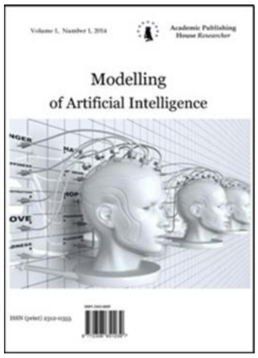

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Multidimensional Limit Theorems in Models with Categorized-Time Absolute Priorities

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Abstract

In the queuing theory, there are single-channel models with a Poisson incoming flow. Among these models, parametric models are considered to be the most suitable for use when priority among incoming flows is specified by functions that depend on one or more parameters. Such models are called parametric models and their research is rarely found in modern scientific literature, since a complex apparatus of the theory of random processes is used.

The article describes the class of all possible limit distributions for a random vector of waiting time in the same parametric system mass service with absolute priorities. In the process of the results, obtained limit theorems for univariate and multivariate characteristics of the system related to the timeouts.

Keywords: queueing system, waiting times, random vector, limit distributions, the period of employment, length of queue, random process, distribution function.

1. Introduction

The parametric model of queueing system considered in this paper is based on the principle of the quantification of the time axis (Bronshtein, 1976).

In a single-server queueing system with waiting there arrive independent Poisson flows of $1-$customers, ... , $r-$customers with parameters$a_1 > 0, \cdots, a_r > 0$, respectively. The service times are independent in their totality, do not depend on the arrival process and for the $i-$customers $(i=\overline{1,r})$ they have the distribution functions $B_i(t), B_i(+0)=0.$. There are no customers in the model at time$t=0$. The time axis is divided into intervals of fixed length, called "quanta":$[0,T),[T,2T),[2T,3T)$,...; an $i-$customer has absolute priority before a $j-$customer $(1 \le i < j \le r)$ if both arrive in the system in the same quantum. In the zones of all the flows, arriving at different quanta, the customers are served in the order of arrival. The indicated model is called a model with categorized-time absolute priorities or a (B,T) scheme (Simonyan, 2014; Danielyan, 1980). The quantity T is the parameter of the model $(0 \le T \le +\infty)$.

We introduce notations: $\rho_{k1} = \sum_{i=1}^{k} a_i \beta_{i1}$ is the load of the model by the $\overline{1,r}$ –customers (1 – customers, ... , $r-$customers), where $\beta_{k1} = \int_0^\infty t dB_k(t)$ $(k=\overline{1,r})$; $\rho_k = 1-\rho_{k1} (k=\overline{1,r})$ is the under-load of the model by the $\overline{1,r}$ – customers; $\overline{w}_k^T(t), (\overline{1,r})$ is the conditional virtual waiting time of a $k-$customer at the moment t under the condition that the accessibility of the customers in the model ceases starting from the moment t.

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The investigation is carried out the following conditions: if $\downarrow 0$, then for $\beta_k(s) = \int_0^\infty e^{-st} dB_k(t) (k = \overline{1,r})$ one has the expansions

$$\beta_k(s) = 1 - \beta_{k1}s + \alpha_k s^{\gamma(k)} + o(s^{\gamma(k)}) \quad (1 < \gamma_{(k)} \le 2), \qquad (1.1)$$

where $\alpha_k > 0$ constants.

For a given T, we represent time t in the form

$$t = nT + \tau \quad (n \ge 0; 0 < \tau \le T,\ n \text{ is an integer}).$$

For $t \to +\infty$ and an "arbitrary variation" of T, the limit distributions for $\overline{w}_k^T(t), k = \overline{1,r}$) depend in an essential manner on the relation between t and τ and on the ratio of the loads ρ_{k1} and ρ_{r1} we are interested in the case $\rho_{r1} = 1$. For the correct formulation of the problem one requires additional explanations. We turn for $t = nT + \tau \to +\infty$ $(n \ge 0; 0 < \tau \le T)$ to the ratio τ/t. The limit points of this ratio fill out completely the interval [0,1] at an arbitrary variation of T. If for τ/t when $t \to +\infty$ one considers the limit point 0, then one has to take the limit points of another ratio $\tau/t^{1/\gamma_r}$, where $\gamma_k = min(\gamma_{(1)}, \cdots, \gamma_{(k)})$ $(k = \overline{1,r})$.

The limit points of the last ratio, under the condition that one considers the limit point 0 of the ratio τ/t fills out completely the semiline [0, +∞). In the usual sense, for $t \to +\infty$, the limit distribution for $\overline{w}_k^T(t)$ does not exist. But if we take a sequence of moments $t_m \to +\infty$ when $m \to +\infty$ such that there exist the limits $\lim_{m\to+\infty} \left(\tau_m / t_m\right)$ and $\lim_{m\to+\infty}\left(\tau_m / t_m^{1/\gamma_r}\right)$, then there exists a limit distribution for $\overline{w}_k^{T_m}(t_m)$.

Consequently, we assume $t \to +\infty$, and thus, that there exists the limit

$$\lim_{t\to+\infty} \tau/t^{1/\gamma_r} \stackrel{\text{def}}{=} b' \ (0 \le b' \le +\infty,\ t = nT + \tau,\ n \ge 0,\ 0 < \tau \le T). \qquad (1.2)$$

Condition (1.2) appears at the investigation of the limiting DF for $\overline{w}_k^T(t), (k = \overline{1,r})$ when $\rho_{r1} = 1$. In (Danielyan, 1982) one solves a series of problems, one of which is formulated in the following manner.

Under the condition of the existence of the limit (1.2), it is necessary to describe the class of all possible limit laws for $\overline{w}_k^T(t), (k = \overline{1,r})$. We formulate the results of (Danielyan, 1982) for $\rho_{r1} = 1$. We introduce notations. Let $\widetilde{G_\alpha}(x) (1 < \alpha \le 2; x \ge 0)$ be the DF of a nonnegative random variable (RV), defined by its Laplace-Stieltes transform (LST)($\mathrm{Re}s \ge 0; \Gamma(x)$ is Euler's Gamma function):

$$\int_0^\infty e^{-sx} d\widetilde{G_\alpha}(x) = e^{s^\alpha}\left\{1 - \frac{s}{\Gamma(1/\alpha)}\int_0^1 e^{-s^\alpha v} v^{-(1-\alpha)^{-1}} dv\right\}.$$

Let $\overline{W}_k(x)$ $(k = \overline{1,r})$ be the stationary DF of the waiting time of a $k-$customer in the scheme $(B, +\infty)$. We set $(k = \overline{1,r})$

$$L_k = \{i : i \le k,\ \gamma_{(i)} = \gamma_k\}, B_k = \sum_{i\in L_k} a_i\alpha_i,\quad p_k = \rho_k\, b' / B_r^{1/\gamma_r}.$$

Table 1.

Load	Limit point	Normalization $N(t)$	Centering $M(t)$	Scheme	Limit distribution
$\rho_{k1} < 1, \rho_{r1} = 1$	$b' = +\infty$	1	0	(B,t)	$\overline{W}_k(x)$
$\rho_{k1} < 1, \rho_{r1} = 1$	$0 \le b' < +\infty$	$(B_r t)^{1/\gamma_r}$	0	(B,t)	$\widetilde{G}_{\gamma_r}(x + p_k)$

Under a unit load there exists the limit

$$\lim_{t\to+\infty} \mathrm{P}\left\{\frac{\overline{w}_k^T(t) + M(t)}{N(t)} < x\right\} = R_k(x), \qquad (1.3)$$

where the functions occurring in (1.3) are given by Table I.

We note that for $\rho_{r1} = 1$ and $k = r$ the condition of type (1.2) is not required. As shown in (Danielyan, 1981), there exists the limit (1.3), where $k = r$, $M(t) = 0$.

The Formulation of the Problem. Assume that in the scheme (B, T)we have $\rho_{r1} = 1$, the loads are fixed, conditions (1.1) hold, and for $t \to +\infty$, the quantity Tcan vary in an arbitrary manner. Under the condition of the existence of the limit (1.2), one has to describe the class of all possible limit laws for the vector $\overline{w}_1^T(t), \dots, \overline{w}_r^T(t)$.

In Sac. 2 we give auxiliary results, some of which will be proved.

In Sac. 3 we formulate and prove the fundamental results of the paper.

2. Auxiliary Results

The analysis of the scheme $(B, T)$is based on the relation between the processes

$$\overline{w}_k^T(t), (k = \overline{1, r}\,,\ t = nT + \tau,\ \ n \geq 0,\ \ 0 < \tau \leq T).$$

and the processes

$$b_k(u), w(u), \pi_k(u),$$

where $b_k(u)$ is the total service time of the $(\overline{1, k})$ – customers, arriving over a time interval of length u ($b_k(u)$ relative to $u$is a process with independent increments); $w(u)$ is the virtual waiting time at the moment u of the model M|G|1|∞ (Simonyan, 2004) with entrance intensity σ_r and with DF $B_r(t)$ of the service time of the customers (hare $\sigma_k = a_1 + \cdots + a_k, k = \overline{1, r}\,$,); $\pi_k(u)$ is the busy period in the servicing of the $(\overline{1, k})$ – customers with lag u in the scheme $(B, +\infty\ )$, i.e. the time interval starting with the lag u, at the beginning of which there are no customers, and ending first time after the gap when the servicing device is free of the$(\overline{1, k})$ – customers ($\pi_k(u)$relative to u is a process with independent increments). Here by the lag we mean the interval time in which the customers accumulate but are not served. The relation between the above described processes in the "terminology of RV" is established in (Danielyan, 1982) and is given by the following statement.

LEMMA 2.1. Let $t = nT + \tau$, $(n \geq 0,\ 0 < \tau \leq T)$. Then for $\overline{w}_k^T(t), (k = \overline{1, r}\,)$ we have the relations

$$\begin{cases} \overline{w}_k^T(t) \overset{d}{=} w(nT) + b_k(\tau) - \tau & \text{if } \pi_k\big(w(nT)\big) \geq \tau, \\ \overline{w}_k^T(t) \overset{d}{=} \overline{w}\left(\tau - \pi_k\big(w(nT)\big)\right) & \text{if } \pi_k\big(w(nT)\big) < \tau. \end{cases} \tag{2.1}$$

The symbol d indicates the equality of the DF of both sides of the random equality on the set indicated in the rlght-hand side of the equality; $\overline{w}_k^T(t)$ is the conditional virtual waiting time of a k – customer at the moment t in the scheme $(B, +\infty)$, i.e. $\overline{w}_k\ (t) \equiv \overline{w}_k^\infty(t)$. By virtue of formulas (2.1), for the proof of the limit theorems in the case $\overline{w}_k^T(t), (k = \overline{1, r}\,)$ for $t \to +\infty$, one has to have available the corresponding limit statements for the processes for $b_k(u), w(u), \pi_k(u)$ for $u \to +\infty$. We denote $(k = \overline{1, r};\ \ t \geq 0)$

$$b_k^*(t) = \frac{b_k(t) - \rho_{k1} t}{(B_k t)^{1/\gamma_k}}\,, \quad \chi(t) = \begin{cases} 1, & t \geq 0, \\ 0, & t < 0, \end{cases} \quad B_k^*(t) = \frac{\overline{b}_k(t) - (\rho_{k1} - \rho_{r-11})t}{(a_k \alpha_k)^{1/\gamma(k)}}.$$

Here $\overline{b}_k(t) = b_k(t) - b_{k-1}(t), (k = \overline{1, r}\,, b_0(t) = 0)$ is the total service time of the k – customer, arriving in an interval of time t.

LEMMA 2.2. Assume that conditions (1.1) hold. Then for $t \to +\infty$ there exist limits $(k = \overline{1, r}, i = \overline{2, r})$

$$\lim_{t \to +\infty} \mathrm{P}\{b_k^*(t) < x\} = G_{\gamma_k}(x) \qquad (-\infty \leq x \leq +\infty), \tag{2.2}$$

$$\lim_{t \to +\infty} \mathrm{P}\{\overline{b}_k^*(t) < x\} = G_{\gamma_{(k)}}(x) \qquad (-\infty \leq x \leq +\infty), \tag{2.3}$$

$$\lim_{t \to +\infty} \mathrm{P}\{b_{i-1}^*(t) < x, b_i^*(t) < y\,\} = G_i(x, y) \qquad (-\infty \leq x, y \leq +\infty), \tag{2.4}$$

where

$$G_i(x, y) = \int_{-\infty}^{x} G_{\gamma_i}\left(\left(\frac{B_i}{a_i \alpha_i}\right)^{1/\gamma_i} y - \left(\frac{B_{i-1}}{a_i \alpha_i}\right)^{1/\gamma_i} u\right) dG_{\gamma_i}(u), \quad (\gamma_{i-1} = \gamma_i), \tag{2.5}$$

$$G_i(x,y) = \int_{-\infty}^{x} \chi(y-u)dG_{\gamma_{i-1}}(u), \qquad (\gamma_{i-1} < \gamma_i), \tag{2.6}$$

$$G_i(x,y) = G_{\gamma_{i-1}}(x)G_{\gamma_i}(y), \quad (\gamma_{i-1} > \gamma_i). \tag{2.7}$$

Here $G_\alpha(x)(1 < \alpha \le 2;\ -\infty \le x \le +\infty)$ is a stable law with parameter α, defined by its characteristic function (i is the imaginary unit, s is a real number):

$$\int_{-\infty}^{+\infty} e^{isx}\, dG_\alpha(x) = e^{(-is)^\alpha}.$$

The proof of (2.2) is given in (Danielyan, 1982) and (2.3) is proved similary to (2.2). We processed to the proof of the relations (2.4) – (2.7).

We perform the computations ($k = \overline{2,r}$; t>0):

$$I_k(t,x) \overset{\text{def}}{=} \mathrm{P}\{b_{k-1}^*(t) < x, b_k^*(t) < y\} =$$

$$= \int_{-\infty}^{x} \mathrm{P}\left\{b_{k-1}^*(t)\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_{k-1}t)^{1/\gamma_k}} + b_r^*(t)\frac{(a_k\alpha_k)^{1/\gamma_{(k)}}}{(B_k t)^{1/\gamma_k}} < y \Big/ b_{k-1}^*(t) = u\right\} \cdot$$

$$\cdot\, d\{b_{k-1}^*(t) < u\} =$$

$$= \left(\int_{-\infty}^{-A} + \int_{-A}^{+\infty}\right) \mathrm{P}\left\{\bar{b}_k^*(t) << \frac{(B_k t)^{1/\gamma_k}}{(a_k\alpha_k)^{1/\gamma_{(k)}}}\left[y - u\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_k t)^{1/\gamma_k}}\right] d\{b_{k-1}^*(t) < u\}\right\}.$$

In this case for every $\varepsilon > 0$ the number $A > 0$ is select sufficiently large so that we should have the inequality $G_{\gamma_{k-1}}(-A) < \varepsilon$. Then by virtue of (2.2) we have ($k = \overline{2,r}$)

$$\lim_{t\to+\infty} I_k(t,x) = \lim_{t\to+\infty} I_k(t,-A) +$$

$$+ \lim_{t\to+\infty} \int_{-A}^{+\infty} \mathrm{P}\left\{b_k^*(t) << \frac{(B_k t)^{1/\gamma_k}}{(a_k\alpha_k t)^{1/\gamma_{(k)}}}\left[y - u\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_k t)^{1/\gamma_k}}\right] d\{b_{k-1}^*(t) < u\}\right\}, \tag{2.8}$$

where

$$\lim_{t\to+\infty} I_k(t,-A) \le \lim_{t\to+\infty} \int_{-\infty}^{-A} d\{b_{k-1}^*(t) < u\} = G_{\gamma_{k-1}}(-A) < \varepsilon \tag{2.9}$$

uniformly with respect to y o Making use of the statement of Theorem 1 from (Shilov, 1965) in the second term of the right-hand side of (2.8) we change the sign of the integral and we interchange the limits; then we let ε go to zero (then $A \to +\infty$), which by virtue of (2.9) yields ($k = \overline{2,r}$)

$$\lim_{t\to+\infty} I_k(t,x) =.$$

$$= \int_{-\infty}^{-A} \lim_{t\to+\infty} \mathrm{P}\left\{b_k^*(t) << \frac{(B_k t)^{1/\gamma_k}}{(a_k\alpha_k t)^{1/\gamma_{(k)}}}\left[y - u\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_k t)^{1/\gamma_k}}\right] dG_{\gamma_{k-1}}(u)\right\}. \tag{2.10}$$

By (2.10), the problem reduces to the computation of the limit ($k = \overline{2,r}$)

$$J_k(x,y) = \lim_{t\to+\infty} \mathrm{P}\left\{b_k^*(t) < \frac{(B_k t)^{1/\gamma_k}}{(a_k\alpha_k t)^{1/\gamma_{(k)}}}\left[y - u\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_k t)^{1/\gamma_k}}\right]\right\}.$$

We consider the mutually exclusive cases, mentioned in Lemma 2.2:

a) $\gamma_{k-1} = \gamma_{(k)}$. Obviously, $\gamma_{k-1} = \gamma_k$. Then by virtue of (2.3), we obtain

$$J_k(x,y) = G_{\gamma_k}\left(\left(\frac{B_k}{a_k\alpha_k}\right)^{1/\gamma_k} y - \left(\frac{B_{k-1}}{a_k\alpha_k}\right)^{1/\gamma_k} x\right),$$

for where there follows (2.5);

b) $\gamma_{k-1} < \gamma_{(k)}$. Obviously, $\gamma_{(k)} > \gamma_{k-1} = \gamma_k$ and $B_{k-1} = B_k$. Then

$$\lim_{t\to+\infty} \frac{(B_k t)^{1/\gamma_k}}{(a_k\alpha_k t)^{1/\gamma_{(k)}}} = +\infty,$$

$$\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_k t)^{1/\gamma_k}} = 1 \ .$$

Consequently, $J_k(x,y) = \chi(y-x)$, from where we o,tain (2.6);

c) $\gamma_{k-1} > \gamma_{(k)}$. Obviously, $\gamma_{k-1} > \gamma_k = \gamma_{(k)}$ and $a_k\alpha_k = B_k$. Then

$$\frac{(B_k t)^{1/\gamma_k}}{(a_k\alpha_k t)^{1/\gamma_k}} = 1\,,$$

$$\lim_{t\to+\infty}\frac{(B_{k-1}t)^{1/\gamma_{k-1}}}{(B_k t)^{1/\gamma_k}} = +\infty\,,$$

and, consequently, $J_k(x,y) = G_{\gamma_k}(y)$, from where there follows (2.7).

COROLLARY 2.1. The limit relation (2.2) is a consequence of the relation (2.4). Namely $(k = \overline{2,r})$

$$G_k(x,+\infty) = G_{\gamma_{k-1}}(x)\,,$$

$$G_k(+\infty,y) = G_{\gamma_k}(y)\,. \qquad (2.11)$$

The proof of the first of the equalities (2.11) follows trivially from (2.5)-(2.7), while the proof of the second one is obvious except in the case $\gamma_{k-1} = \gamma_k == \gamma_{(k)}$. On the basis of (2.5) we have

$$G_k(+\infty,y) = \int_{-\infty}^{+\infty} G_k\left(\left(\frac{B_k}{a_k\alpha_k}\right)^{1/\gamma_k}(y-z)\right) d_z G_{\gamma_k}\left(\left(\frac{B_k}{B_{k-1}}\right)^{1/\gamma_k} z\right) =$$

$$= G_{\gamma_k}\left(\left(\frac{a_k\alpha_k}{B_k}\right)^{1/\gamma_k} y\right) * G_{\gamma_k}\left(\left(\frac{B_{k-1}}{B_k}\right)^{1/\gamma_k} y\right). \qquad (2.11)$$

where * is the convolution sign, while the symbol above the convolution sign indicates the variable with respect to which the convolution is taken. The right-hand side of (2.12) is the DF of the random variable

$$\eta = \left(\frac{a_k\alpha_k}{B_k}\right)^{1/\gamma_k}\xi_1 + \left(\frac{B_{k-1}}{B_k}\right)^{1/\gamma_k}\xi_2\,,$$

where the RV ξ_1 and ξ_2 in our case we have has a stable DF $G_{\gamma_k}(y)$. Sinc in our case we have $B_k = a_k\alpha_k + B_{k-1}$, on the basis of Theorem 2 (Feller, 1950), we conclude that the RV η has a stable DF $G_{\gamma_k}(y)$; this proves the second of the equalities (2.11).

It is known (Feller, 1950) that the DF $G_\alpha(x)(1 < \alpha \le 2;\ -\infty \le x \le +\infty)$ has density $g_\alpha(x)$. It turns out that the DF $G_k(x,y)$ has density

$$g_k(x,y) = \frac{d^2}{dxdy}G_k(x,y)\,.$$

COROLLARY 2.2. a) For $\gamma_{k-1} = \gamma_{(k)}$ we have

$$g_k(x,y) = \left(\frac{B_k}{a_k\alpha_k}\right)^{1/\gamma_k} g_{\gamma_k}\left(\left(\frac{B_k}{a_k\alpha_k}\right)^{1/\gamma_k} y - \left(\frac{B_{k-1}}{a_k\alpha_k}\right)^{1/\gamma_k} x\right) g_{\gamma_k}(y)\,;$$

b) for $\gamma_{k-1} < \gamma_{(k)}$ we have

$$g_k(x,y) = \chi(y-x)g_{\gamma_k}(x)\,,$$

where

$$\chi(y-x)\begin{cases}1 \text{ if } x = y,\\ 0 \text{ if } x\neq y.\end{cases}$$

c) for $\gamma_{k-1} > \gamma_{(k)}$ we have

$$g_k(x,y) = g_{\gamma_{k-1}}(x)g_{\gamma_k}(y).$$

COROLLARY 2.3. Assume that conditions (1.1) hold. Then there exists the limit

$$\lim_{t\to+\infty} \mathrm{P}\{b_k^*(t) < x_k\ (k = \overline{1,r})\} = G(x_1,\ldots,x_r),$$

where $G(x_1,\ldots,x_r)$ has the multidimensional density

$$G(x_1,\ldots,x_r) = \int_{-\infty}^{x_1}\ldots\int_{-\infty}^{x_r} g(u_1,\ldots,u_r)du_1\ldots du_r.$$

Moreover,

$$g(x_1,\dots,x_r)=\frac{\prod_{k=2}^{r} g_k(x_{k-1},x_k)}{\prod_{k=2}^{r} g_{\gamma_k}(x_k)}\ .$$

The proof is similar to the proof of Corollary I from (Grigoryan, 1982).

LEMMA 2.3. Assume that in the scheme $(B,+\infty)$of the model M$_r$ |G$_r$|1|∞ we have $\rho_{r1}=1$, the loads are fixed, and conditions (1.1) are satisfied. Then there exists the limit

$$\lim_{t\to+\infty} \mathrm{P}\left\{\frac{\rho_k\pi_k(w(t))}{(B_r t)^{1/\gamma_r}}<x\right\}=\tilde{G}_{\gamma_r}(x)\quad (x\ge 0)\ .$$

We consider the vector function $(u_1\le u_2\le\cdots\le u_{r-1})$:

$$\left(\pi_1(u_1),\cdots,\pi_{r-1}(u_{r-1})\right)=\left(\pi_1^{(1)}(u_1),\cdots,\pi_{r-1}^{(1)}(u_{r-1})\right)+$$

$$+\left(0,\pi_2^{(2)}(u_2-u_1),\cdots,\pi_{r-1}^{(2)}(u_2-u_1)\right)+$$

$$+\left(\underbrace{0,\cdots,0}_{r-2},\pi_{r-1}^{(r-1)}(u_{r-1}-u_{r-2})\right).\qquad(2.13)$$

Here the processes $\pi_k^{(i)}(u)$ $\left(i=\overline{1,k}\right)$ and $\pi_k(u)$ are independent and identically distributed. IN the right-hand side of (2.13) the vectors are added component wise.

Assume that the following conditions hold:

for $u_1\to+\infty$ $(0<\rho_1u_1\le\cdots\le\rho_{r-1}u_{r-1})$ ther exist the limits

$$\lim_{u_1\to+\infty}\frac{u_k}{u_{k-1}}\ .\qquad(2.14)$$

Then a consequence of a theorem of E. A. Danielyan (Danielyan, 1980) holds

LEMMA 2.4. . Assume that in the scheme $(B,+\infty)$of the model M$_r$ |G$_r$|1|∞ we have $\rho_{r1}=1$, the loads are fixed, and conditions (1.1) and (2.14) are satisfied. Then there exists the limit

$$\lim_{u_1\to+\infty} \mathrm{P}\left\{\frac{\pi_k(\rho_k u_k)}{u_k}<x_k\ (k=\overline{1,r-1})\right\}=E(x_1,\dots,x_{r-1}),$$

where

$$E(x_1,\dots,x_{r-1})=\begin{cases}1 & \text{for } x_1>1,\cdots,x_{r-1}>1,\\ 0 & \text{otherwise.}\end{cases}$$

3. Fundamental Results

We describe the class of all possible limit DF of the vector process $\left(\overline{w}_1^T(t),\cdots,\overline{w}_r^T(t)\right)$ for $t=nT+\tau,\ (n\ge 0,\ 0<\tau\le T)$, $\rho_{r1}=1$, fixed loads and for an arbitrary variation of T.

TEOREM 3.1. Let $\rho_{r1}=1$, assume that the loads are fixed, conditions (1.1) are satisfied, and for $t\to+\infty$ there exists the limit (1.2).

a) If $b'=+\infty$ then there exist the limit $(x_i\ge 0; i=\overline{1,r})$

$$\lim_{t\to+\infty}\mathrm{P}\left\{\overline{w}_k^T(t)<x_k\ (k=\overline{1,r-1}),\frac{\overline{w}_r^T(t)}{(B_r t)^{1/\gamma_r}}<x_r\right\}=$$

$$=\overline{W}(x_1,\cdots,x_{r-1})\tilde{G}_{\gamma_r}(x),\qquad(3.1)$$

where $\overline{W}(x_1,\cdots,x_{r-1})$ is the limit distribution for $t\to+\infty$ and $\rho_{r-11}<1$ jf the vector $\left(\overline{w}_1^{\infty}(t),\cdots,\overline{w}_r^{\infty}(t)\right)$;

b) If $0\le b'<+\infty$ then there exist the limit $(x_i\ge 0; i=\overline{1,r})$

$$\lim_{t\to+\infty}\mathrm{P}\{\overline{w}_k^T(t)<x_k\ (k=\overline{1,r})\}=G_{\gamma_k}\left(\min_{1\le i\le r}(x_i+p_i)\right).\qquad(3.2)$$

PROOF. The proof of the theorem follows immediately from Lemmas 2.1-2.4.

References

Bronshtein, 1976 – *Bronshtein O.I., Dukhovnyi I.M.* (1976). Modeli prioritetnogo obsluzhivaniya v informatsionnovychislitel'nykh sistemakh [Models of priority maintenance in information and computing systems]. M.: Nauka, 220 p.

Simonyan, 2014 – *Simonyan A.R., Simonyan R.A.* (2014). Modern Trends in the Study of Single-channel parametric Models of Queueing // *Modeling of Artificial Intelligence*. № 4 (4). pp. 184-188.

Simonyan, 2005 – *Simonyan A.R., Ulitina E.I.* (2005). O parametricheskikh modelyakh massovogo obsluzhivaniya [On parametric queuing models]. // *Obozrenie prikladnoi i* *promyshlennoi matematiki*. T. 12. S. 184.

Danielyan, 1980 – *Danielyan E.A.* (1980). Waiting time in a model with categorized-time priorities // *Kibernetika*, No. 6, pp. 103-109.

Danielyan, 1982 – *Danielyan E.A., Khostikyan P.T.* (1982). Limit theorems in the models M_r $|G_r$ $|1|$ ∞ with time-categorized absolute and relative priorities for fixed // *Intercollegiate Transactions in Mathematics*, No. 1, Yerevan , pp. 134-164.

Danielyan, 1981 – *Danielyan E.A.* (1981). The mathematical theory of the priority models M_r $|G_r$ $|1|$ ∞. Abstract of Doctoral Dissertation, Moscow State Univ.

Simonyan, 2004 – *Simonyan A.R, Ulitina E.I.* (2004). A Theorem on the convergence to a stableaw in the $M|G|1|\infty$ model // *Russian Mathematical Surveys*. T. 59. № 3. C. 589-590.

Shilov, 1965 – *Shilov G.E.* (1965). Mathematical analysis. Oxford, New York, Pergamon Press, 485 p.

Feller, 1950 – *Feller W.* (1950). Introduction to Probability Theory and its Applications (Volume 2). Published by John Wiley and Sons, 626 p.

Grigoryan, 1982 – *Grigoryan G., Danielyan E.A.* (1982). On a method of analysis of priority systems // *Intercollegiate Transactions in Mathematics*, No. 1, Yerevan, pp.165-196.