Figure 8. **Critical Damping**

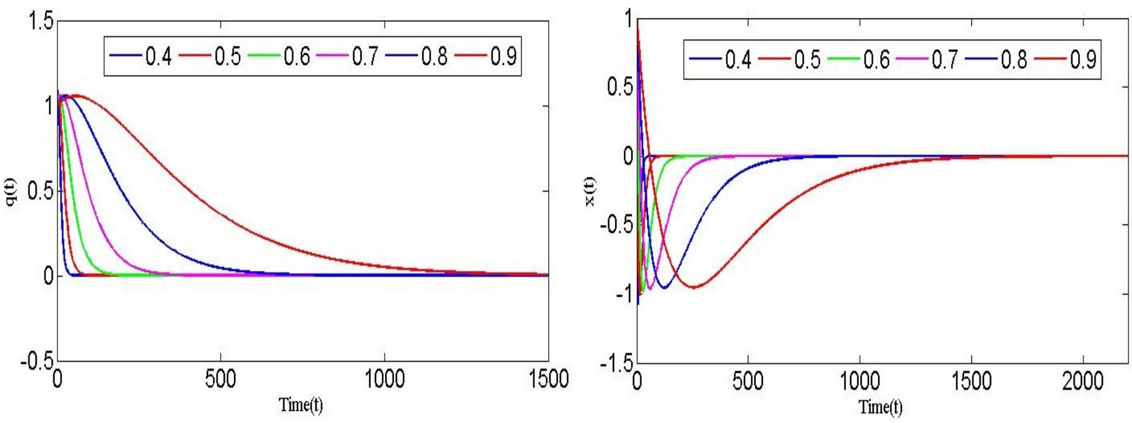

Figure 9. **Critically damped motions of (6) for different values of ν**

3. Conclusion

The Stability properties of both continuous and Discrete Fractional 2-D model describing damped series RLC circuit is analyzed. Time plots and phase portraits are presented to show the Stability of the Series RLC circuit.

References

[1] A. A. Elsadany and A. E. Matouk, *Dynamical behaviors of fractional - order Lotka - Volterra predator - prey model and its discretization*, J. Appl. Math. Comput., DOI 10.1007/s12190-014-0838-6.

[2] https://science.howstuffworks.com/environmental/energy/circuit3.htm

[3] Ivo Petras, *Fractional order Nonlinear Systems-Modeling, Analysis and Simulation*, Higher Education Press, Springer International Edition, April (2010).

[4] V. K. Mehta and Rohit Mehta, *Principles of Electronics*, S.Chand & Company Pvt.Limited, Eleventh Edition, (2013).

[5] R. Murali and A. Ponmana Selvan, *Ulam stability of a LCR Electric Circuit with Electromotive Force*, Int. J. Math. And Appl., 6(1)(2018)(special Issue), 61-66.

[6] Paul Blachard, Robert L. Levaney and Glen R. Hall, *Differential Equations*, Brooks/cole, Fourth Edition, (2012).

[7] B. L. Theraja, *Basic Electronics: Solid State*, S. Chand & Company Pvt. Limited, Fifth Edition, (2012).