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CARTESIAN / TENSOR PRODUCT OF SOME NEW CLASS OF STAR – IN – COLORING GRAPHS

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ABSTRACT

A proper coloring of a graph $G = (V, E)$ is a mapping $f: V \to \{1,2,3,\ldots\}$such that if $e = v_i v_j \epsilon E$, then $f(v_i) \neq f(v_j)$. A graph G is said to admit star – in – coloring if it satisfies the following conditions.

- *No path of length three(P_4) is bicolored.*
- *If any path of length two (P_3) with end vertices are of the same color, then the edges of $P_3$are directed towards the middle vertex.*

In this paper, we have proved that the splitting graph of fan graph, the splitting graph of double fan graph, Cartesian product of path and fan graph, Cartesian product of path and double fan graph, tensor product of path and fan graph, tensor product of path and double fan graph and Cartesian product of K_2 and the path graph is star – in – coloring graphs. In addition, we have given the general pattern of colors for all these graphs and their star – in – chromatic number.

KEYWORDS: *Star – In – Coloring, Splitting Graph, Cartesian Product of Two Graphs, Tensor Product of Two Graphs*

INTRODUCTION

In 1973, Grunbaum [1] was introduced by the concept of star–coloring of graphs. A *star – coloring* of a graph G is a proper coloring of the graph with the condition that no path of length three (P_4) is bicolored. The star – coloring of graphs have been investigated by Fertin, et al. [2] and Nesetril, et al. [3]. Splitting graph $S(G)$ was defined by Sampathkumar and Walikar [4]. The tensor product of graphs was defined by Alfred North Whitehead, et al. [5] in their Principia Mathematica. Motivated by the concepts of star – coloring and in – coloring, Sudha and Kanniga [6,7] have introduced the star – in – coloring of graphs. Sugumaran and Kasirajan [8] have found the lower and upper bounds of star – in – chromatic number of the graphs such as cycle, regular cyclic, gear, fan, double fan, web and complete binary tree.

Definition 1: A *star – coloring* of a graph G is a proper coloring of the graph with the condition that no path of length three (P_4) is 2–colored.

An*n – star – in – coloring* of a graph G is a star – coloring of G using at most n colors.

Definition 2: An *in – coloring* of a graph G is a proper coloring of the graph G if there exists any path $P_3$of length two with the end vertices are of the same color, then the edges of P_3 are oriented towards the central vertex.

Definition 3: A graph G is said to be a *star – in – coloring* graph if the graph G admits both star – coloring and in – coloring.

Definition 4: The minimum number of colors required for the star – in – coloring of a graph G is called the *star – in – the chromatic number* of G and is denoted by $\chi_{si}(G)$.

First, we describe the star – in – coloring of a simple graph as shown in Figure 1. Let $v_1, v_2, v_{3,} v_4$ be the vertices, and let the number within the circle indicates, that particular color is assigned to that vertex.

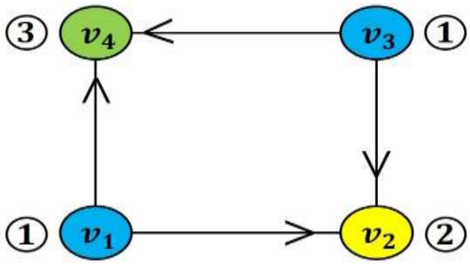

Figure 1: Star – in – Coloring of Cycle C_4

In this graph we see that no two adjacent vertices have the same color, no path on four vertices is bicolored, each and every edge in a path of length two in which end vertices have same color are oriented towards the central vertex. Hence it is a star – in – colored with orientation. Further the star – in – chromatic number of the above graph is 3.

Definintion5: For any graph G, the *splitting graph* $S(G)$ is obtained by adding to each vertex v_i in G a new vertex $v_i^{'}$ such that $v_i^{'}$ is adjacent to the neighbors of v_i in G.

Definintion 6: Suppose G and H are two graphs with $V(G) = \{u_1, u_2, \dots, u_m\}$ and $V(H) = \{v_1, v_2, \dots, v_n\}$. Then the Cartesian product $G \times H$ is the graph with vertex set $V(G \times H) = V(G) \times V(H) = \{(u_i, v_j) : u_i \in V(G), v_j \in V(H)\}$ and e is an edge of $G \times H$ iff $e = (u_i, v_j)(u_k, v_l)$,where either$i = k \text{ and } v_j v_l \in E(H)$or $j = l \text{ and } u_i u_k \in E(G)$.

Definintion 7: The tensor product of two graphs G and Hdenoted by $G \otimes H$ has the vertex set $V(G \otimes H) = V(G) \times V(H)$ and the edge set $E(G \otimes H) = \{(u_i, v_j)(u_k, v_l) : u_i u_k \in E(G) \text{ and } v_j v_l \in E(H)\}$.

MAIN RESULTS

Theorem 1: The splitting graph of fan graph $S(F_n)$admit star – in – coloring and its star – in – chromatic number is $\chi_{si}[S(F_n)] = 7$, where $n \geq 9$ and n is odd.

Proof: Consider a fan graph $F_n = P_n + K_1$ which consists of $n + 1$ vertices and $2n - 1$ edges. The splitting graph $S(F_n)$ consists of $2(n + 1)$ vertices and $3(2n - 1)$ edges. Let $V(P_n) = \{v_1, v_2, v_3, \dots, v_n\}$ and let $v_0 \in K_1$. Let $v_i^{'}$ be a new vertex (corresponding to a vertex v_i) Added in F_n to get $S(F_n)$ for each $i = 0,1,2, \dots, n$.

Let V be the vertex set of $S(F_n)$ and E be the edge set of $S(F_n)$. We define a function$f: V \to \{1, 2, 3, \dots\}$ such that $f(v_i) \neq f(v_j)$ if $v_i v_j \epsilon E$, as follows:

$f(v_0) = 6, f(v_0^{'}) = 7$ and

$$f(v_i) = \begin{cases} 1, if\ i \equiv 1\ (mod\ 2) \\ 2, if\ i \equiv 2\ (mod\ 4) \\ 3, if\ i \equiv 0\ (mod\ 4)\ and\ i > 0 \end{cases}$$

$$f(v_i^{'}) = \begin{cases} 1, if\ i \equiv 1\ (mod\ 2) \\ 4, if\ i \equiv 2\ (mod\ 4) \\ 5, if\ i \equiv 0\ (mod\ 4)\ and\ i > 0 \end{cases}$$

Hence the star – in – chromatic number of $S(F_n)$ is $\chi_{si}[S(F_n)] = 7$.

Illustration 1 Consider a fan graph F_9. By definition of splitting graph $S(F_9)$ consists of 20 vertices and 51 edges.

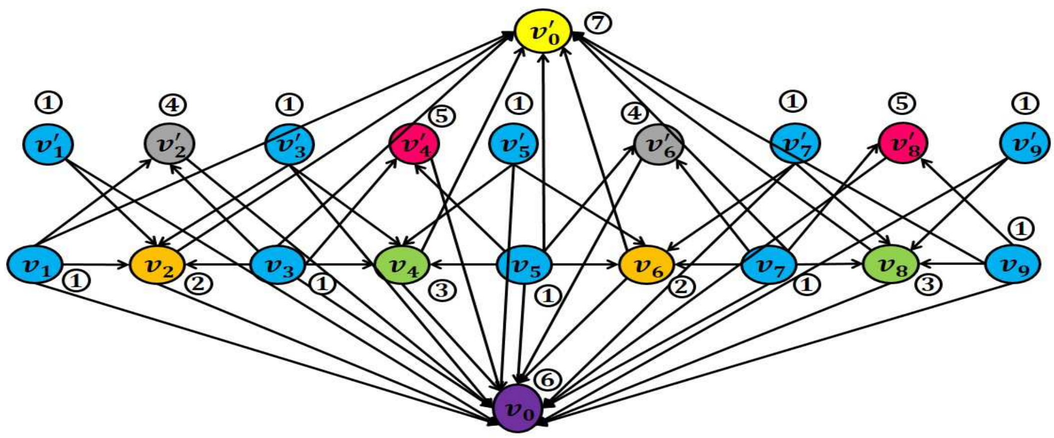

Figure 2: Star – in – Coloring of $S(F_9)$

To satisfy star – in – coloring of $S(F_9)$, the minimum number of colors required to the vertices of $S(F_9)$ are 1,2,3,4,5,6 and 7. Hence the star – in – chromatic number of $S(F_9)$ is $\chi_{si}[S(F_9)] = 7$.

Theorem 2: The splitting graph of double fan graph $S(\ DF_n)$ Admits star – in – coloring and its star – in – chromatic number is $\chi_{si}[S(\ DF_n)] = 9$, where $n \geq 9$ and n is odd.

Proof: Consider a double fan graph $F_n = P_n + \overline{K_2}$ which consists of $n + 2$ vertices and $3n - 1$ edges. The splitting graph $S(DF_n)$ consists of $2(n + 2)$ vertices and $3(3n - 1)$ edges. Let $V(P_n) = \{v_1, v_2, v_3, \dots, v_n\}$ and let $V(K_2) = \{u_0, v_0\}$. Let $v_i^{'}$ $(0 \leq i \leq n)$be a new vertex (corresponding to a vertex v_i) Added in DF_n.

Let V be the vertex set of $S(DF_n)$ and E be the edge set of $S(DF_n)$.We define a function$f: V \rightarrow \{1, 2, 3, \dots\}$ such that $f(v_i) \neq f(v_j)$ if $v_i v_j \epsilon\ E$, as follows:

$$f(v_0) = 6, f(v_0^{'}) = 7, f(u_0) = 8, f(u_0^{'}) = 9$$

$$f(v_i) = \begin{cases} 1, if\ i \equiv 1\ (mod\ 2) \\ 2, if\ i \equiv 2\ (mod\ 4) \\ 3, if\ i \equiv 0\ (mod\ 4)\ and\ i > 0 \end{cases}$$

$$f(v_i^{'}) = \begin{cases} 1, if\ i \equiv 1\ (mod\ 2) \\ 4, if\ i \equiv 2\ (mod\ 4) \\ 5, if\ i \equiv 0\ (mod\ 4)\ and\ i > 0 \end{cases}$$

By using the above pattern of coloring, the graph$S(DF_9)$ isstar – in – colored and$\chi_{si}[S(DF_9)] = 9$.

Illustration 2: Consider a double fan graph DF_9. By definition of splitting graph $S(DF_9)$ consists of 22 vertices and 78edges.

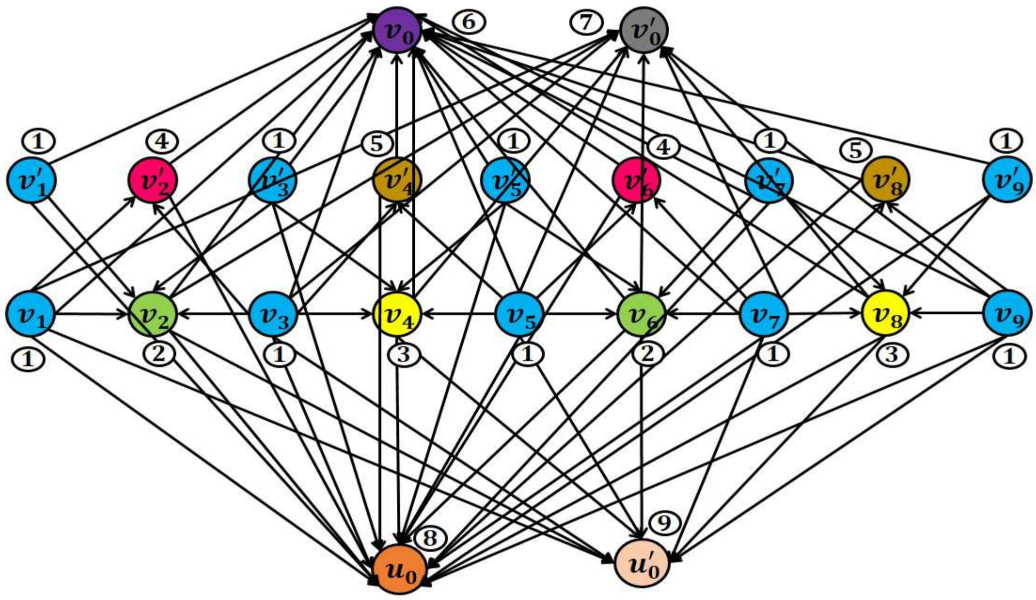

Figure 3: Star – in – Coloring of $S(DF_9)$

To satisfy star – in – coloring of $S(DF_9)$ The minimum number of colors required to the vertices of $S(DF_9)$ are 1,2,3,4,5,6,7,8 and 9. Hence the star – in – chromatic number of $S(DF_9)$ is $\chi_{si}[S(DF_9)] = 9$.

Theorem 3: The Cartesian product of path graph and a fan admits star – in – coloring and its star – in – chromatic number satisfies the inequality $7 \leq \chi_{si}\ [P_m \times F_n] \leq 8$, where m is odd and $n \geq 8$.

Proof: Consider a path graph P_m which consists of m vertices denoted by $u_1, u_2, \ldots, u_m$ and $m-1$ edges and the fan graph F_n which consists of $(n+1)$ vertices denoted by $v_0, v_1, v_2, \ldots, v_n$ and $(2n-1)$ edges. The Cartesian product $P_m \times F_n$ consists of $m(n+1)$ vertices and $n(3m-1)-1$ edges.

Let V be the vertex set of $P_m \times F_n$ and E be the edge set of $P_m \times F_n$. We define a function$f: V \rightarrow \{1, 2, 3, \ldots\}$ such that $f(u_i v_j) \neq f(u_k v_l)$ if $(u_i v_j)(u_k v_l) \epsilon\ E$, as follows:

Case 1: Let$m = 3$.

$$f(u_1 v_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 2, if\ j\ \equiv 2\ (mod\ 4) \\ 3, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

$$f(u_2 v_j) = \begin{cases} 1, if\ j\ \equiv 0\ (mod\ 2)\ and\ j > 0 \\ 4, if\ j\ \equiv 1\ (mod\ 4) \\ 5, if\ j\ \equiv 3\ (mod\ 4) \end{cases}$$

$$f(u_3 v_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 3, if\ j\ \equiv 2\ (mod\ 4) \\ 2, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

$f(u_1 v_0) = f(u_3 v_0) = 6$ and $f(u_2 v_0) = 7$.

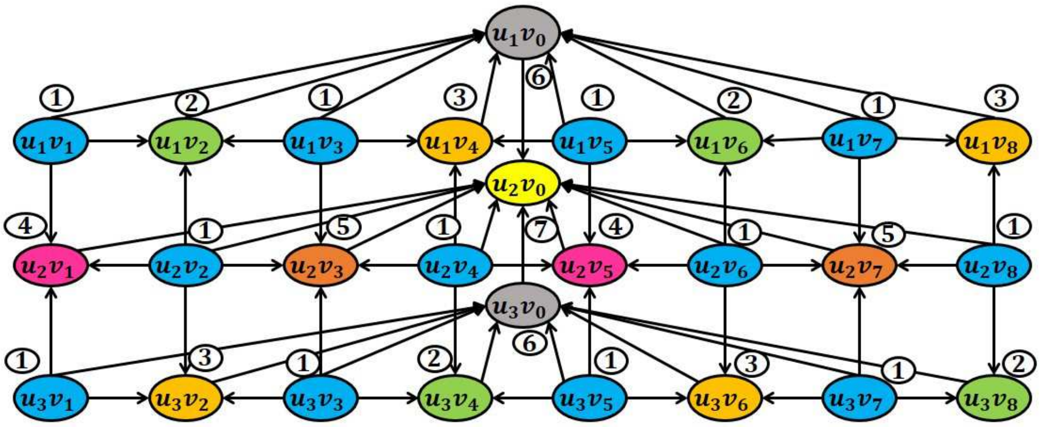

Figure 4: Star – in – Coloring of $P_3 \times F_8$

According to Case 1, the vertices in $P_3 \times F_8$ are assigned with colors 1,2,3,4,5,6 and 7 which satisfy the conditions of star – in – coloring.

Case 2: Let $m > 3$.

Subcase 2.1: $For\ i \equiv 1\ (mod\ 4)$

$$f(u_iv_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 2, if\ j\ \equiv 2\ (mod\ 4) \\ 3, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

Subcase 2.2: $For\ i \equiv 2\ (mod\ 4)$

$$f(u_iv_j) = \begin{cases} 1, if\ j\ \equiv 0\ (mod\ 2)\ and\ j > 0 \\ 4, if\ j\ \equiv 1\ (mod\ 4) \\ 5, if\ j\ \equiv 3\ (mod\ 4) \end{cases}$$

Subcase 2.3: $For\ i \equiv 3\ (mod\ 4)$

$$f(u_iv_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 3, if\ j\ \equiv 2\ (mod\ 4) \\ 2, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

Subcase 2.4: $For\ i \equiv 0\ (mod\ 4)$

$$f(u_iv_j) = \begin{cases} 1, if\ j\ \equiv 0\ (mod\ 2)\ and\ j > 0 \\ 5, if\ j\ \equiv 1\ (mod\ 4) \\ 4, if\ j\ \equiv 3\ (mod\ 4) \end{cases}$$

and

$$f(u_iv_0) = \begin{cases} 6, if\ i\ \equiv 1\ (mod\ 2) \\ 7, if\ i\ \equiv 2\ (mod\ 4) \\ 8, if\ i\ \equiv 0\ (mod\ 4) \end{cases}$$

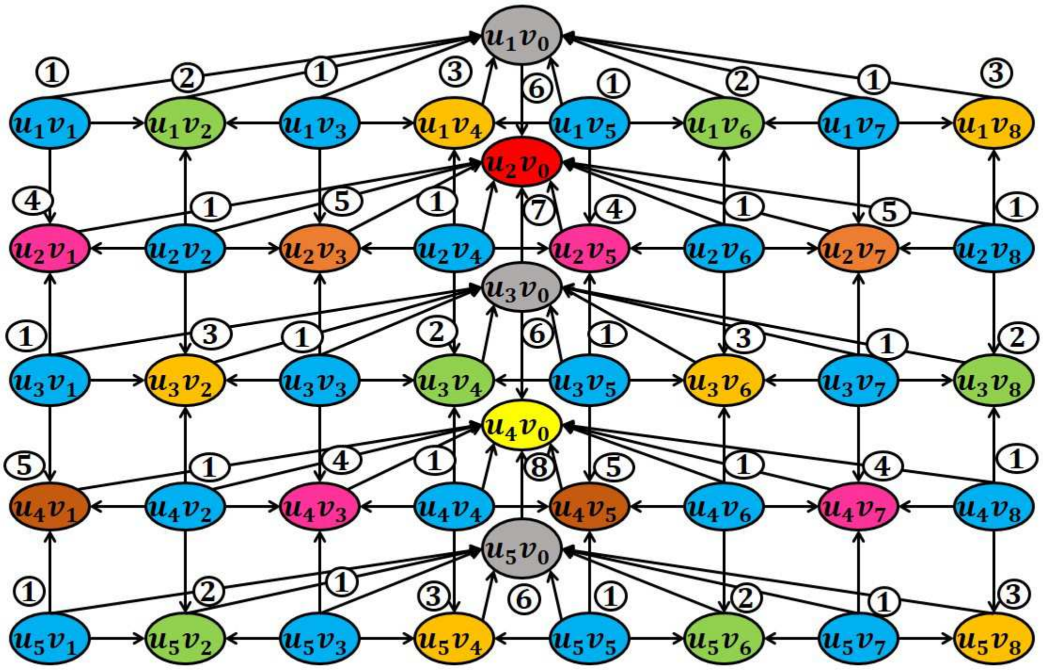

Figure 5: Star – in – Coloring of $P_5 \times F_8$

According to Case 2, the vertices in $P_5 \times F_8$ is assigned with colors 1,2,3,4,5,6,7 and 8 which satisfy the conditions of star – in – coloring.By using the above pattern of coloring, the Cartesian product $P_m \times F_n$ is star – in – colored and its star – in – the chromatic number satisfies the inequality $7 \leq \chi_{si}\ [P_m \times F_n] \leq 8$.

Theorem 4: The tensor product of path graph and a fan admits star – in – coloring and its star – in – chromatic number is given by $\chi_{si}[P_m \otimes F_n] = \begin{cases} n+2, if\ m = 2,3 \\ 2n+3, if\ m \geq 4 \end{cases}$

Proof: Consider a path graph P_m which consists of m vertices denoted by $u_1, u_2, \dots, u_m$ and $m-1$ edges and the fan graph F_n which consists of $(n+1)$ vertices denoted by $v_0, v_1, v_2, \dots, v_n$ and $(2n-1)$ edges. The tensor product $P_m \otimes F_n$ consists of $m(n+1)$ vertices and $(m-1)(4n-2)$ edges.

Let V be the vertex set of $P_m \otimes F_n$ and E be the edge set of $P_m \otimes F_n$.We define a function$f: V \rightarrow \{1, 2, 3, \dots\}$ such that $f(u_iv_j) \neq f(u_kv_l)$ if $(u_iv_j)(u_kv_l) \epsilon\ E$, as follows:

Case 1: Let$m = 2,3$.

$$f(u_iv_j) = \begin{cases} 1, if\ i = 1,3 \\ j+2, if\ i = 2 \end{cases}$$

Case 2: Let$m \geq 4$.

$$f(u_iv_j) = \begin{cases} 1, if\ i \equiv 1\ (mod\ 2) \\ j+2, if\ i \equiv 2\ (mod\ 4) \\ j+n+3, if\ i \equiv 0\ (mod\ 4) \end{cases}$$

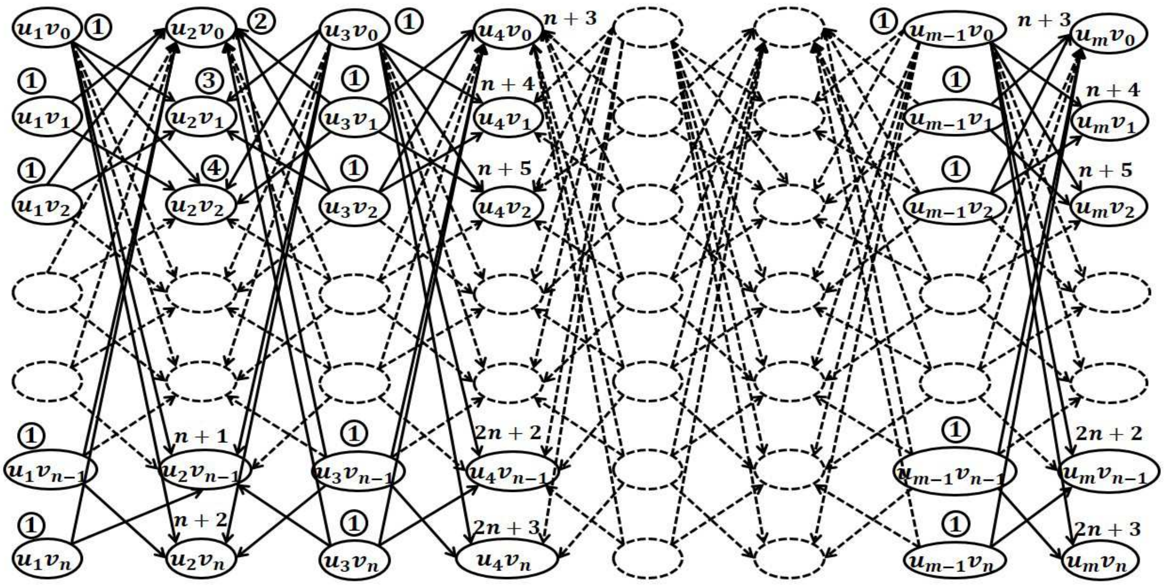

Figure 6: Star – in – Coloring of $P_m \otimes F_n, m \equiv 0(mod\ 4)$

According to Case 1 and Case 2, the tensor product $P_m \otimes F_n$ is star – in – colored and its star – in – chromatic number is $\chi_{si}[P_m \otimes F_n] = n + 2$, when $m = 2,3$ and $\chi_{si}[P_m \otimes F_n] = 2n + 3$ when $m \geq 4$.

Theorem 5: The Cartesian product of path graph and a double fan admits star – in – coloring and its star – in – chromatic number satisfies the inequality $7 \leq \chi_{si}\ [P_m \times DF_n] \leq 8$, where m is odd and $n \geq 8$.

Proof: Consider a path graph P_m which consists of m vertices denoted by $u_1, u_2, \ldots, u_m$ and $m - 1$ edges and the double fan graph F_n which consists of $(n + 2)$ vertices denoted by $v_0, v_0^{'}, v_1, v_2, \ldots, v_n$ and $(3n - 1)$ edges. The Cartesian product $P_m \times DF_n$ consist of $m(n + 2)$ vertices and $m(4n + 1) - n - 2$ edges.

Let V be the vertex set of $P_m \times DF_n$ and E be the edge set of $P_m \times DF_n$. We define a function$f: V \rightarrow \{1, 2, 3, \ldots\}$ such that $f(u_i v_j) \neq f(u_k v_l)$ If $(u_i v_j)(u_k v_l) \epsilon\ E$,as follows:

Case 1: Let$m = 3$.

$$f(u_1 v_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 2, if\ j\ \equiv 2\ (mod\ 4) \\ 3, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

$$f(u_2 v_j) = \begin{cases} 1, if\ j\ \equiv 0\ (mod\ 2)\ and\ j > 0 \\ 4, if\ j\ \equiv 1\ (mod\ 4) \\ 5, if\ j\ \equiv 3\ (mod\ 4) \end{cases}$$

$$f(u_3 v_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 3, if\ j\ \equiv 2\ (mod\ 4) \\ 2, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

$f(u_1 v_0) = f(u_3 v_0) = 6, f(u_2 v_0) = 7$ and

$f(u_1 v_0^{'}) = f(u_3 v_0^{'}) = 7, f(u_2 v_0^{'}) = 6.$

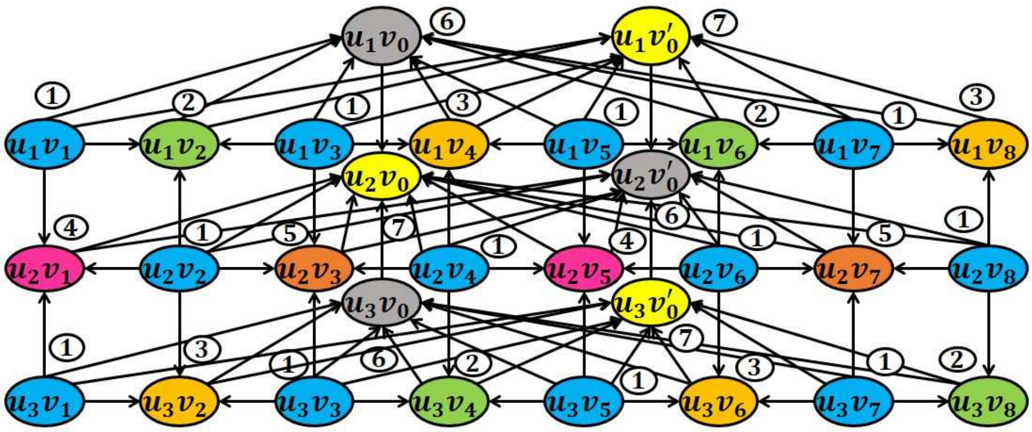

Figure 7: Star – in – Coloring of $P_3 \times DF_8$

According to Case 1, the vertices in $P_3 \times DF_8$ are assigned with colors 1,2,3,4,5,6 and 7 which satisfy the conditions of star – in – coloring.

Case 2: Let$m > 3$.

Subcase 2.1:$For\ i \equiv 1\ (mod\ 4)$

$$f(u_i v_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 2, if\ j\ \equiv 2\ (mod\ 4) \\ 3, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

Subcase 2.2:$For\ i \equiv 2\ (mod\ 4)$

$$f(u_i v_j) = \begin{cases} 1, if\ j\ \equiv 0\ (mod\ 2)\ and\ j > 0 \\ 4, if\ j\ \equiv 1\ (mod\ 4) \\ 5, if\ j\ \equiv 3\ (mod\ 4) \end{cases}$$

Subcase 2.3:$For\ i \equiv 3\ (mod\ 4)$

$$f(u_i v_j) = \begin{cases} 1, if\ j\ \equiv 1\ (mod\ 2) \\ 3, if\ j\ \equiv 2\ (mod\ 4) \\ 2, if\ j\ \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

Subcase 2.4:$For\ i \equiv 0\ (mod\ 4)$

$$f(u_i v_j) = \begin{cases} 1, if\ j\ \equiv 0\ (mod\ 2)\ and\ j > 0 \\ 5, if\ j\ \equiv 1\ (mod\ 4) \\ 4, if\ j\ \equiv 3\ (mod\ 4) \end{cases}$$

$$f(u_i v_0) = \begin{cases} 6, if\ i\ \equiv 1\ (mod\ 2) \\ 7, if\ i\ \equiv 2\ (mod\ 4) \\ 8, if\ i\ \equiv 0\ (mod\ 4) \end{cases}$$

$$f(u_i v_0') = \begin{cases} 7, if\ i\ \equiv 1\ (mod\ 2) \\ 8, if\ i\ \equiv 2\ (mod\ 4) \\ 6, if\ i\ \equiv 0\ (mod\ 4) \end{cases}$$

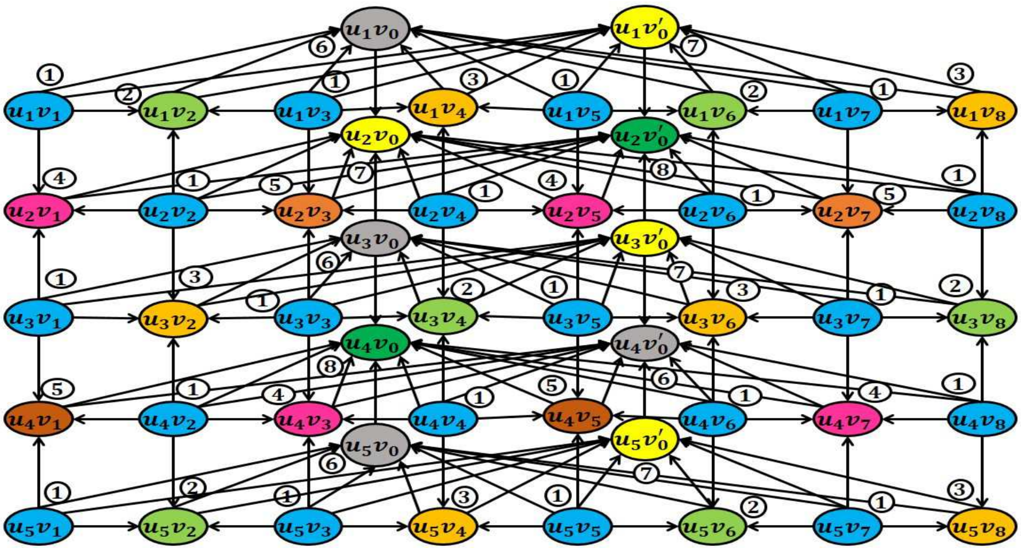

Figure 8: Star – in – Coloring of $P_5 \times DF_8$

According to Case 2, the vertices in $P_5 \times DF_8$ are assigned with colors 1,2,3,4,5,6,7 and 8 which satisfy the conditions of star – in – coloring.By using the above pattern of coloring, the Cartesian product $P_m \times DF_n$ are star – in – colored and its star – in – chromatic number satisfies the inequality $7 \leq \chi_{si}\ [P_m \times DF_n] \leq 8$.

Theorem 6: The tensor product of path graph and a double fan admits star – in – coloring and its star – in – chromatic number is given by $\chi_{si}[P_m \otimes DF_n] = \begin{cases} n+3, if\ m = 2,3 \\ 2n+5, if\ m \geq 4 \end{cases}$

Proof: Consider a path graph P_m which consists of m vertices denoted by $u_1, u_2, \ldots, u_m$ and $m-1$ edges and the double fan graph DF_n which consists of $(n+2)$ vertices denoted by $v_0, v_0', v_1, v_2, \ldots, v_n$ and $(3n-1)$ edges. The tensor product $P_m \otimes DF_n$ consists of $m(n+2)$ vertices and $2(m-1)(3n-1)$ edges.

Let V be the vertex set of $P_m \otimes DF_n$ and E be the edge set of $P_m \otimes DF_n$.We define a function$f: V \rightarrow \{1, 2, 3, \ldots\}$ such that $f(u_i v_j) \neq f(u_k v_l)$ if $(u_i v_j)(u_k v_l) \epsilon\ E$, as follows:

Case 1: Let$m = 2, 3$.

$$f(u_i v_j) = \begin{cases} 1, if\ i = 1,3 \\ j+3, if\ i = 2\ and\ j > 0 \end{cases}$$

$$f(u_2 v_0') = 2, f(u_2 v_0) = 3$$

Case 2: Let$m \geq 4$.

$$f(u_i v_j) = \begin{cases} 1, if\ i \equiv 1\ (mod\ 2) \\ j+3, if\ i \equiv 2\ (mod\ 4)\ and\ j > 0 \\ j+n+5, if\ i \equiv 0\ (mod\ 4)\ and\ j > 0 \end{cases}$$

$$f(u_i v_0') = \begin{cases} 2, if\ i \equiv 2\ (mod\ 4) \\ n+4, if\ i \equiv 0\ (mod\ 4) \end{cases}$$

$$f(u_i v_0) = \begin{cases} 3, if\ i \equiv 2\ (mod\ 4) \\ n+5, if\ i \equiv 0\ (mod\ 4) \end{cases}$$

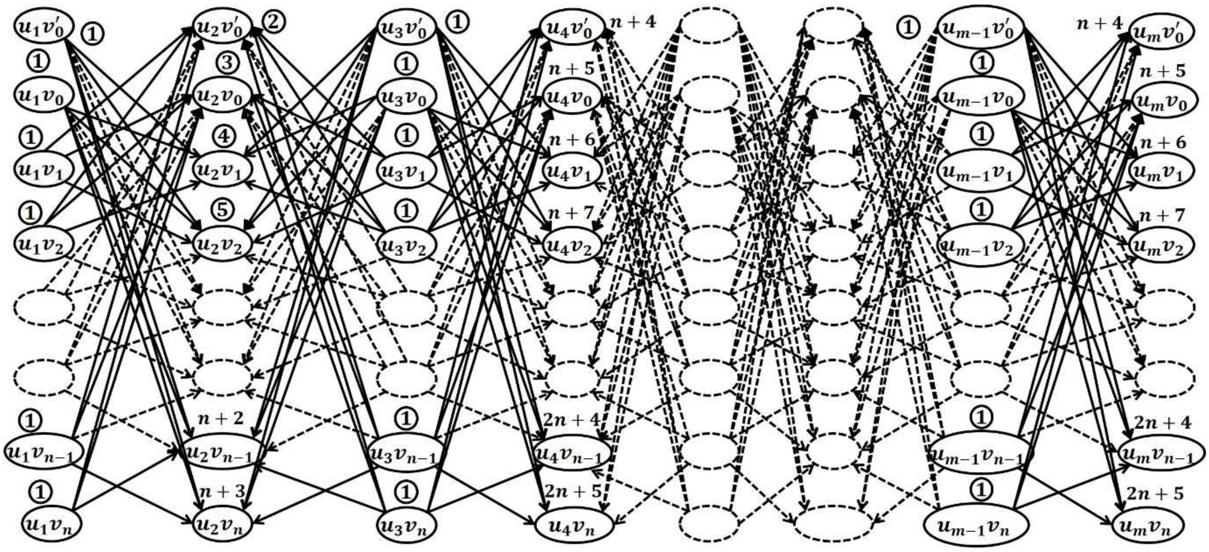

Figure 9: Star – in – Coloring of $P_m \otimes DF_n, m \equiv 0(mod\ 4)$

According to Case 1 and Case 2, the tensor product $P_m \otimes DF_n$ is star – in – colored and its star – in – chromatic number is $\chi_{si}[P_m \otimes DF_n] = n + 3$, when $m = 2,3$ and $\chi_{si}[P_m \otimes DF_n] = 2n + 5$ when $m \geq 4$.

Theorem 7: The Cartesian product of a complete graph K_2 and a path admits star – in – coloring and its star – in – chromatic number is $\chi_{si}[K_2 \times P_n] = 4$.

Proof: Consider a complete graph K_2 which consists of 2 vertices denoted by u_1, u_2 and 1 edge and the path graph P_n which consists of n vertices denoted by $v_1, v_2, \dots, v_n$ and $n-1$ edges. The Cartesian product $K_2 \times P_n$ consists of $2n$ vertices and $3n-2$ edges.

Let V be the vertex set of $K_2 \times P_n$ and E be the edge set of $K_2 \times P_n$. We define a function $f: V \rightarrow \{1, 2, 3, \dots\}$ such that $f(u_i v_j) \neq f(u_k v_l)$ if $(u_i v_j)(u_k v_l) \epsilon\ E$, as follows:

$$f(u_1 v_i) = \begin{cases} 1, if\ i\ \equiv 1\ (mod\ 2) \\ 2, if\ i\ \equiv 2\ (mod\ 6) \\ 3, if\ i\ \equiv 4\ (mod\ 6) \\ 4, if\ i\ \equiv 0\ (mod\ 6) \end{cases}$$

$$f(u_2 v_i) = \begin{cases} 1, if\ i\ \equiv 2\ (mod\ 2) \\ 2, if\ i\ \equiv 5\ (mod\ 6) \\ 3, if\ i\ \equiv 1\ (mod\ 6) \\ 4, if\ i\ \equiv 3\ (mod\ 6) \end{cases}$$

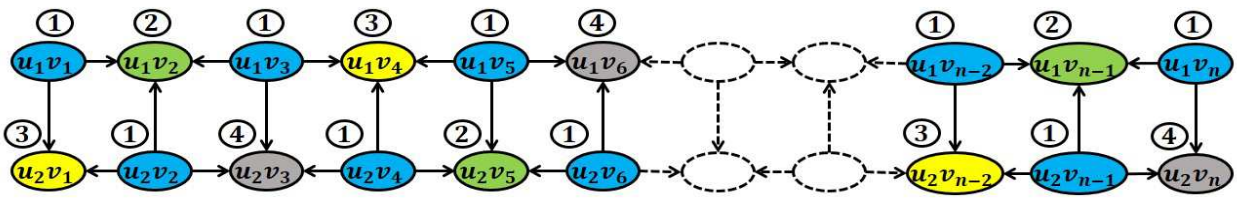

Figure 10: Star – in – Coloring of $K_2 \times P_n, n \equiv 0\ (mod\ 3)$ and n is Odd

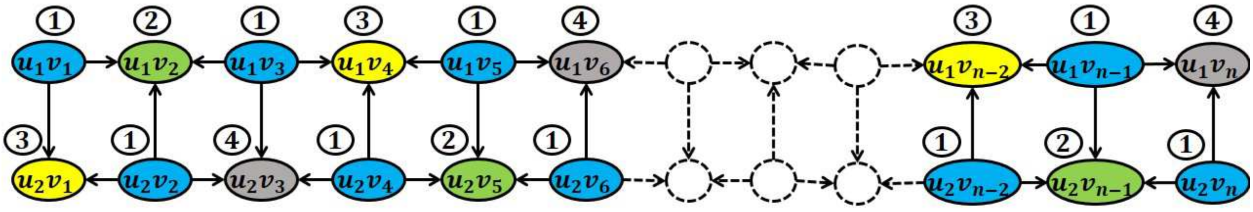

Figure 11: Star – in – Coloring of $K_2 \times P_n, n \equiv 0\ (mod\ 3)$ and n is Even

By using the above pattern of coloring, the Cartesian product $K_2 \times P_n$ is star – in – colored and its star – in – chromatic number is $\chi_{si}[K_2 \times P_n] = 4$.

CONCLUSIONS

In this paper, we have shown that the lower and upper bounds of star – in – chromatic number of some of the graphs are as follows:

- The star – in – chromatic number of $S(F_n)$ is 7
- The star – in – chromatic number of$S(\ DF_n)$ is 9
- $7 \leq \chi_{si}\ [P_m \times F_n] \leq 8$
- $\chi_{si}[P_m \otimes F_n] = \begin{cases} n+2, if\ m = 2,3 \\ 2n+3, if\ m \geq 4 \end{cases}$
- $7 \leq \chi_{si}\ [P_m \times DF_n] \leq 8$
- $\chi_{si}[P_m \otimes DF_n] = \begin{cases} n+3, if\ m = 2,3 \\ 2n+5, if\ m \geq 4 \end{cases}$
- The star – in – chromatic number of $K_2 \times P_n$ is 4

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