

Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHII (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	

SOI: [1.1/TAS](#) DOI: [10.15863/TAS](#)

International Scientific Journal Theoretical & Applied Science

p-ISSN: 2308-4944 (print) e-ISSN: 2409-0085 (online)

Year: 2016 Issue: 6 Volume: 38

Published: 30.06.2016 <http://T-Science.org>

Alexandr Shevtsov

candidate of technical sciences, member of PILA (USA),
member of European Academy of Natural History (UK),
corresponding member of the Kazakhstan National
Academy of Natural Sciences,
President of International Academy TAS,
Department of «Mathematics», Deputy Director on
Science of faculty of information technologies,
automation and telecommunications,
Taraz state University named after M.Kh. Dulati,
Kazakhstan

Shev_AlexXXXX@mail.ru

SECTION 2. Applied mathematics. Mathematical modeling.

SOME ALGORITHMS BUILD THE BIFURCATION CURVES OF THE LORENZ ATTRACTOR IN MAPLE

Abstract: Here are investigated some problems of constructing bifurcation curves for the Lorenz attractor on Maple.

Key words: Lorenz, maple, bifurcation curves.

Language: English

Citation: Shevtsov AN (2016) SOME ALGORITHMS BUILD THE BIFURCATION CURVES OF THE LORENZ ATTRACTOR IN MAPLE. ISJ Theoretical & Applied Science, 06 (38): 166-191.

Soi: <http://s-o-i.org/1.1/TAS-06-38-33> *Doi:*  <http://dx.doi.org/10.15863/TAS.2016.06.38.33>

Introduction

The construction of the Lorenz model associated with the forecasting processes, stochastic attractors, turbulence, etc. In fact, the Lorenz model is too simple, not to expect stochasticity in much more complex systems. Bifurcation analysis it was shown that sometimes there is a real opportunity to

build a "bifurcation tree," indicating the sequence of the various metamorphoses with the solutions in the parameter space of the system. Thus, it is possible to find a way of appearance of turbulence and build a scenario for its development.[1-12]

Model

Consider the model of the Lorenz attractor:

```
> restart;
with(plots) :
with(plottools, line) :
with(Detools) :
n := 1000;
s := 10;
r := 28;
b := 8/3;
x[0] := 5;
y[0] := 3;
z[0] := 5;
t := 0.01;
for i from 1 to n do
x[i] := x[i-1] + s·(y[i-1] - x[i-1])·t;
y[i] := y[i-1] + (x[i-1]·(r - z[i-1]) - y[i-1])·t;
z[i] := z[i-1] + (x[i-1]·y[i-1] - b·z[i-1])·t;
od
points := {seq([x[i], y[i], z[i]], i = 0 ..n)} :

pointplot3d(points, axes = boxed);
spacecurve([cos(t), sin(t), t], t = 0 ..4·Pi);
```



Impact Factor:

ISRA (India)	= 1.344	SIS (USA)	= 0.912	ICV (Poland)	= 6.630
ISI (Dubai, UAE)	= 0.829	PIHHI (Russia)	= 0.234	PIF (India)	= 1.940
GIF (Australia)	= 0.564	ESJI (KZ)	= 1.042	IBI (India)	= 4.260
JIF	= 1.500	SJIF (Morocco)	= 2.031		

```

n:= 1000
s:= 10
r:= 28
b:= 8/3
x0:= 5
y0:= 3
z0:= 5
t:= 0.01

```

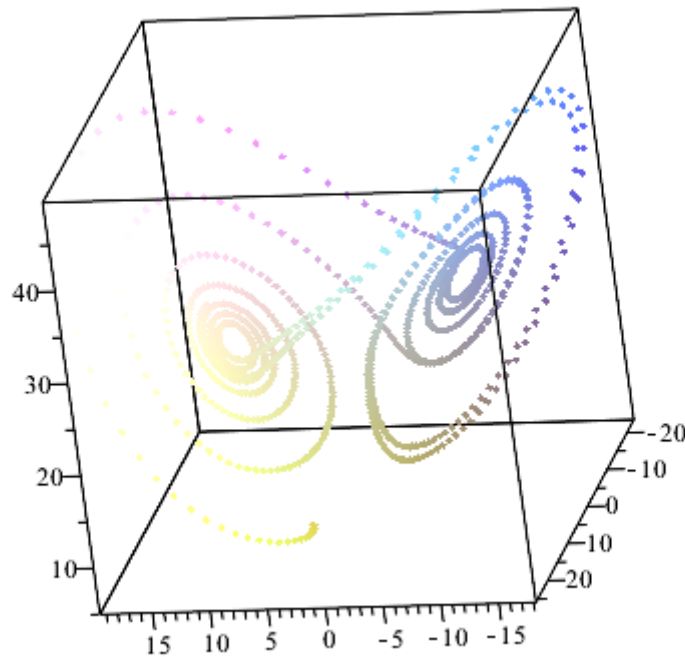


Figure 1 - Model of the Lorenz attractor.

```

> restart;
with(plots):
  with(plottools, line):
    n:= 150;
    t0:= 2.5;
    t:= 0.01;
    x[0]:= 3.051523;
    y[0]:= 1.582542;
    z[0]:= 15.62388;

    for s from 10 to 10 do
      for b from 0 to 9 do
        for r from 0 to 9 do
          for i from 1 to n do
            x[i]:= x[i-1] + s*(y[i-1] - x[i-1])*t;
            y[i]:= y[i-1] + (x[i-1]*(14 + r - z[i-1]) - y[i-1])*t;
            z[i]:= z[i-1] + (x[i-1]*y[i-1] - (b/3)*z[i-1])*t;
          od;
          px[100*s + 10*b + r] := {seq([t0 + t*i, x[i]*t], i=1..n)};
          py[100*s + 10*b + r] := {seq([t0 + t*i, y[i]*t], i=1..n)};
          pz[100*s + 10*b + r] := {seq([t0 + t*i, z[i]*t], i=1..n)};

          a[100*s + 10*b + r] := {seq([x[i], y[i], z[i]], i=1..n)}:
        od;
      od;
    od;

```

Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	

```
q1 := 1000;
q2 := 1099;
for k from q1 to q2 do
c[k] := pointplot3d(a[k], axes=boxed);
ccx[k] := plot(px[k]);
ccy[k] := plot(py[k]);
ccz[k] := plot(pz[k])
od:
display(seq(c[j], j=q1..q2), color=[red]);
display(seq(ccx[j], j=q1..q2));
display(seq(ccy[j], j=q1..q2));
display(seq(ccz[j], j=q1..q2));
n:=150
t0:=2.5
t:=0.01
x0:=3.051523
y0:=1.582542
z0:=15.62388
q1:=1000
q2:=1099
```

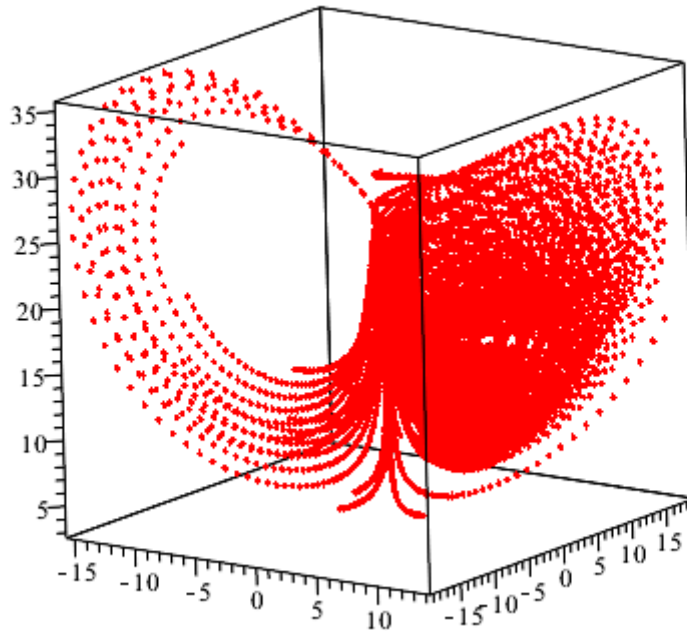


Figure 2 - Model of the Lorenz attractor, $r = 0.9$, $b = 0.9$, $s = 10$.

Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PИИИ (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	

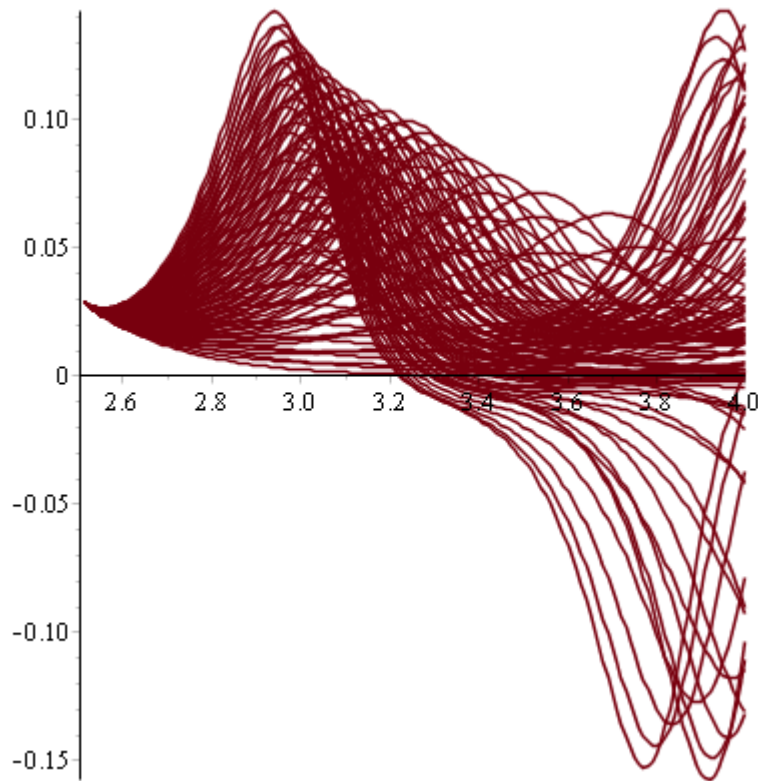


Figure 3 - Bifurcation of the function $x[t]$.

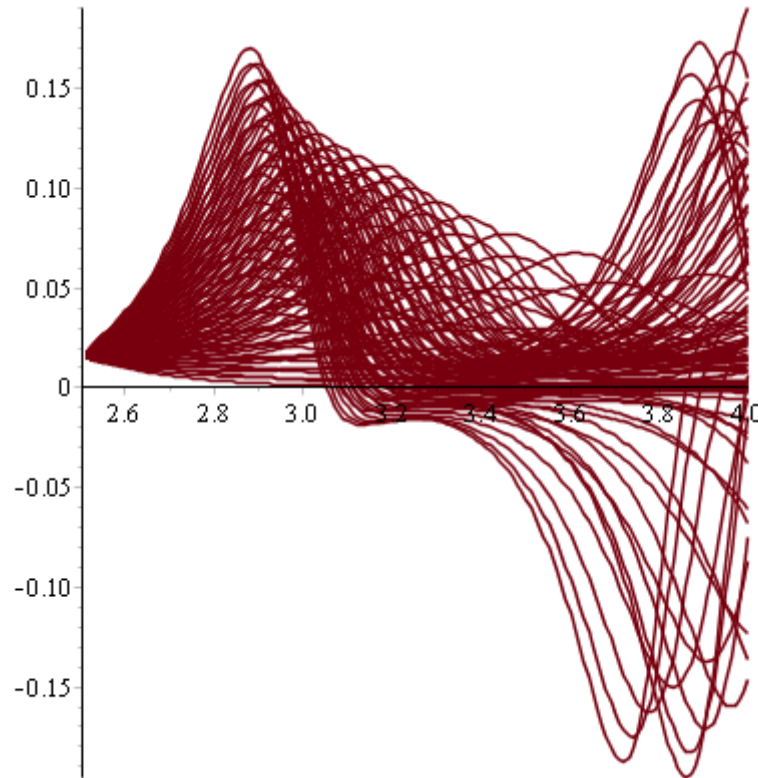


Figure 4 - Bifurcation of the function $y[t]$.

Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	

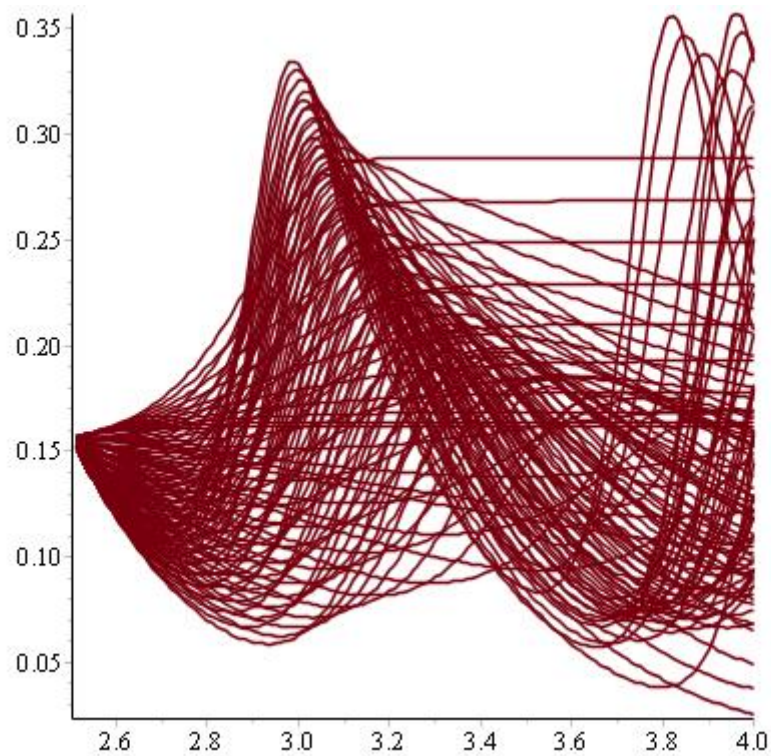


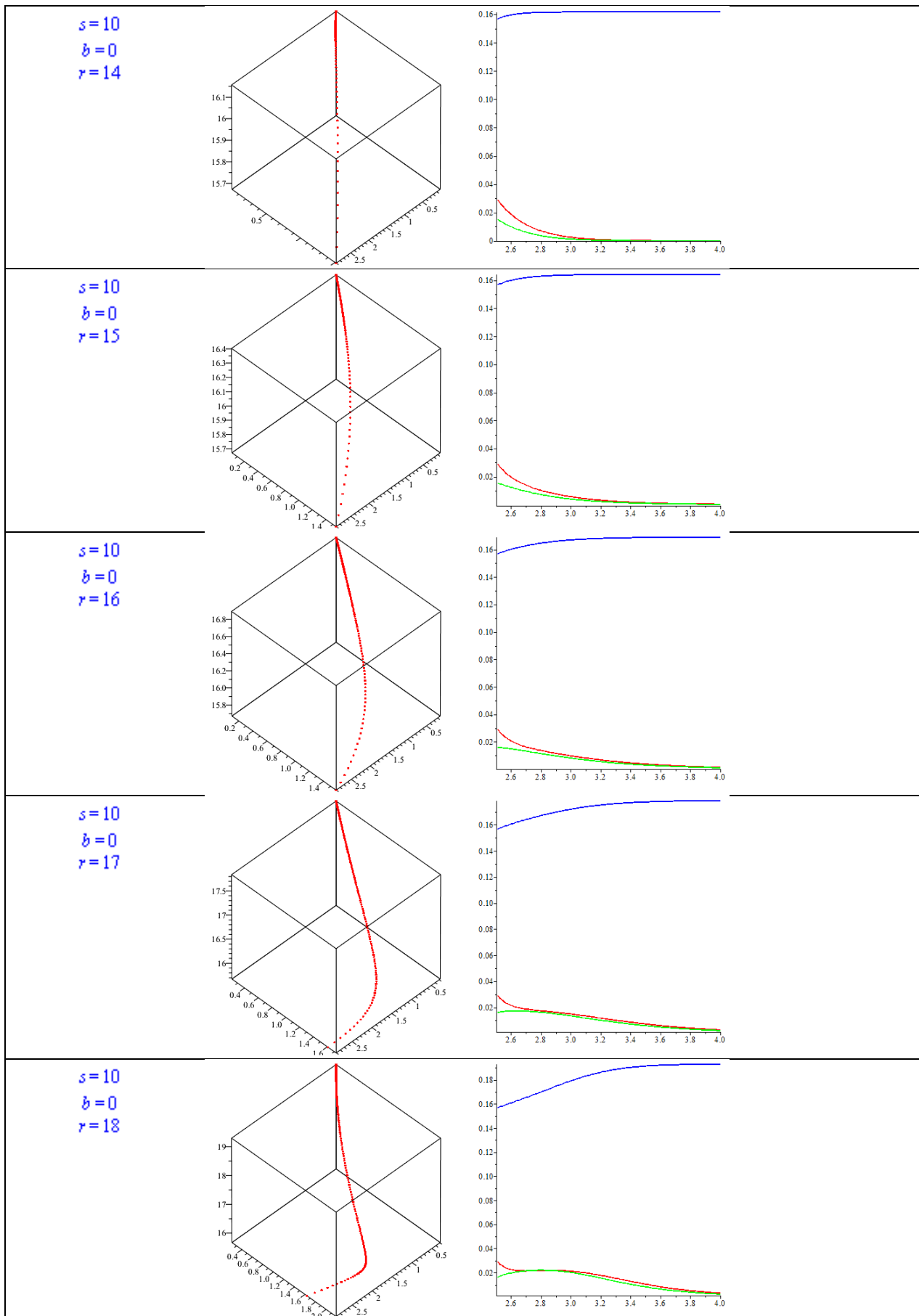
Figure 5 - Bifurcation of the function $z[t]$.

```
q1 := 1000;
q2 := 1099;
for k from q1 to q2 do
  c[k] := pointplot3d(a[k], axes=boxed);
  ccx[k] := plot(px[k], color=red);
  ccy[k] := plot(py[k], color=green);
  ccz[k] := plot(pz[k], color=blue)
od;

for i from 1000 to 1099 do
  q1 := i;
  q2 := i;
  display(seq(c[j], j=q1..q2), color=[red]);
  display(ccx[i], ccy[i], ccz[i]);
od;
```

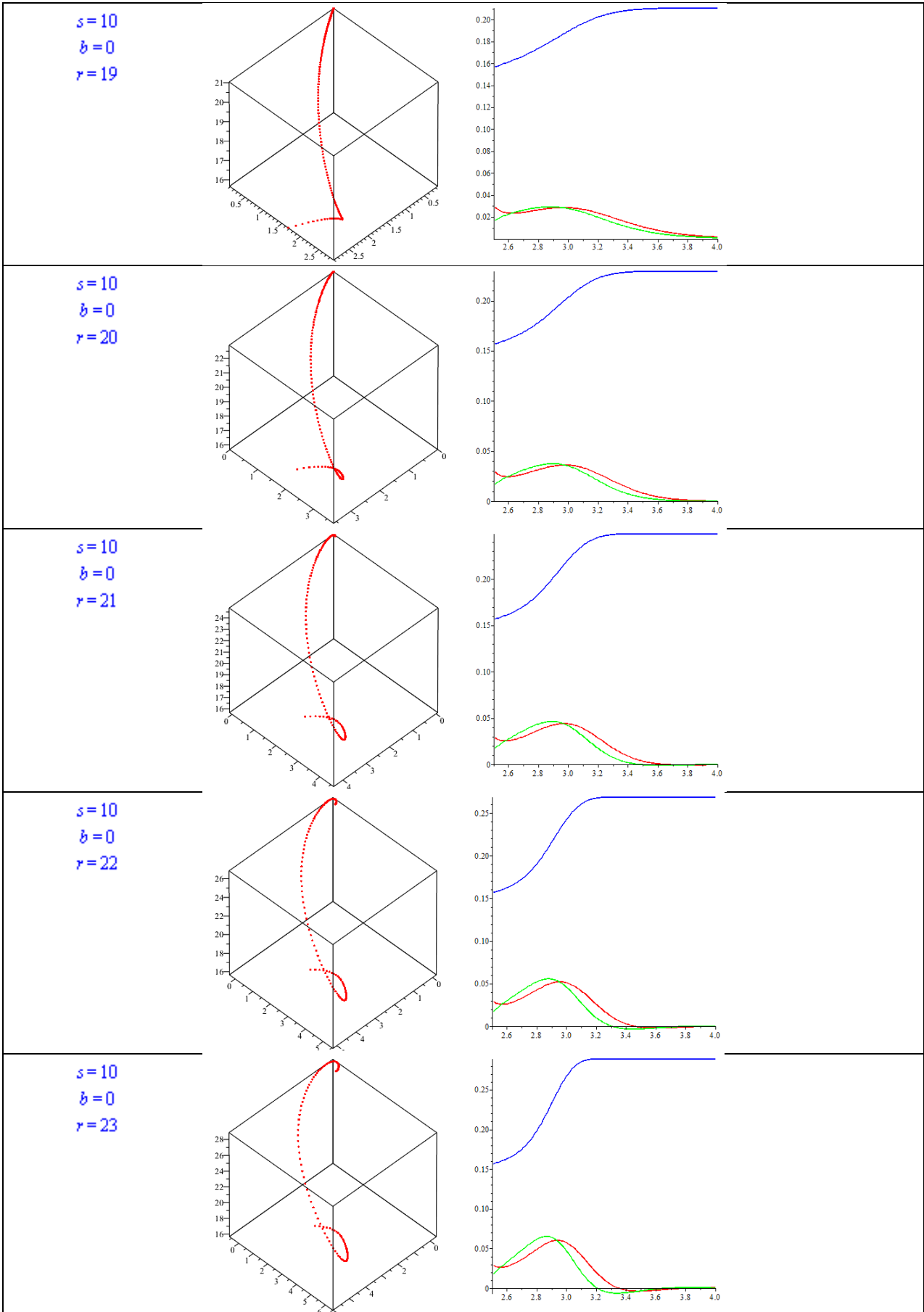
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



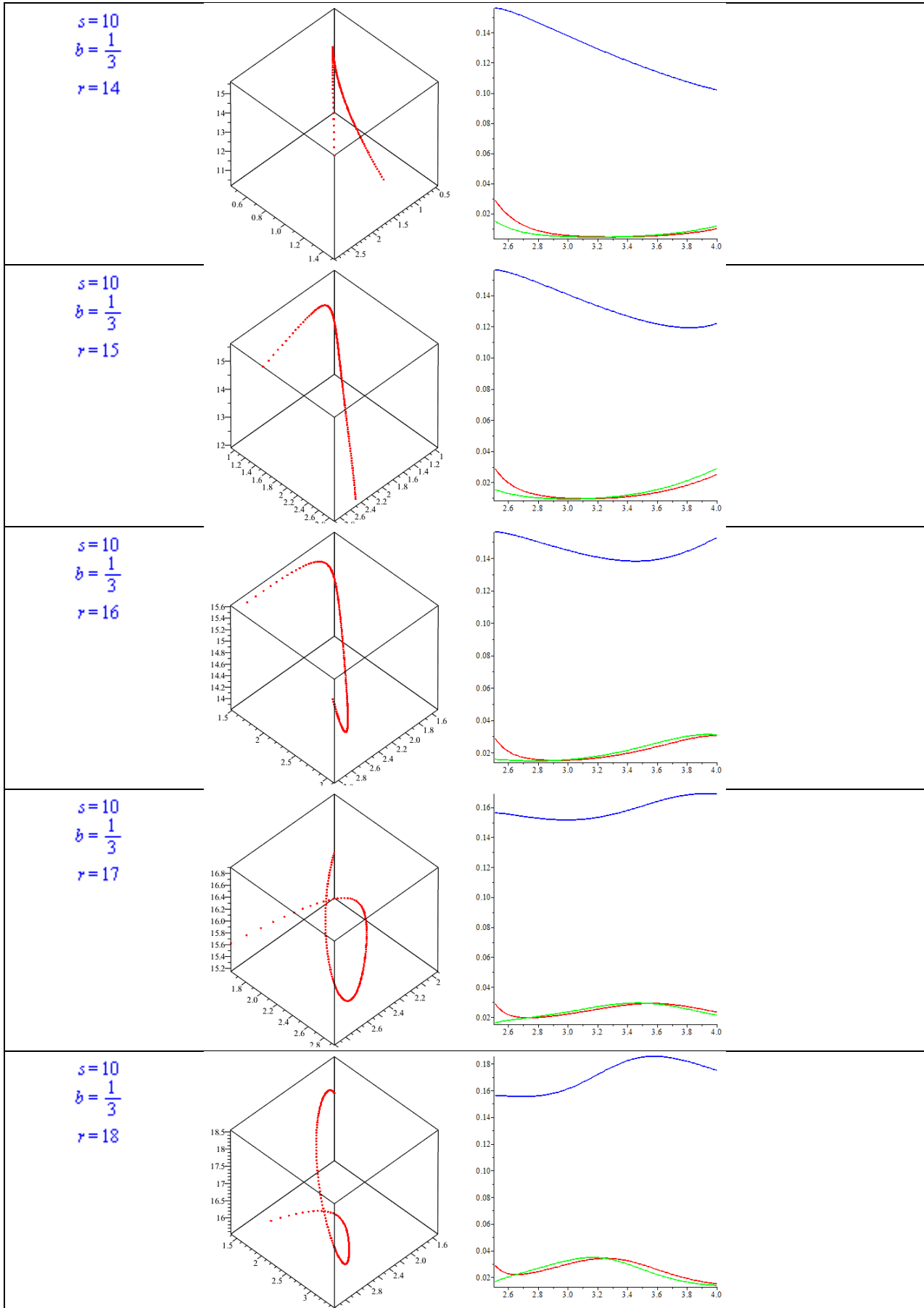
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



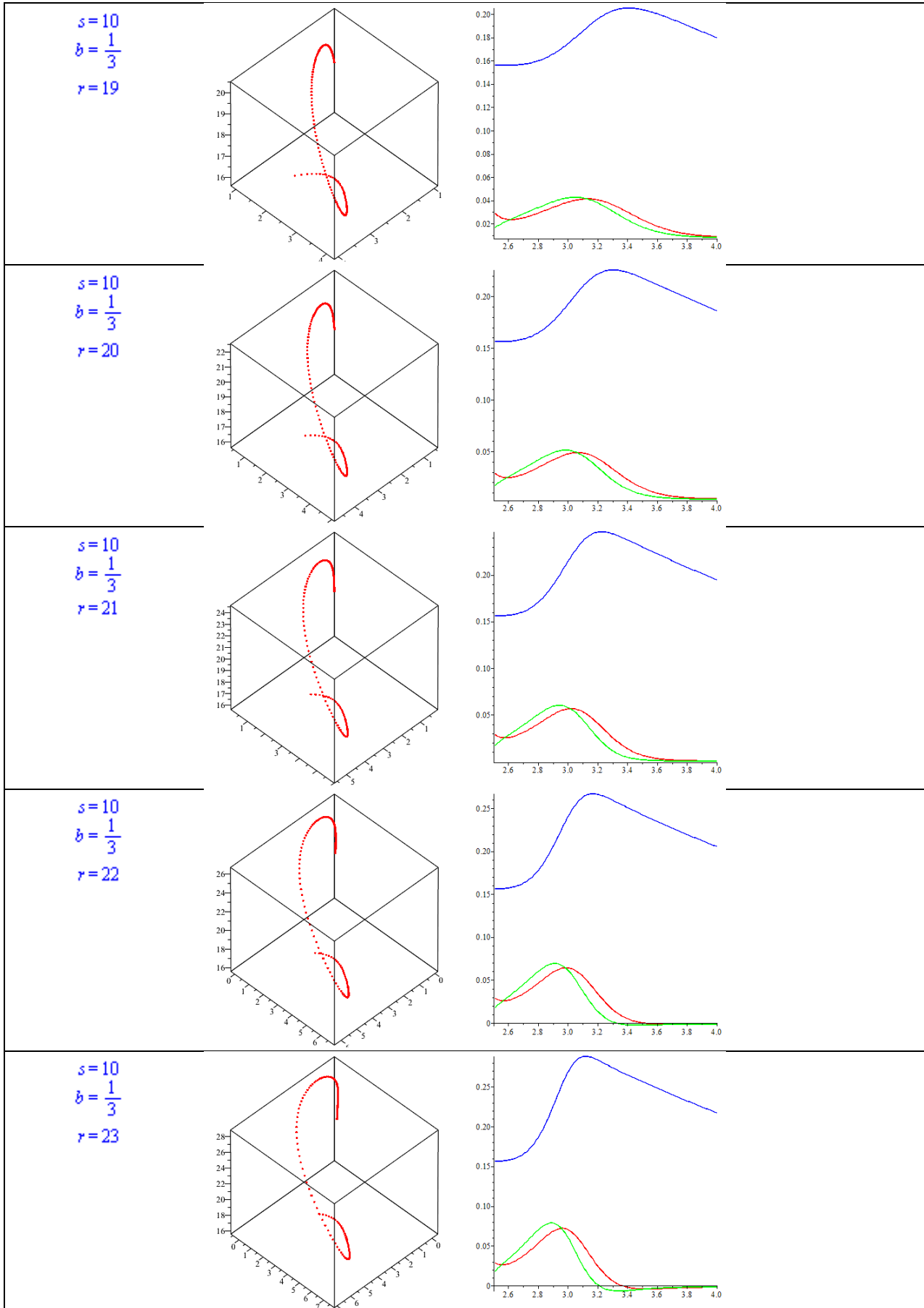
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



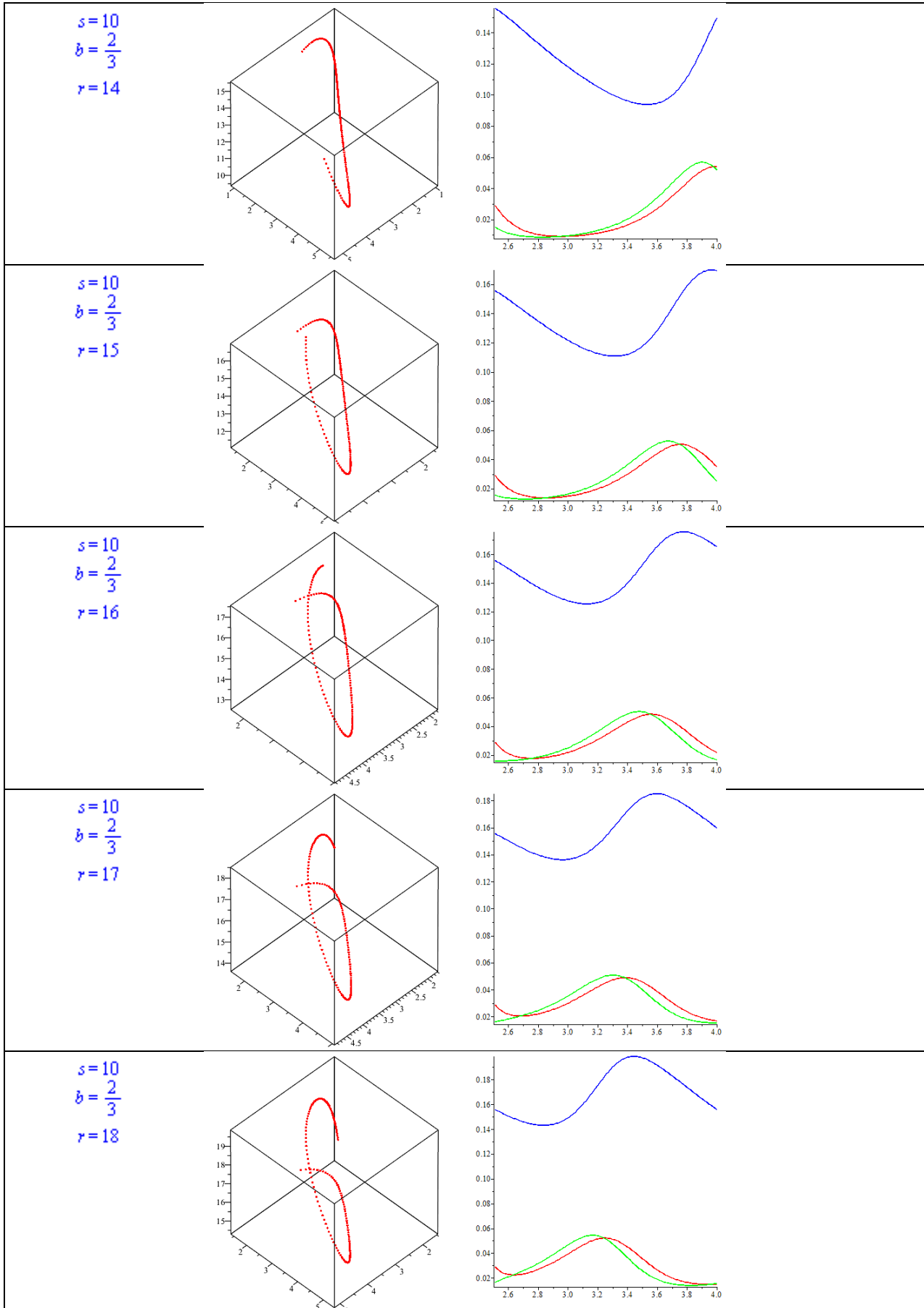
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



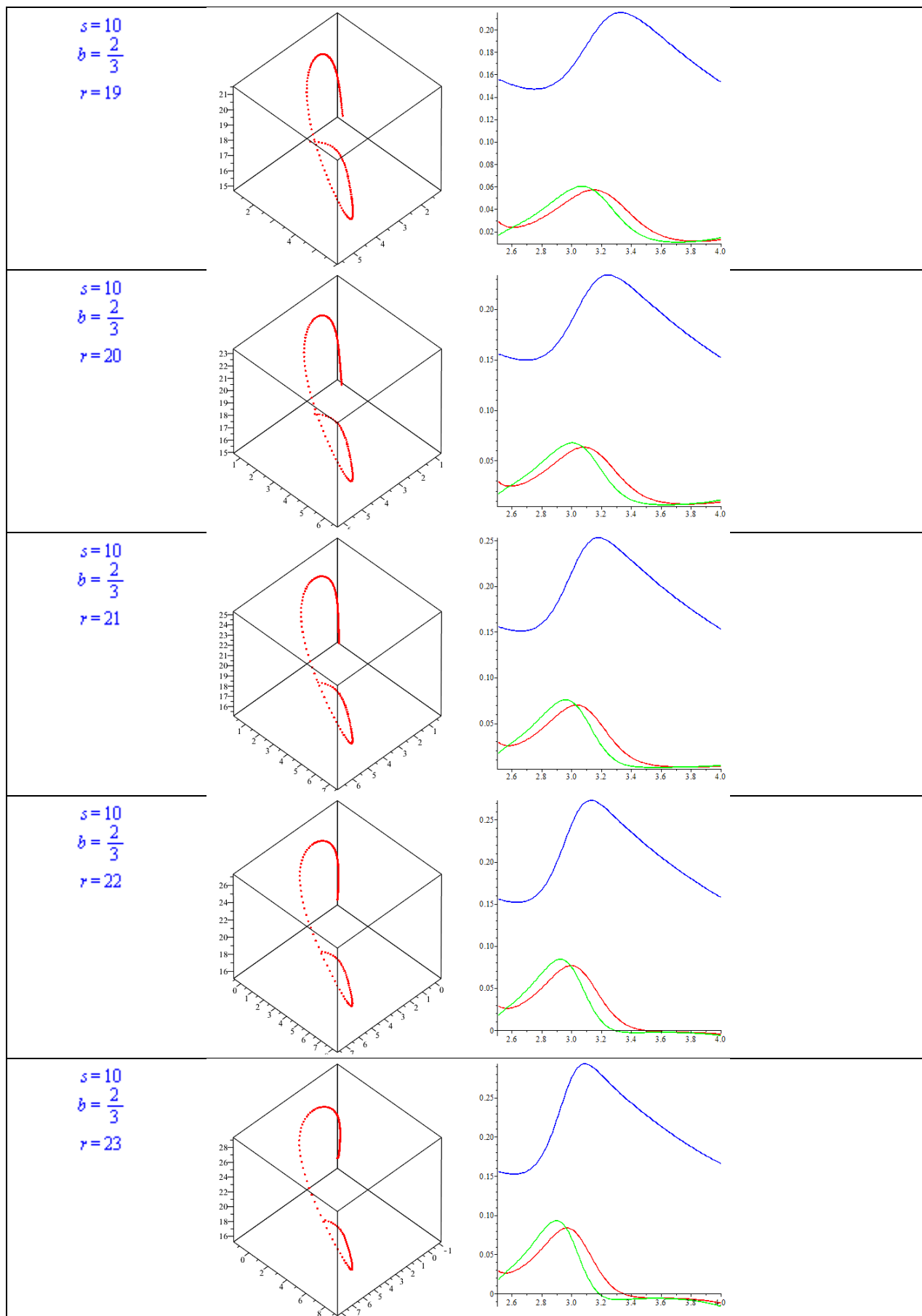
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



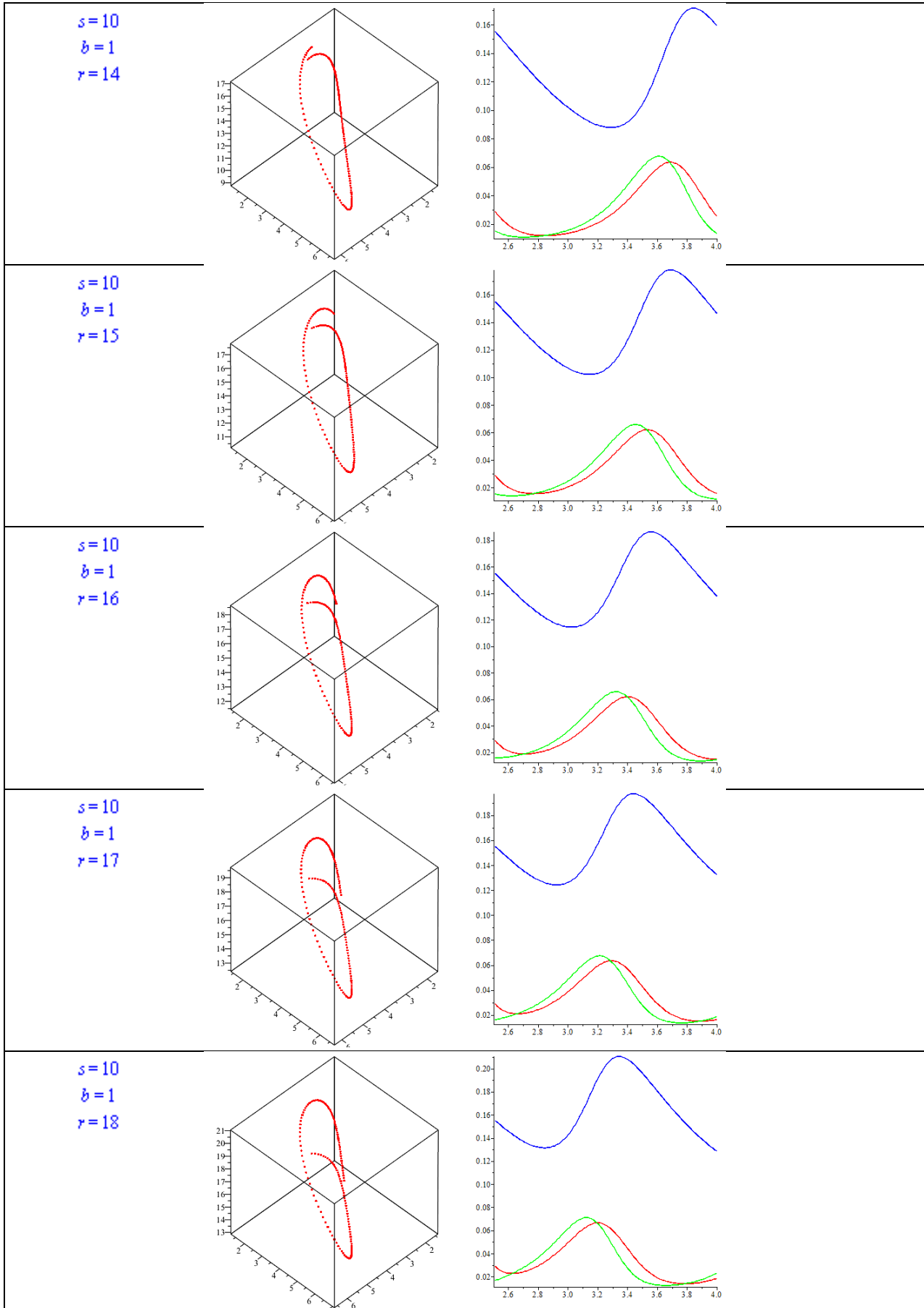
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



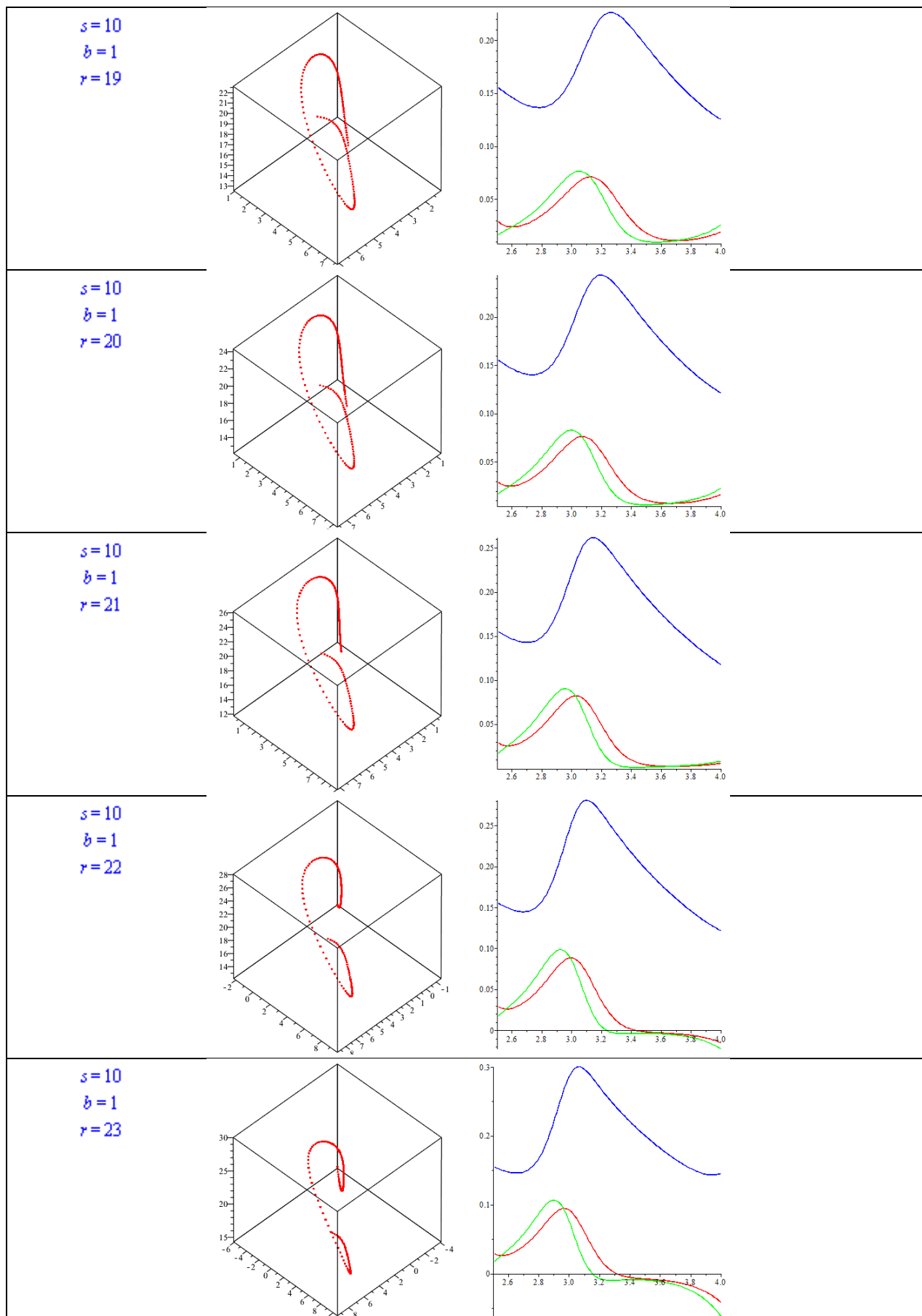
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



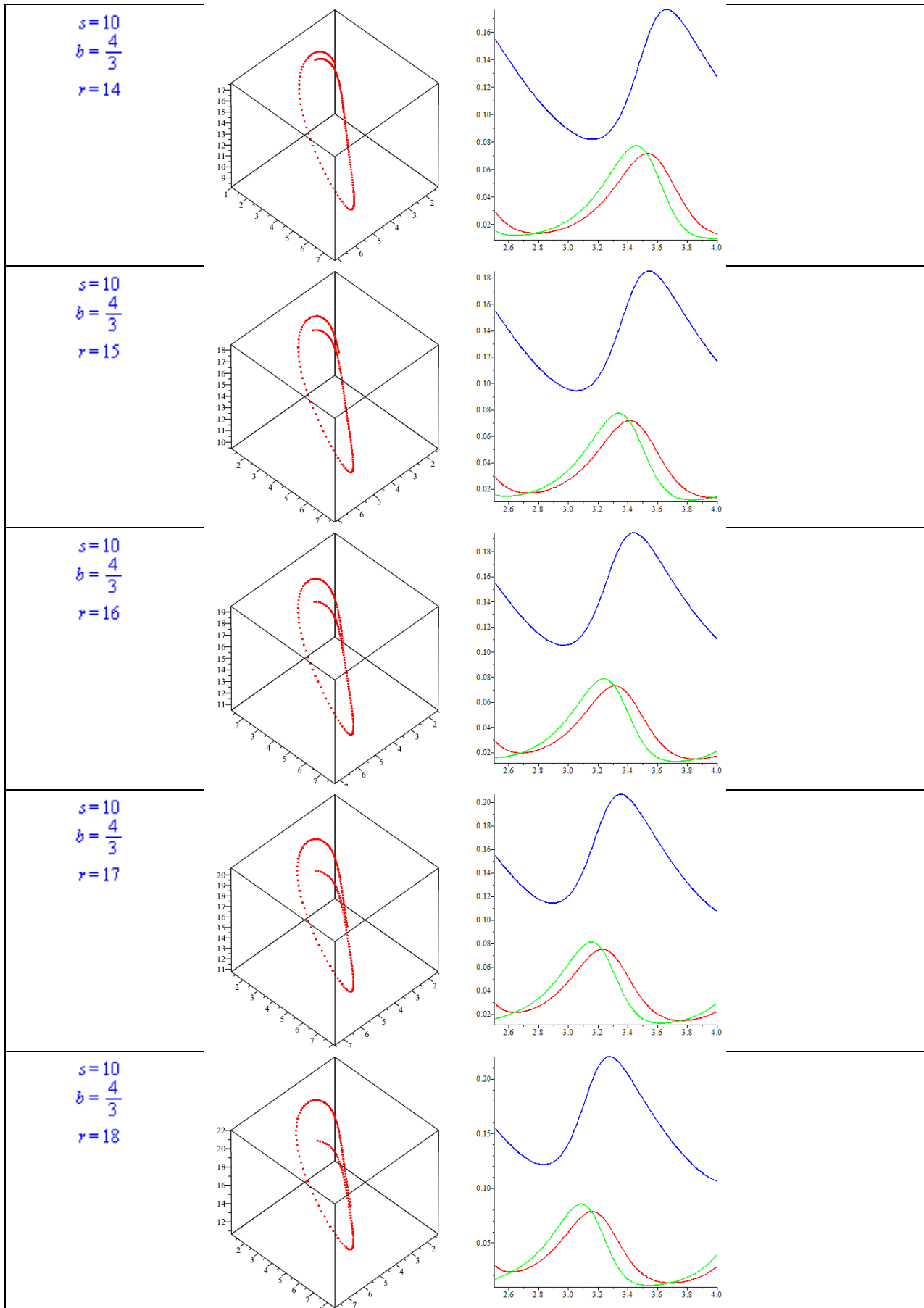
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



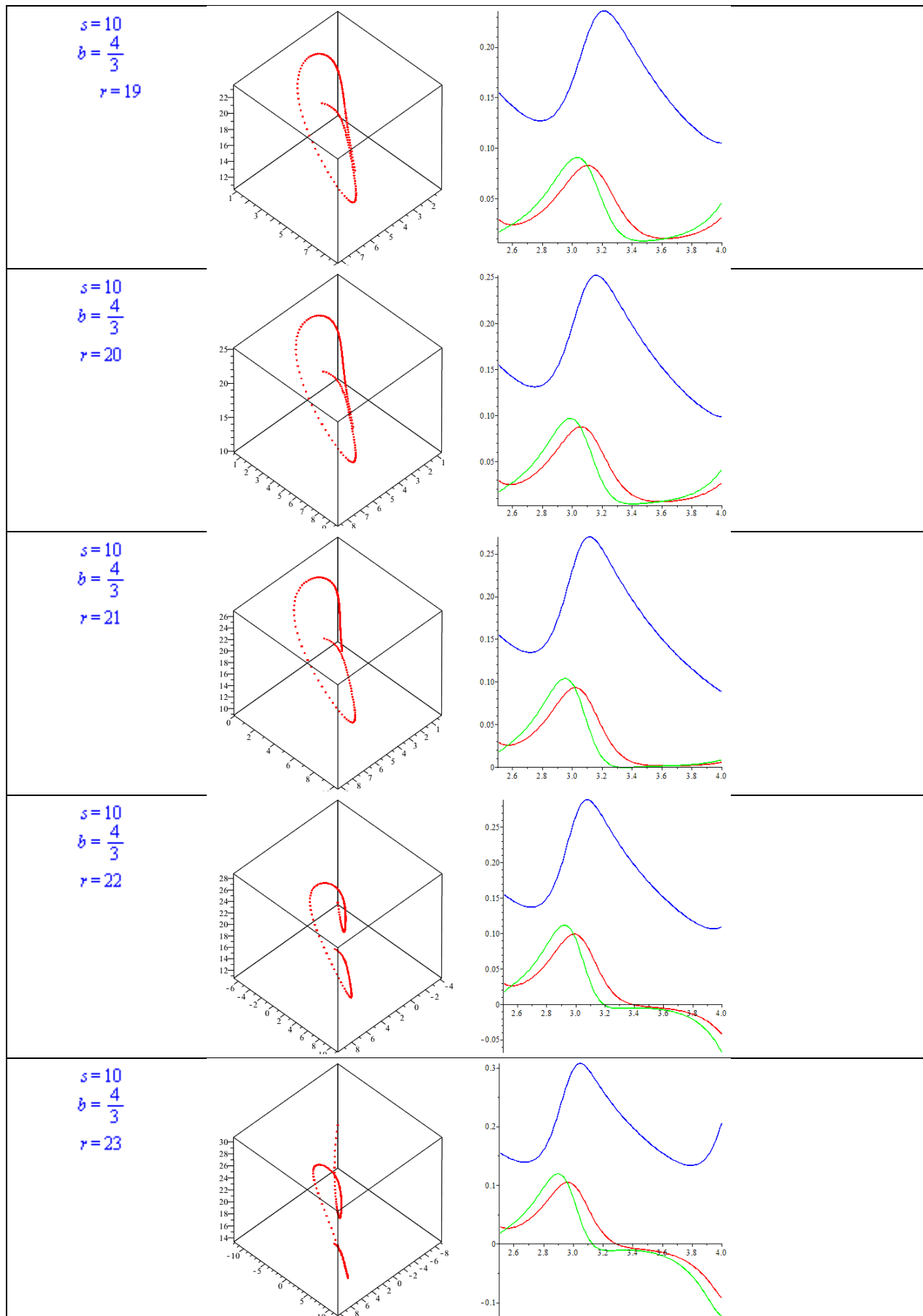
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



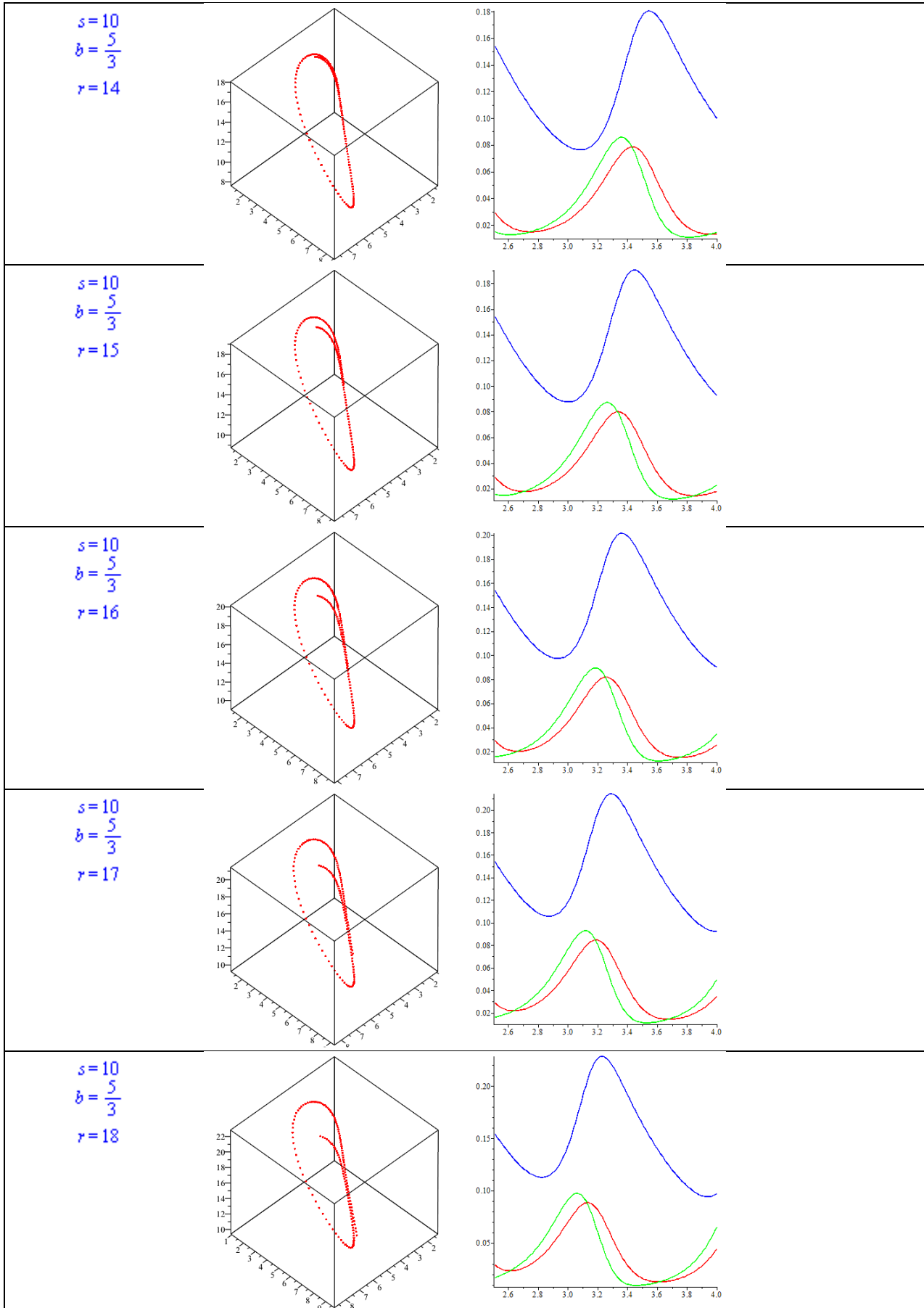
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHII (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



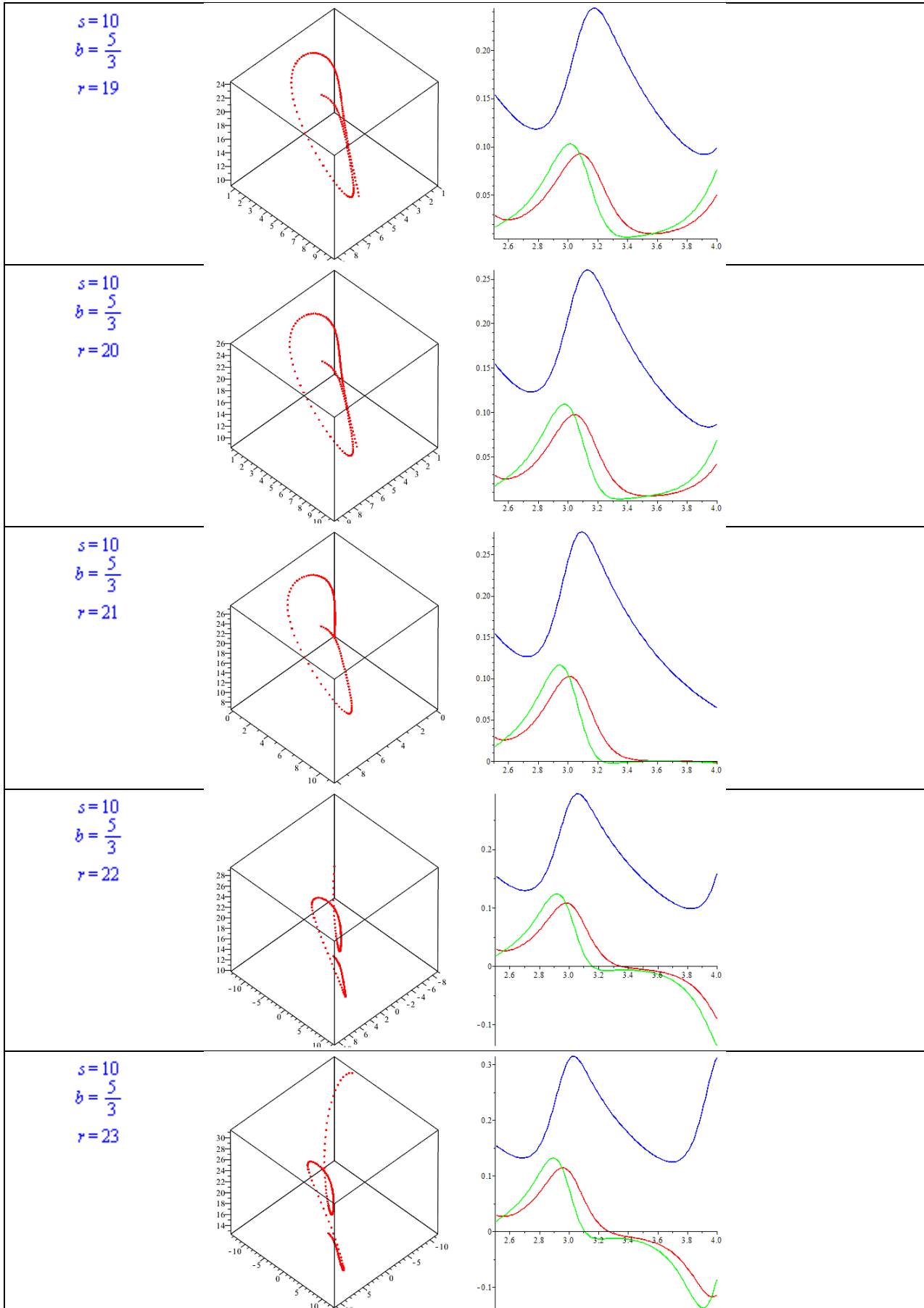
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



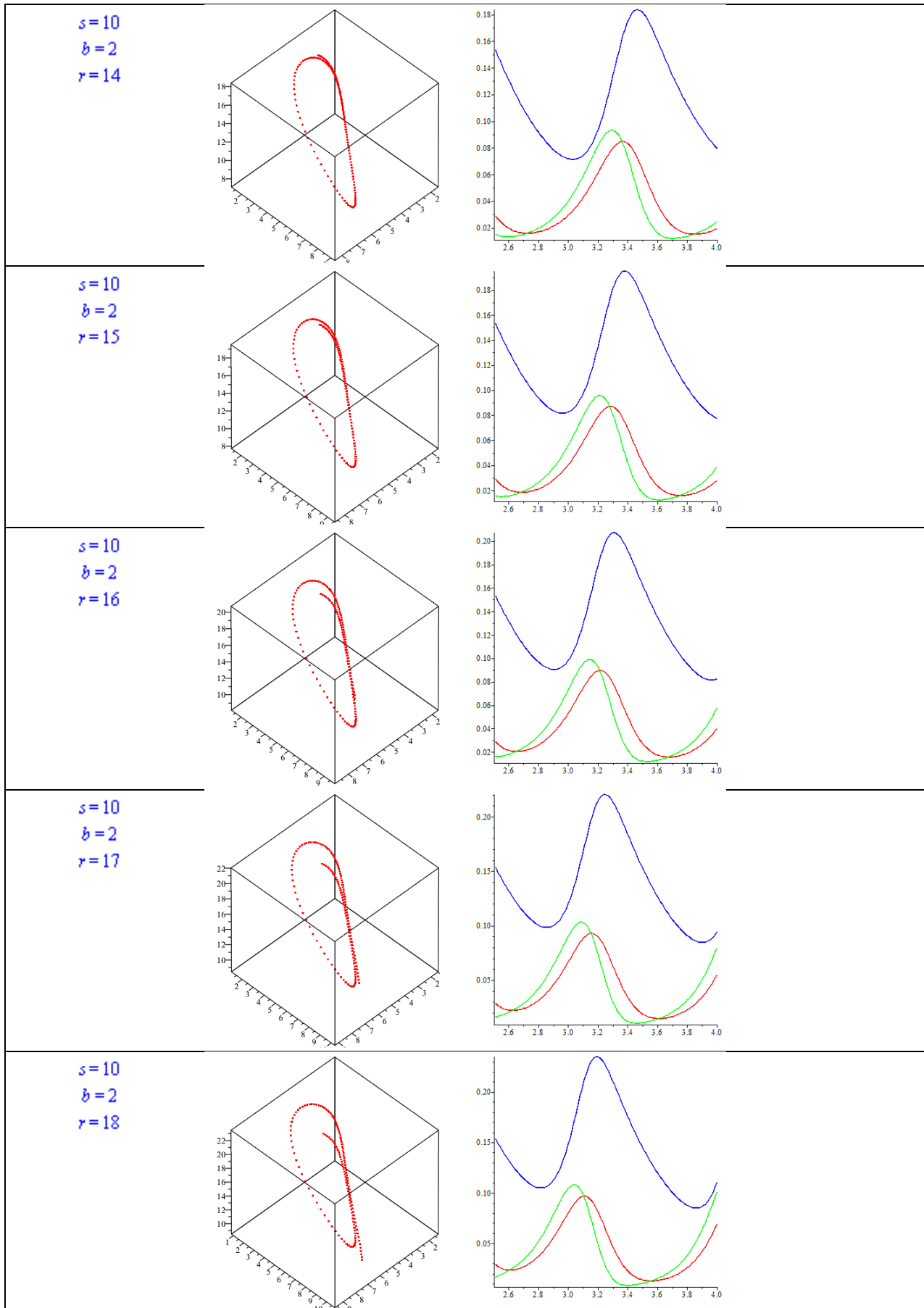
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



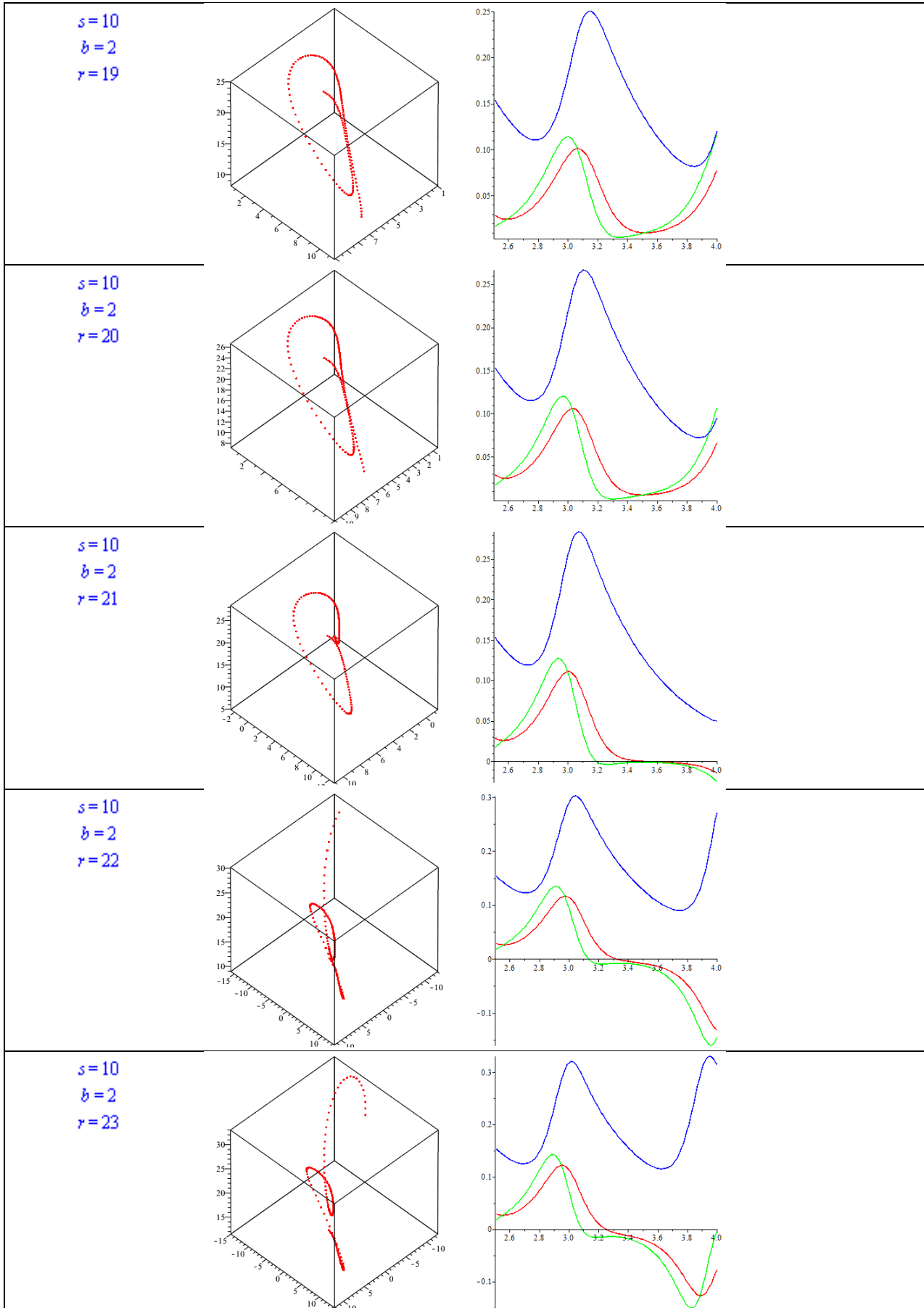
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



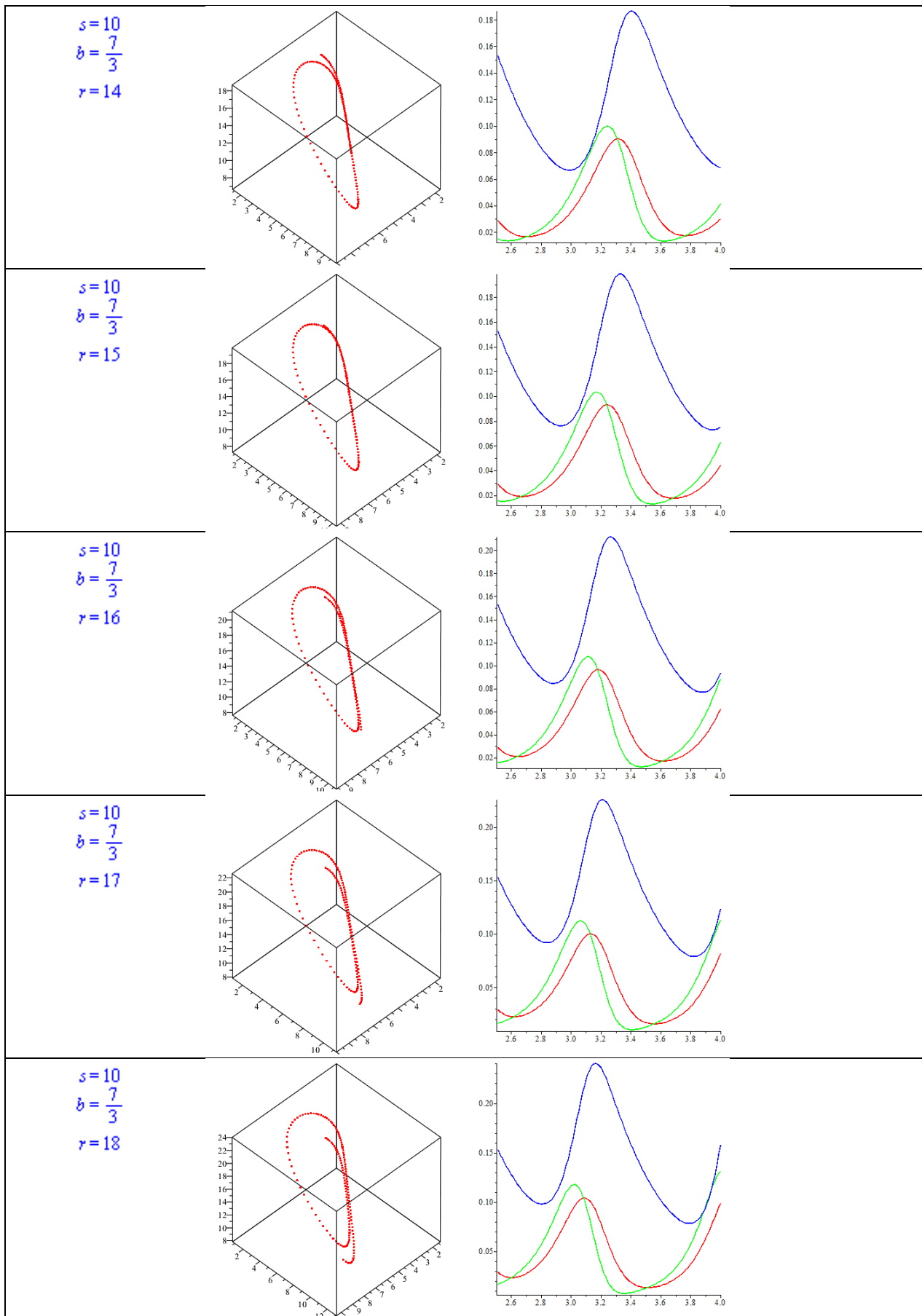
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



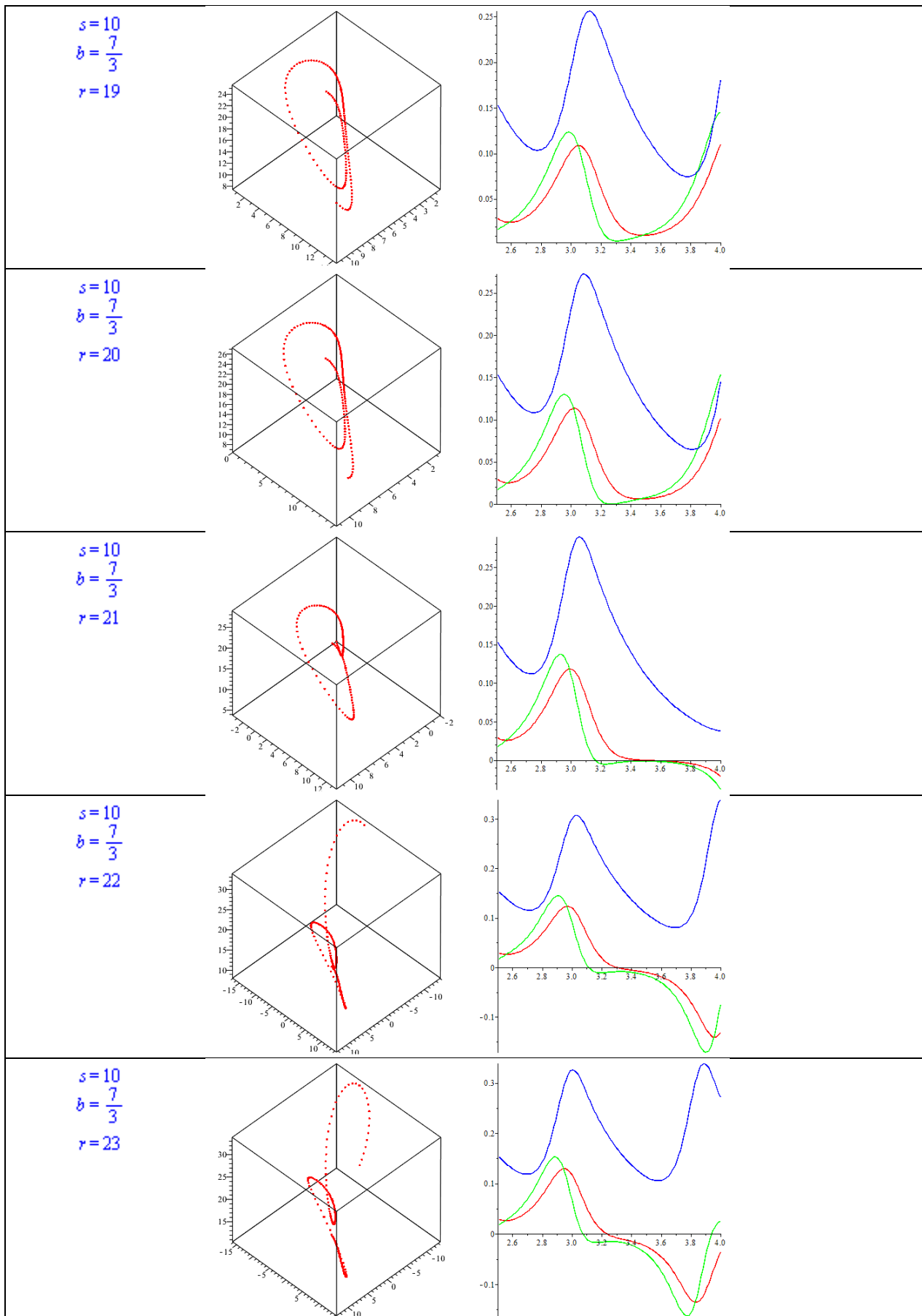
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



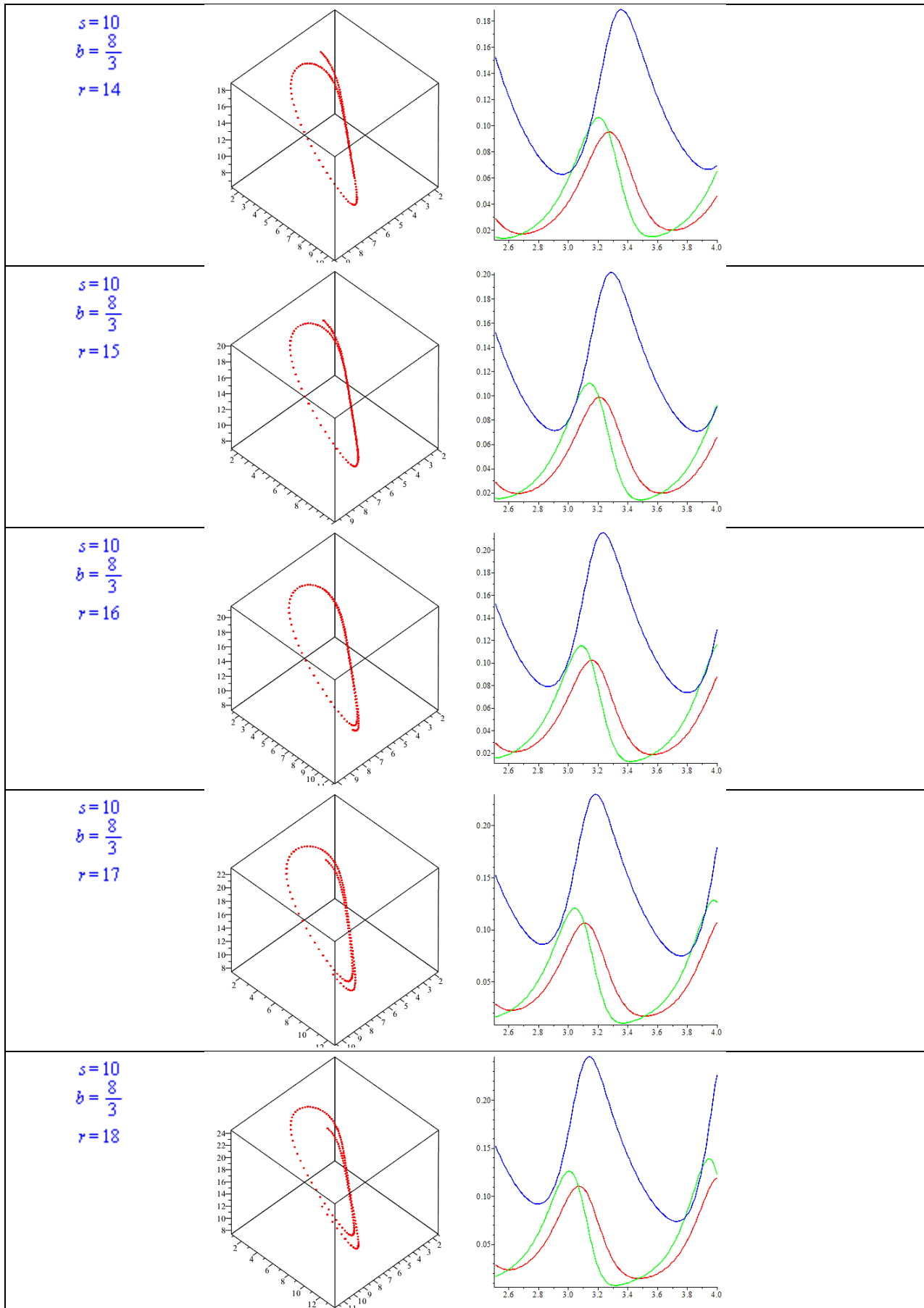
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



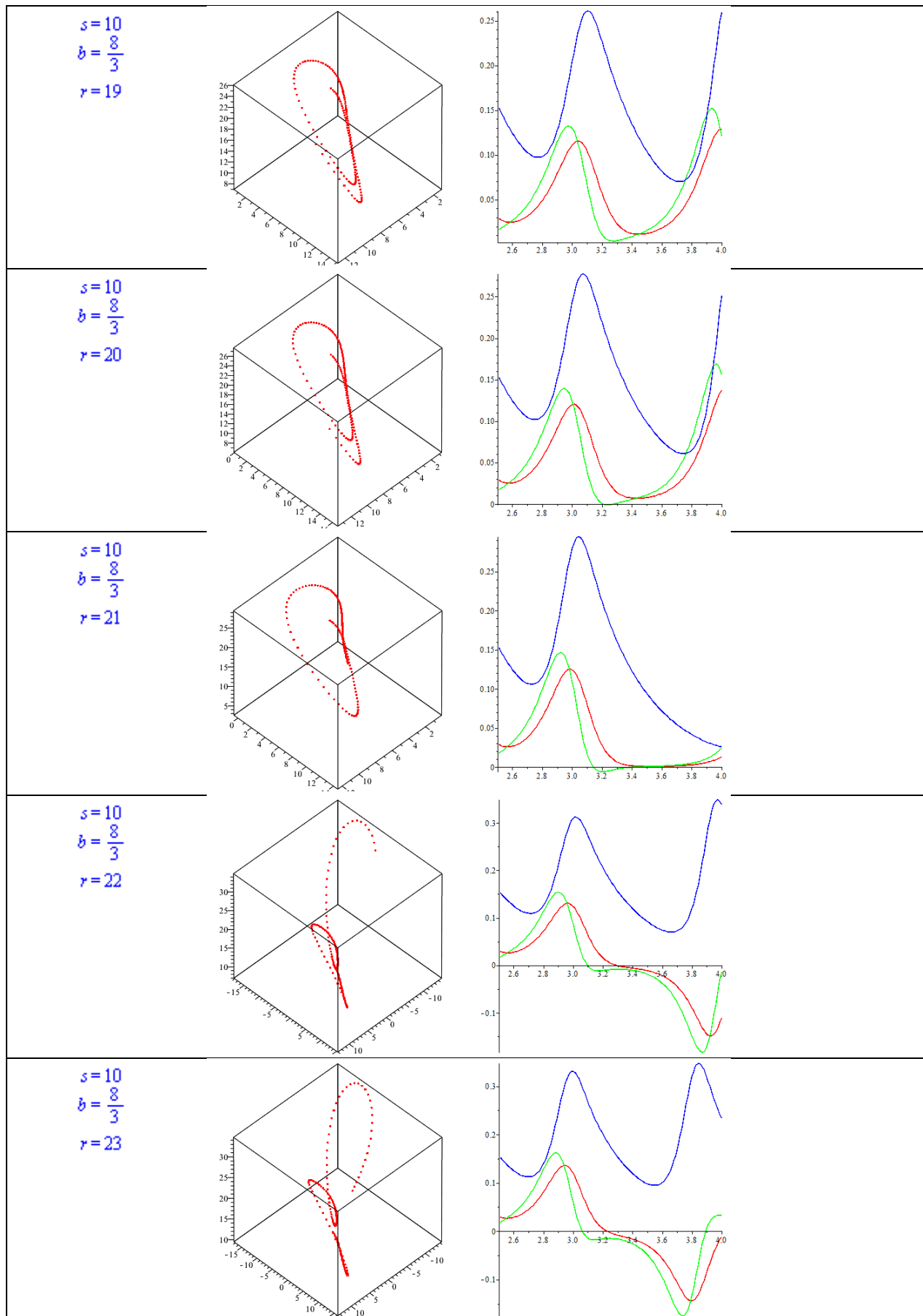
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



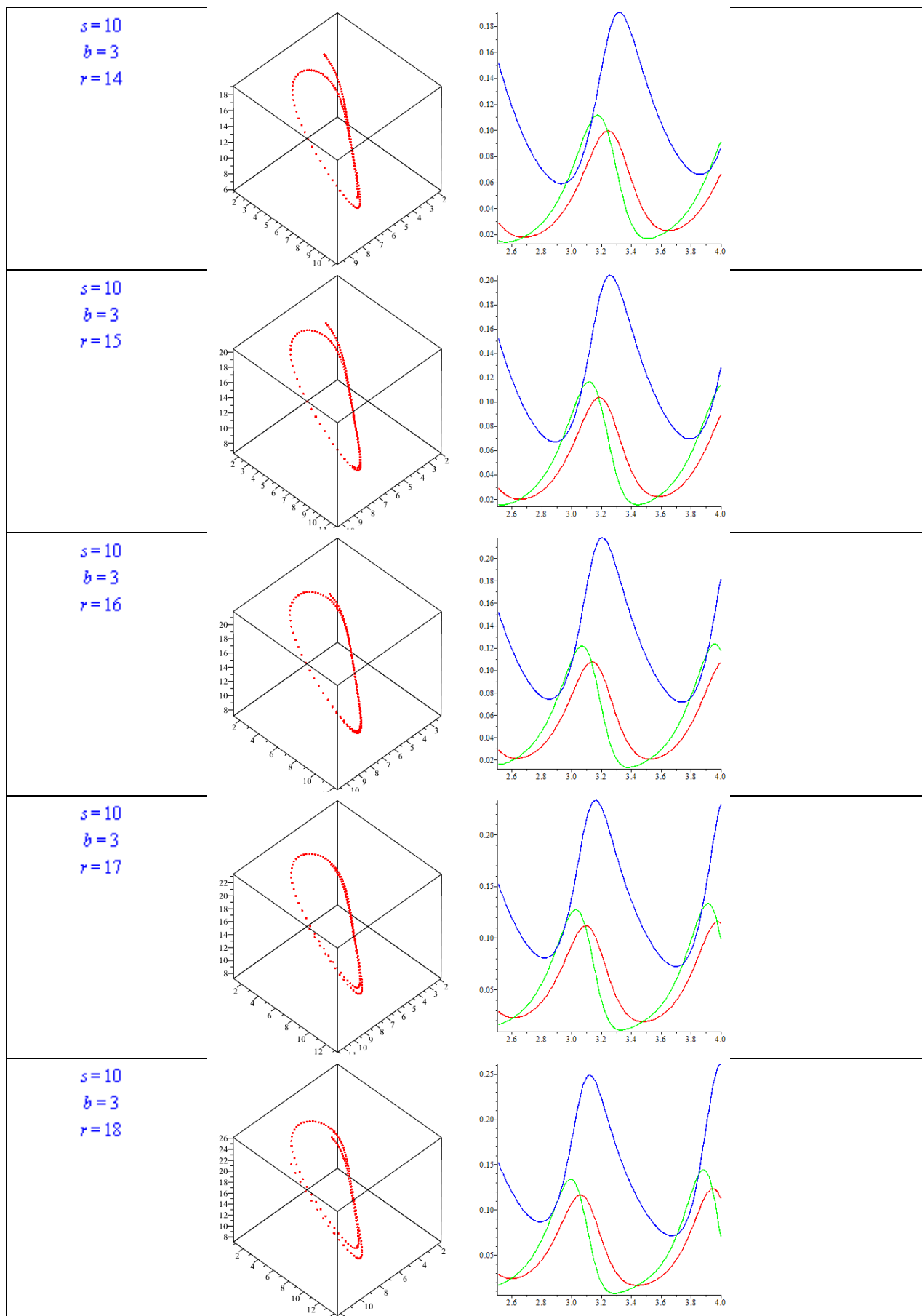
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



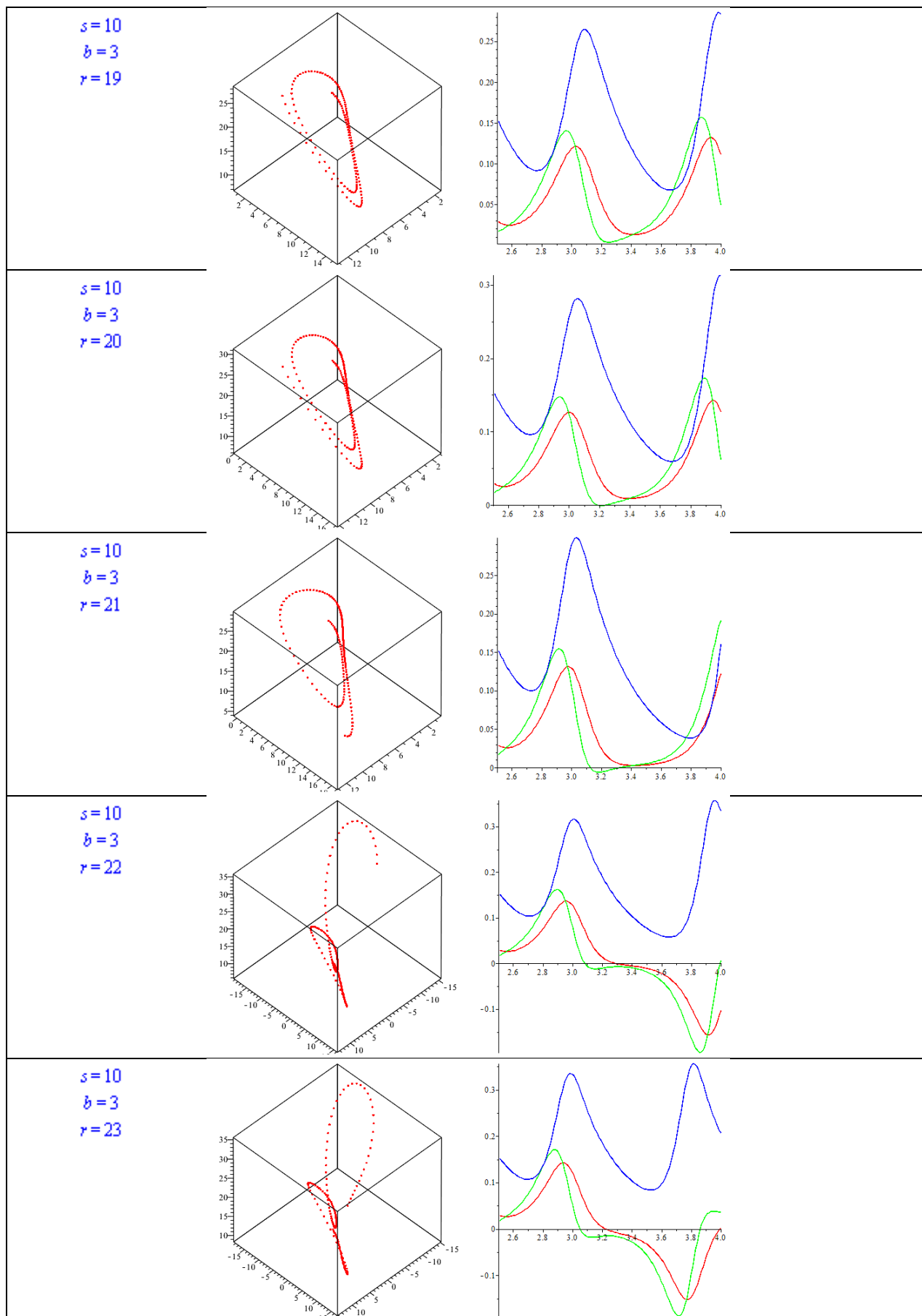
Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHHI (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHII (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	



Impact Factor:

ISRA (India) = 1.344	SIS (USA) = 0.912	ICV (Poland) = 6.630
ISI (Dubai, UAE) = 0.829	PIHII (Russia) = 0.234	PIF (India) = 1.940
GIF (Australia) = 0.564	ESJI (KZ) = 1.042	IBI (India) = 4.260
JIF = 1.500	SJIF (Morocco) = 2.031	

Results

The study of the bifurcation of the oscillation functions x, y, z for the Lorenz attractor for different values of the coefficients b, r .

References:

1. Sparrow S (1982) The Lorenz equations : Bifurcations, chaos and strange attractors. - Springer Verlag, N. - Y. 1982.
2. Khaken G (1985) Sinergetika: ierarkhii neustoychivostey v samoorganizuyushchikhsya sistemakh i ustroystvakh. Moscow: Mir, 1985. 423 p.
3. Nikolis G, Prigozhin I (1979) Samoorganizatsiya v neravnovesnykh sistemakh. Moscow: Mir, 1979. 512 p.
4. Marpl-ml SL (1990) Tsifrovoy spektral'nyy analiz i ego prilozheniya. Moscow: Mir, 1990. 584 p.
5. Likhtenberg A, Liberman M (1984) Regulyarnaya i stokhasticheskaya dinamika. Moscow: Mir, 1984. 528 p.
6. Shuster G (1988) Determinirovanny khaos. Vvednie. Moscow: Mir, 1988. 240 p.
7. Gorley PN, Gorley PP, Tomchuk PM (1996) Pis'ma v ZhTF. 1996. T. 22. V. 20. pp. 82–86.
8. Korn G, Korn T (1970) Spravochnik po matematike dlya nauchnykh rabotnikov i inzhenerov. Moscow: Nauka, 1970. 720 p.
9. Williams RF (1979) The structure of the Lorenz attractors - Publ. Math. IHES, 1979, 50, pp. 321-347.
10. Yorke JA, Yorke ED (1979) Metastable chaos : the transition to sustained chaotic oscillations in a model of Lorenz - J. Stat. Phys., 1979, 21, pp. 263-267.
11. Rychlik M (1989) Lorenz attractors through a Shilnikov-type bifurcation, Part 1. Ergodic theory dynamical systems, 1989, 10, pp. 793-821.
12. Tucker W (2002) A rigorous ODE solver and Smale's 14th problem -Found. Comput. Math., 2002, 2, pp. 53-117.

