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Hypersurface-Homogeneous Universe with Λ in $f(R,T)$ Gravity by Hybrid Expansion Law

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Abstract. In the present study we investigate, Hypersurface-Homogeneous cosmological model in $f(R,T)$ theory of gravity with a term Λ. We obtain the gravitational field equations in the metric formalism, which follow from the covariant divergence of the stress-energy tensor. The field equations correspond for a specific choice of $f(R,T)=f_1(R)+f_2(T)$, with the individual superior functions $f_1(R)=\lambda R$ and $f_2(T)=\lambda T$. In this paper, we consider a simple form of expansion history of Universe referred to as the hybrid expansion law − a product of power-law and exponential type of functions. Einstein's field equations have been solved by taking into account the hybrid expansion law for scale factor that yields time dependent deceleration parameter (DP). Some physical and geometric properties of the model along with physical acceptability of the solutions have also been discussed in detail.

Keywords: Hypersurface-Homogeneous universe, variable cosmological constant, modified theory of gravity.

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1 Introduction

The analysis of observational data shows that our Universe for later stages of evolution indicates accelerated expansion. This conclusion is based on the observations of high redshift type SN Ia supernovae [1–4]. Recent astronomical observations indicate that about 70 % of the Universe consists of dark energy with negative pressure .In recent years, modified gravities have recently been verified to explain the late-time accelerated expansion of the Universe. Different modified theories of gravitation are $f(R)$ gravity [5-8] and Gauss–Bonnet gravity or $f(G)$ gravity [9-12]. Another approach to modified gravity is so-called $f(T)$ gravity [13-15], where T is the scalar torsion. Recently, Harko et al. [16] proposed $f(R,T)$ gravity theory by taking into account the gravitational Lagrangian as the function of Ricci scalar R and of the trace of energy-stress tensor T. They have obtained the equation of motion of test particle and the gravitational field equation in metric formalism both. The $f(R,T)$ gravity models could justify the late time cosmic accelerated enlargement of the Universe. Many Authors [17-46] studied completely different cosmological models in $f(R,T)$ theory of gravity.

Motivated by the above observational facts, in this paper, we propose to study cosmological model represented by Hypersurface -Homogenous reference system for perfect fluid distribution within the framework of $f(R,T)$ gravity. We choose a specific choice of the functional $f\left(R,T\right)=f_1\left(R\right)+f_2\left(T\right)$ with $f_1\left(R\right)=\lambda R$ and $f_2\left(T\right)=\lambda T$.

2 Gravitational Field Equations of Modified Gravity Theory

The $f(R,T)$ theory of gravity is the modification of General Relativity (GR). In this theory the modified gravity action is given by

$$s = \frac{1}{16\pi}\int f(R,T)\sqrt{-g}\ d^4x + \int L_m \sqrt{-g}\ d^4x \tag{1}$$

where $f(R,T)$ is an arbitrary function of the Ricci scalar R and the trace T of the stress energy tensor T_{ij} of the matter, L_m is the matter Lagrangian density. If $f(R,T)$ is replaced by $f(R)$, we get the action for $f(R)$ gravity and replacement of $f(R,T)$ by R leads to the action of general relativity.

Varying the action S about metric tensor g_{ij}, the field equations of $f(R,T)$ gravity are obtained as

$$f_R(R,T)R_{ij} - \frac{1}{2}f\left(R,T\right)g_{ij} + f_R\left(R,T\right)\left(g_{ij}\nabla^i\nabla_i - \nabla_i\nabla_j\right) = 8\pi T_{ij} - f_T\left(R,T\right)T_{ij}\text{-}f_T\left(R,T\right)\theta_{ij} \tag{2}$$

where

$$\theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta}\frac{\partial^2 L_m}{\partial g_{ij}\partial g^{\alpha\beta}} \tag{3}$$

Here $f_R = \dfrac{\delta\ f\left(R,T\right)}{\delta R}$, $f_T = \dfrac{\delta\ f\left(R,T\right)}{\delta T}$, ∇_i is the co-variant derivative and T_{ij} is the standard matter energy momentum tensor derived from the Lagrangian L_m.

If the matter is regarded as a perfect fluid the stress energy tensor of the matter Lagrangian is given by $T_{ij} = \left(\rho + p\right)u_i u_j - pg_{ij}$. Here $u^i = (0,0,0,1)$ is the velocity vector in co-moving coordinates that satisfies the condition $u^i u_i = 1$ and $u^i\nabla_j u_i = 0$. Here ρ, p are energy density and pressure of the fluid respectively.

For perfect fluid, the matter Lagrangian density may have two choices either $L_m = -p$ or $L_m = \rho$ that has widely been studied in literature [47-49]. Here we have assumed the matter Lagrangian as $L_m = -p$. Now θ_{ij} in equation (3) can be reduced to

$$\theta_{ij} = -2T_{ij} - g_{ij}p \tag{4}$$

It is mentioned here that this field equation depends on the physical nature of the matter field. There are three classes of these models

$$f\left(R,T\right) = \begin{cases} R + 2f\left(T\right) \\ f_1\left(R\right) + f_2\left(T\right) \\ f_1(R) + f_2\left(R\right)\ f_3\left(T\right) \end{cases} \tag{5}$$

We consider it in the form $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$, where λ is an arbitrary constant.

The field equations (2), for the specific choice of $f(R,T) = \lambda(R+T)$, reduces to

$$G_{ij} = \left(\frac{8\pi + \lambda}{\lambda}\right)T_{ij} + \left(p + \frac{1}{2}T\right)g_{ij} \tag{6}$$

We choose a small negative value for the arbitrary λ to draw a better analogy with the usual Einstein field equations and we intend to keep this choice of λ throughout.

We have the Einstein field equation with cosmological constant as

$$G_{ij} - \Lambda g_{ij} = -8\pi T_{ij} \tag{7}$$

Comparing equations (6) and (7), we yield

$$\Lambda = \Lambda\left(T\right) = p + \frac{1}{2}T \tag{8}$$

and

$$8\pi = \frac{8\pi + \lambda}{\lambda} \tag{9}$$

The dependence of the cosmological constant Λ on the trace of the energy momentum tensor T has been proposed before by Poplawski [50] where the cosmological constant in the gravitational

Lagrangian is considered as a function of the trace of the energy-momentum tensor. Since we have considered the perfect fluid as the source, according to Poplawski [51], the trace of energy-momentum tensor is function of isotropic pressure and energy density i.e.

$$T = -3p + \rho \tag{10}$$

3 Metric and Field Equations

We consider the Hypersurface-homogeneous space time of the form,

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)\left[\, dy^2 + \Sigma^2\,(y, K)dz^2 \,\right] \tag{11}$$

where $A(t)$ and $B(t)$ are the cosmic scale functions, $\Sigma(y,K) = \sin y, y, \sinh y$ respectively when $K = 1,\ 0,\ -1$.

Stewart and Ellis[52] obtained general solutions of Einstein's field equations for a perfect fluid distribution satisfying a barotropic equation of state for the Hypersurface-homogeneous space time. Hajj-Boutros [53] developed a method to find exact solutions of field equations in case of the metric (11) in presence of perfect fluid and obtained exact solutions of the field equations which add to the rare solutions not satisfying the barotropic equation of state. Hypersurface-homogeneous bulk viscous fluid cosmological models with time-dependent cosmological term have been discussed by Chandel et al. [54]. Katore and Shaikh [55] obtained the exact solutions of the field equations for Hypersurface-homogeneous space time under the assumption on the anisotropy of the fluid (dark energy), which are obtained for exponential and power-law volumetric expansions in a scalar-tensor theory of gravitation. Katore and Shaikh [56] presented a class of solutions of Einstein's field equations describing two-fluid models of the universe in Hypersurface-Homogenous space time.

In a co-moving coordinate system, the field equations (8), for the metric (11), can be explicitly written as

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = \left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \tag{12}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \left(\frac{8\pi + \lambda}{\lambda}\right)p - \Lambda \tag{13}$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{K}{B^2} = -\left(\frac{8\pi + \lambda}{\lambda}\right)\rho - \Lambda \tag{14}$$

where an overhead dot hereafter, denotes ordinary differentiation with respect to cosmic time t only. The trace in our model is given by equation (10) i.e. $T = -3p + \rho$, so that the effective cosmological constant in equation (8) reduces to

$$\Lambda = \frac{1}{2}\left(\rho - p\right) \tag{15}$$

The spatial volume is given by

$$V = a^3 = AB^2 \tag{16}$$

where a is the mean scale factor.

4 Solution of Field Equations

The anisotropy of the expansion can be parameterized after defining the directional Hubble parameters and the mean Hubble parameter of the expansion. The directional Hubble parameters, which determine the universe expansion rates in the directions of the x, y, z axes, are defined as

$$H_x = \frac{\dot{A}}{A}, H_y = H_z = \frac{\dot{B}}{B} \tag{17}$$

and for the average scale factor $a = (AB^2)^{\frac{1}{3}}$, the Hubble parameter H, that determines the volume expansion rate of the universe , can be generalized to anisotropic cosmological models:

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\frac{\dot{V}}{V} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{18}$$

The physical quantities of observational interest are the expansion scalar θ, the average anisotropy parameter A_m and the shear scalar σ^2. These are defined as

$$\theta = u^i_{;i} = \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right) \tag{19}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 \tag{20}$$

$$\sigma^2 = \frac{3}{2}A_m H^2 \tag{21}$$

Now we have a set of three equations with five unknown functions A, B, p, ρ, Λ. To get a determinate solution of field equations, we need extra conditions. One can introduce more conditions either by an assumption corresponding to some physical situation of an arbitrary mathematical supposition. Pradhan et. al. [57] discussed the law of variation of scale factor $a = (t^k e^t)^{\frac{1}{n}}$ which yields a time-dependent deceleration parameter (DP).

We consider the generalized hybrid expansion law for scale factor as following

$$a = (t^n e^{kt})^{\frac{1}{m}} \tag{22}$$

where, m, n and k are non-negative constants. The proposed law gives the time dependent deceleration parameter (DP) which describes the transitioning universe. The scale factor given by Eq. (22) yields a time-dependent deceleration parameter which exhibits a transition of the universe from the early decelerating phase to the present accelerating phase.

We also assume that the Expansion Scalar θ is proportional to the Shear Scalar σ which gives us

$$A = B^{\alpha} \tag{23}$$

where α is a constant.

The work of Thorne [58] motivates us to consider such assumption given by Eq. (23). According to him, observations of velocity red shift relation for extragalactic sources suggest that Hubble expansion of the universe is isotropic within about 30% range approximately (Kantowski and Sachs [59]; Kristian and Sachs [60]) and red shift studies place the limit $\frac{\sigma}{H} \leq 0.3$ where σ and H are shear scalar and Hubble parameter respectively. The physical significance of this condition for perfect fluid and barotropic EoS in a more general case has been discussed by Collins et. al.[61]. The condition (23) is used by many researchers [62-65] to find exact solutions of cosmological models.

Now using equations (16), (22), and (23) the expansion for the metric coefficient are

$$A = (t^n e^{kt})^{\frac{3\alpha}{m(\alpha+2)}} \tag{24}$$

$$B = (t^n e^{kt})^{\frac{3}{m(\alpha+2)}} \tag{25}$$

With the suitable choice of coordinates and constants, the metric (1) with the help of (24) and (25) can be

$$ds^2 = dt^2 - (t^n e^{kt})^{\frac{6\alpha}{m(\alpha+2)}} dx^2 - (t^n e^{kt})^{\frac{6}{m(\alpha+2)}} \left[\, dy^2 + \Sigma^2\,(y, K) dz^2 \,\right] \tag{26}$$

5 Some Physical Properties of the Model

The Spatial Volume is obtained as

$$V = (t^n e^{kt})^{\frac{3}{m}} \tag{27}$$

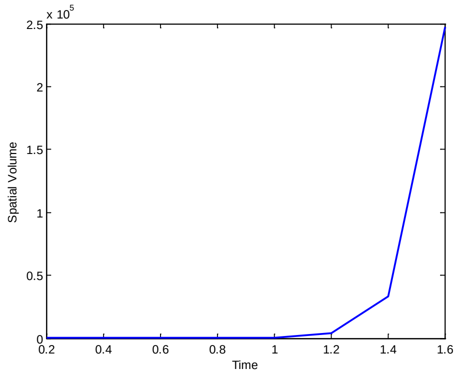

Figure 1. Spatial Volume vs time.

From Eq. (27), we observe that the spatial volume is zero at $t = 0$, which shows that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario.

The Hubble parameter is obtained as

$$H = \frac{1}{m}\left(k + \frac{n}{t}\right) \tag{28}$$

It shows that Hubble parameter is a decreasing function of time.

The deceleration parameter yields as

$$q = \frac{d}{dt}\left(\frac{1}{H}\right) - 1 = \frac{m^2}{(kt+n)^2} - 1 \tag{29}$$

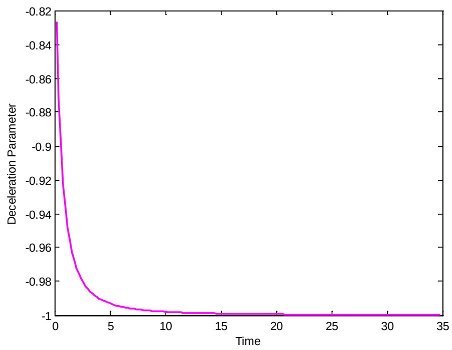

Figure 2. Deceleration Parameter vs time.

Recent observations of SNe Ia, expose that the present universe is accelerating and the value of Deceleration Parameter lies to some place in the range $-1 < q < 0$. It follows that in our derived model, one can choose the value of Deceleration Parameter consistent with the observation.

The mean anisotropic parameter becomes

$$A_m = \frac{2(\alpha - 1)^2}{\left(\alpha + 2\right)^2} \tag{30}$$

Since A_m is constant, the mean anisotropic parameter is uniform throughout the evolution of the universe.

The expansion scalar yields

$$\theta = \frac{3}{m}\left(k + \frac{n}{t}\right) \tag{31}$$

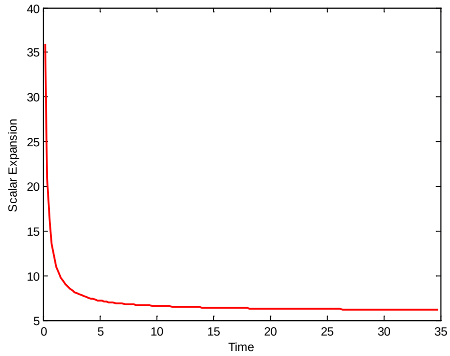

Figure 3. Scalar Expansion vs time.

From Eqs. (27) and (31), we observe that the spatial volume is zero at $t = 0$ and the expansion scalar is infinite, which show that the universe starts evolving with zero volume at $t = 0$ which is big bang scenario which resembles with the investigations of Pradhan et. al. [66], Katore and Shaikh [67].

The Shear Scalar is found to be

$$\sigma^2 = \frac{3(\alpha-1)^2}{m^2(\alpha+2)^2}\left(k+\frac{n}{t}\right)^2 \tag{32}$$

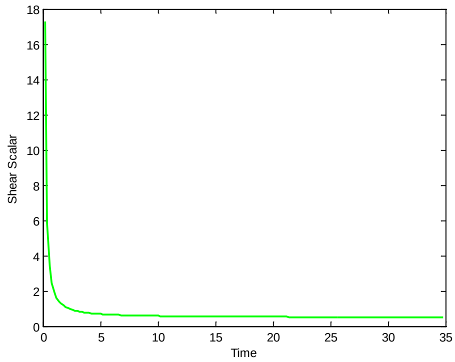

Figure 4. Shear Scalar vs time.

For large value of t, the shear scalar vanishes; hence the shape of the universe remains unchanged during evolution which resembles with the investigations of Katore et. al. [68].

The Pressure can be obtained as

$$p = \frac{1}{\beta}\left\{\left(\frac{\alpha-4\beta-3}{2(1+\beta)}\right)\frac{3n}{m(\alpha+2)t^2} + \left(\frac{3\beta-\alpha+5}{2(1+\beta)}\right)\frac{9}{m^2(\alpha+2)^2}\left(k+\frac{n}{t}\right)^2 + \frac{K}{2B^2}\right\} \tag{33}$$

Here $\beta = \frac{8\pi+\lambda}{\lambda}$.

The Energy Density is given by

$$\rho = \frac{1}{\beta}\left\{\frac{-1}{2(1+\beta)}\frac{3n(\alpha+1)}{m(\alpha+2)}\frac{1}{t^2} + \frac{9}{m^2(\alpha+2)^2}\left(k+\frac{n}{t}\right)^2\left[\frac{(\alpha+1)}{2(1+\beta)} - 2\alpha - 1\right] + \frac{K}{B^2}\left[\frac{1}{2(1+\beta)} - 1\right]\right\} \tag{34}$$

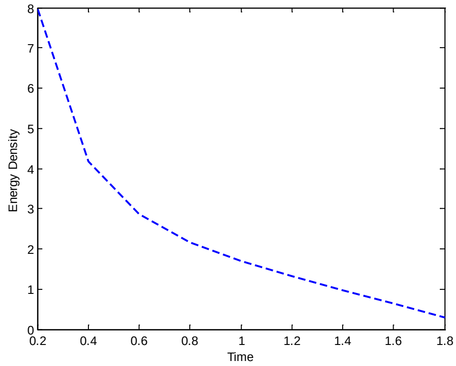

Figure 5. Energy Density vs time.

From Eqs. (34), we observe that the energy density ρ is always positive and decreasing function of time and approaches to zero as $t \to \infty$. Figures 5 depict ρ versus time t showing the positive decreasing function of t and approaching to zero at $t \to \infty$.

The cosmological constant

$$\Lambda = \frac{1}{2(1+\beta)}\left\{\frac{3n(\alpha+1)}{m(\alpha+2)t^2} - \frac{9(\alpha+1)}{m^2(\alpha+2)}\left(k+\frac{n}{t}\right)^2 - \frac{K}{(t^n e^{kt})^{\frac{6}{m(\alpha+2)}}}\right\} \tag{35}$$

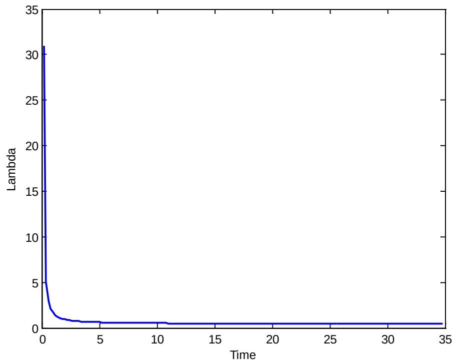

Figure 6. Cosmological Constant vs time ($K = 1$).

A negative effective mass density (repulsion) is corresponded by a positive value of Λ. Hence, we expect that in the universe with a positive value of Λ the expansion will tend to accelerate whereas in the universe with negative value of Λ the expansion will slow down, stop and reverse. Figure 6,7,8 are the plots of cosmological term Λ versus time for $K = 1, 0, -1$ .In all three figures, we observe that Λ is decreasing function of time t and it approaches a small positive value at late time (i.e. at present epoch).Recent cosmological observations suggest the existence of a positive cosmological constant Λ with the magnitude $\Lambda(G\hbar / c^3) \approx 10^{-123}$ .These observations on magnitude and red-shift of type Ia supernova suggest that our universe may be an accelerating one with induced cosmological density through the cosmological Λ-term. Thus, the nature of Λ in our derived model is supported by recent observations.

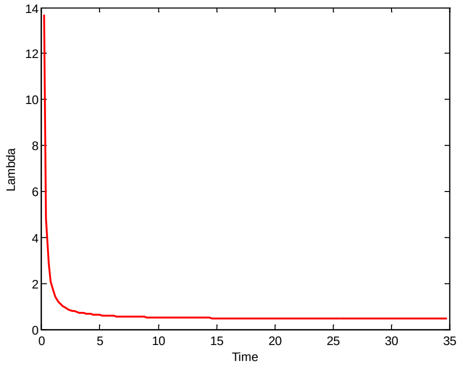

Figure 7. Cosmological Constant vs time ($K = 0$).

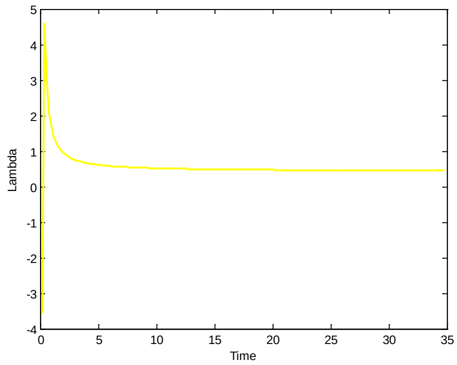

Figure 8. Cosmological Constant vs time ($K = -1$).

6 Conclusion

To deal with the problems of late time acceleration of the universe, Harko et al. [16] proposed a new theory known as $f(R,T)$ theory of gravity by modifying general theory of relativity. We chose a specific choice of the functional $f(R,T) = f_1(R) + f_2(T)$ with $f_1(R) = \lambda R$ and $f_2(T) = \lambda T$ and investigated the exact solutions of Hypersurface − Homogenous cosmological model. We observed that the hybrid expansion law gives time dependent DP, representing a model which generates a transition of universe from an early decelerating phase to a recent accelerating phase. We observe that the energy density ρ is always positive and decreasing function of time and approaches to zero as $t \to \infty$. The mean anisotropic parameter is uniform throughout the evolution of the universe. The spatial scale factor and volume scalar vanish at $t = 0$. The nature of Λ in our derived model is supported by recent observations. Christian Corda [69] has shown that, by assuming that advanced projects on the detection of Gravitational Waves (GWs) will improve their sensitivity allowing to perform a GWs astronomy, accurate angular and frequency dependent response functions of interferometers for gravitational waves arising from various Theories of Gravity, i.e. General Relativity and Extended Theories of Gravity, will be the definitive test for General Relativity.

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References

1. A.G. Riess et al., Astron. J. 116, 1009 (1998).
2. S. Perlmutter et al., Astrophys. J. 517, 565 (1999).
3. C.L. Bennet et al., Astrophys. J. Suppl. Ser. 148, 1 (2003).
4. T. Padmanabham, Phys. Rep. 380, 235 (2003).
5. S.M. Carrol, V. Duvvuri, M. Trodden, and M.S. Turner. Phys. Rev. D, 70,043528 (2004). doi:10.1103/PhysRevD.70.043528.
6. S. Capozziello, S. Nojiri, S.D. Odintsov, and A. Troisi. Phys. Lett. B, 639, 135(2006). doi:10.1016/j.physletb.2006.06.034.
7. S.Nojiri and S.D. Odintsov. Phys. Rev. D, Part. Fields, 74, 086005 (2006).doi:10.1103/PhysRevD.74.086005.
8. M. Sharif and S. Arif. Astrophys. Space Sci. 342, 237 (2012). doi:10.1007/s10509-012-1150-2.
9. S.Nojiri and S.D. Odintsov. Phys. Lett. B, 631, 1 (2005). doi:10.1016/j.physletb.2005.10.010.
10. S.Nojiri and S.D. Odintsov. Phys. Rep. 505, 59 (2011). doi:10.1016/j.physrep.2011.04.001.
11. S. Nojiri, S.D. Odintsov, and P.V. Tretyakov. Prog. Theor. Phys. Suppl. 172, 81(2008). doi:10.1143/PTPS.172.81.
12. N.M. Garcia, F.S.N. Lobo, and J.P. Mimoso. J. Phys. Conf. Ser. 314, 012056(2011). doi:10.1088/1742-6596/314/1/012056.
13. K. Bamba, S. Capozziello, S. Nojiri, and S.D. Odintsov. Astrophys. Space Sci.342, 155 (2012). doi:10.1007/s10509-012-1181-8.
14. M. Jamil, D. Momeni, and R. Myrzakulov. Eur. Phys. J. C, 72, 2137 (2012).doi:10.1140/epjc/s10052-012-2137-4.
15. M. Jamil, K. Yesmakhanova, D. Momeni, and R. Myrzakulov. Cent. Eur. J.Phys. 10, 1065 (2012). doi:10.2478/s11534-012-0103-2.
16. Harko, T., Lobo, F.S.N., Nojiri, S., Odintsov, S.D.: Phys. Rev. D 84,024020 (2011)
17. K.S. Adhav: Astrophys Space Sci (2012) 339:365–369.
18. Reddy, D.R.K., Naidu, R.L., Satyanarayan, B.: Int. J. Theor. Phys. **51**,3222 (2012)
19. Shamir, M.F., Jhangeer, A., Bhatti, A.A.: arXiv:1207.0708v1 [gr-qc].(2012)
20. Chaubey, R., Shukla, A.K.: Astrophys. Space Sci. 343, 415 (2013).
21. Kiran, M., Reddy, D.R.K.: Astrophys. Space Sci. 346, 521 (2013).
22. Reddy, D.R.K., Bhuvana Vijaya, R., Vidya Sagar, T., Naidu, R.L.: Astrophys.Space Sci. 350, 375 (2014a)
23. Reddy, D.R.K., Santikumar, R., Naidu, R.L.: Astrophys. Space Sci.342, 249 (2014b)
24. G.C. Samanta: Int J Theor Phys (2013) 52:2647–2656.
25. R.L.Naidu, D.R.K. Reddy,T. Ramprasad,K.V. Ramana: Astrophys Space Sci (2013) 348:247–252.
26. Shri Ram and Priyanka: Astrophys Space Sci (2013) 347:389–397.
27. A.K.Yadav,P. K. Srivastava, L.Yadav: Int J Theor Phys(2014). DOI 10.1007/s10773-014-2368-2
28. G. P. Singh, B.K. Bishi: RJP v.1.1 r2013b Romanian Academy Publishing House(2014).
29. N.Ahmed and A.Pradhan: Int J Theor Phys (2014) 53:289–306.
30. D.R.K. Reddy ,R. Bhuvana Vijaya,T. Vidya Sagar,R.L. Naidu: Astrophys Space Sci (2014) 350:375–380.
31. N. K. Sharma and J. K. Singh: Int J Theor Phys(2014)DOI 10.1007/s10773-014-2089-6
32. C. P. Singh and V. Singh: Gen RelativGravit (2014) 46:1696.
33. J.K. Singh and N.K. Sharma: Int J Theor Phys (2014) 53:1424–1433.
34. V.U.M. Rao, K.V.S. Sireesha, and D.Ch. Papa Rao: Eur. Phys. J. Plus (2014) 129: 17.
35. K.L. Mahanta: Astrophys Space Sci (2014)DOI 10.1007/s10509-014-2040-6.
36. M. Sharif , Z. Yousaf: Astrophys Space Sci(2014).DOI 10.1007/s10509-014-2113-6.
37. A. Y. Shaikh : Prespacetime Journal,7(15),1950-1961 (2016).
38. Sharif, M., Zubair, M.: Astrophys. Space Sci. 349, 457 (2014)
39. P K Sahoo, B Mishra and G Chakradhar Reddy Eur. Phys.J. Plus. 129 49 (2014)
40. Shaikh, A.Y. & Katore, S.D.: Pramana - J Phys (2016) 87: 83. doi:10.1007/s12043-016-1299-2
41. A.Y.Shaikh: Carib.j.SciTech, Vol.4, 983-991(2016).
42. B. Mishra,P.K. Sahoo,Sankarsan Tarai: Astrophys Space Sci (2015) 359:15
43. A.Y. Shaikh, S. R. Bhoyar: Prespacetime Journal,6(11),1179(2015).
44. A.Y.Shaikh, K. S. Wankhade: Prespacetime Journal,6(11),1213(2015).

45. Shaikh, A.Y.: Int J Theor Phys 55: 3120, (2016). doi:10.1007/s10773-016-2942-x
46. Katore S, Shaikh A.Y: Prespacetime Journal3 (11), 1087-1096 (2012)
47. Bertolami, O., Bohmer, C.G., Harko, T., Lobo, F.S.N.: Phys. Rev. D75, 104016 (2007)
48. Bisabr, Y.: Gen. Relativ. Gravit. 45, 1559 (2013)
49. Sotiriou, T.P., Faraoni, V.: Class. Quantum Gravity 25, 205002 (2008)
50. Poplawski, N.J.: arXiv:gr-qc/0608031 (2006)
51. Poplawski, N.J.: Class. Quant. Grav. 23, 2011 (2006)
52. Stewart, J.M., Ellis, G.F.R.: J. Math. Phys. 9, 1072 (1968)
53. Hajj-Boutros, J.: J. Math. Phys. 28, 2297 (1985)
54. Chandel, S., Singh, M.K., Shri Ram: Adv. Stud. Theor. Phys. 6(24),1189–1198 (2012a)
55. S.D. Katore, A.Y. Shaikh: Astrophys Space Sci (2015) 357:27.
56. S.D. Katore, A.Y. Shaikh: Prespacetime Journal:6(3),207-217,(2015).
57. Anirudh Pradhan, Ajay Kumar Singh, D.S. Chouhan: Int J Theor Phys (2012).DOI 10.1007/s10773-012-1329-x
58. Thorne, K.S.: Astrophys. J. 148, 51 (1967)
59. Kantowski, R., Sachs, R.K.: J. Math. Phys. 7, 433 (1966)
60. Kristian, J., Sachs, R.K.: Astrophys. J. 143, 379 (1966)
61. Collins, C.B., Glass, E.N., Wilkinson, D.A.: Gen. Relativ. Gravit. 12,805 (1980)
62. S.D. Katore, A.Y. Shaikh: Bulg. J. Phys. 42 (2015) 29–41.
63. A Y Shaikh and S D Katore: Pramana – J. Phys. (2016) 87:88 .DOI 10.1007/s12043-016-1272-0.
64. S.D. Katore, A.Y. Shaikh, S.A. Bhaskar: Bulg. J. Phys. 41 (2014) 34–59.
65. S.D. Katore, A.Y. Shaikh: Bulg.J.Phys. 39 (2012) 241-247.
66. Anirudh Pradhan,Ajay Kumar Singh,H. Amirhashchi: Int J Theor Phys (2012).DOI 10.1007/s10773-012-1262-z.
67. S.D.Katore,A.Y.Shaikh:Int J Theor Phys (2012) 51:1881–1888.DOI10.1007/s10773-011-1064-8.
68. S.D. Katore,K.S. Adhav ,A.Y. Shaikh,N.K. Sarkate: Int J Theor Phys 49: 2358–2363, (2010) DOI 10.1007/s10773-010-0422-2.
69. Chirstian Corda: Int. J. Mod. Phys. D, 18, 2275 (2009).

