FAST CLMS ALGORITHM

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Abstract: A computationally simplified modification of the Coupled LMS (CLMS) algorithm is derived. Instead of each iteration, the adaptive filter coefficients are updated periodically. On the other hand, the gradient is estimated in each iteration. Mean error analysis and simulation results show that the proposed algorithm exhibits a similar performances as to original one, while the computational complexity is significantly reduced.

1. INTRODUCTION

The Filtered-x Least Mean Square (FxLMS) is a modification of the LMS algorithm which is mostly applied to the Active Noise Canceler (ANC) systems [1]. In an ANC system, the algorithm convergence speed is one of the key performance criteria. The convergence speed of the FxLMS depends on the eigenvalue spread of autocorrelation matrix of the filtered input signal [2], [3]. Therefore, even for the white inputs the FxLMS exhibits slow convergence speed. The characteristics of the convergence, as well as the influence of the modeling errors in the secondary path on the stability of the FxLMS are analyzed in [4]-[6].

Steady state error of an FxLMS is another important criterion for evaluation of the quality of an ANC system performance. It depends on a number of factors: algorithm's step size, minimal system error, adaptive filter order, and secondary path order [4]. With the FxLMS algorithm there is always an excess mean square error, regardless of the noise characteristics of the desired signal [4].

Numerous modifications of the FxLMS are proposed, aimed to increase the convergence speed. One of them, described in [8], is based on the modified block diagram of the FxLMS (MFxLMS). The Self Orthogonalizing FxLMS (SOFxLMS), described in [9], increases the convergence speed in the white noise environment. A computationally simplified modification of this algorithm is presented in [9].

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In [10] a new algorithm for adaptation of the ANC system is proposed, which is referred to as CLMS. The CLMS algorithm uses two adaptive filters, where the first one is used for identification of the unknown system. The second one is adapted using the coefficient of the first filter, with the purpose to minimize the output error. The CLMS exhibits faster convergence speed and smaller steady state error than the FxLMS and its modifications. The CLMS algorithm has a higher computational complexity than the FXLMS.

In this paper computationally simplified version of the CLMS is derived. The adaptive filter coefficients are updated every L-th iteration, instead of at each iteration. In this way the computational complexity of the CLMS is reduced, while the performances are same.

The paper is organized as follows. A brief description of the FxLMS, SOFxLMS and CLMS is given in Section 2. Sections 3 and 4 deal with the Fast CLMS (BCLMS) and computational complexity, respectively. Finally, the simulation results and conclusion are given in Section 7 and 8.

2. FXLMS AND ITS MODIFICATIONS

Block diagram of the FxLMS is shown in Fig. 1, where **s** is the impulse response of the secondary path, $\hat{\mathbf{s}}$ is the estimation of the secondary path, **h** is the impulse response of the unknown system, and **w** is the adaptive controller.

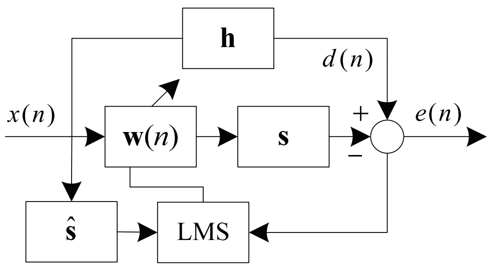

Figure 1. Block diagram of the FxLMS algorithm

The FxLMS algorithm minimizes the error signal by adjusting the coefficients of the adaptive filter, which is followed by the filter of the secondary path, (Fig. 1). The error signal is defined as follows:

$$e(n) = d(n) - \mathbf{w}(n)^T \mathbf{x}_s(n), \tag{1}$$

where $d(n)$ is the desired signal, $\mathbf{x}_s(n)$ is the vector of the filtered input signal, and $\mathbf{w}(n)$ is the adaptive weight vector. The optimal solution which minimizes the error signal is [3]:

$$\mathbf{w}^* = \mathbf{R}_{ss}^{-1}\mathbf{p}_s. \tag{2}$$

where $\mathbf{R}_{ss}$ is the autocorrelation matrix of the filtered input signal, and $\mathbf{p}_s$ is the cross-correlation vector between the filtered input signal and the desired signal.
To minimize the error, the FxLMS iteratively updates the coefficients of the adaptive filter by using the iterative formula:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu e(n)\mathbf{x}_{\hat{s}}(n). \tag{3}$$

where μ is the adaptation step-size, and $\mathbf{x}_{\hat{s}}(n)$ is obtained by filtering the input signal with an estimate of the secondary path. The FxLMS filter in steady state achieves [3]:

$$\mathbf{w}(\infty) = \mathbf{R}_{\hat{s}s}^{-1}\mathbf{p}_s, \tag{4}$$

where $\mathbf{R}_{\hat{s}s}$ is the cross-correlation matrix between the estimated and filtered input signal, and $\mathbf{p}_{\hat{s}}$ is the cross-correlation vector between estimated filtered input signal and the desired signal.

The maximal algorithm step size is given by [3]:

$$\mu_{\max} = \frac{1}{N\overline{x}_s^2}, \tag{5}$$

where N is the adaptive filter order, and $\overline{x}_s^2$ is the mean-square value of the filtered input signal.

The SOFxLMS algorithm, proposed in [8], uses the following iterative formula:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu\mathbf{C}\mathbf{x}_{\hat{s}}(n)e(n), \tag{6}$$

where $\mathbf{C}$ is the inverse autocorrelation matrix of the filtered input signal:

$$\mathbf{C} = \begin{bmatrix} \sum_{i=0}^{M-1} s_i^2 & \sum_{i=0}^{M-2} s_i s_{i+1} & \cdots & \sum_{i=0}^{M-N} s_i s_{i+N-1} \\ \sum_{i=0}^{M-2} s_i s_{i+1} & \sum_{i=0}^{M-1} s_i^2 & \cdots & \sum_{i=0}^{M-N+1} s_i s_{i+N-2} \\ \cdots & \cdots & \cdots & \cdots \\ \sum_{i=0}^{M-N} s_i s_{i+N-1} & \sum_{i=0}^{M-N+1} s_i s_{i+N-2} & \cdots & \sum_{i=0}^{M-1} s_i^2 \end{bmatrix}^{-1}. \tag{7}$$

The order of the secondary path is denoted by M. For white inputs, the SOFxLMS performs similarly to the LMS algorithm [9].

In [10] the Coupled LMS algorithm (CLMS) is proposed. The CLMS consists of two adaptive filters $\mathbf{w}_h$ and $\mathbf{w}_F$, (Fig. 2). The vector $\mathbf{w}_h$ is the LMS filter, with dimensions $(N+M-1)\times 1$, aimed to identify the unknown system $\mathbf{h}$. The second filter $\mathbf{w}_F$, with dimensions $N\times 1$, is aimed to minimize the output system error $e(n)$.

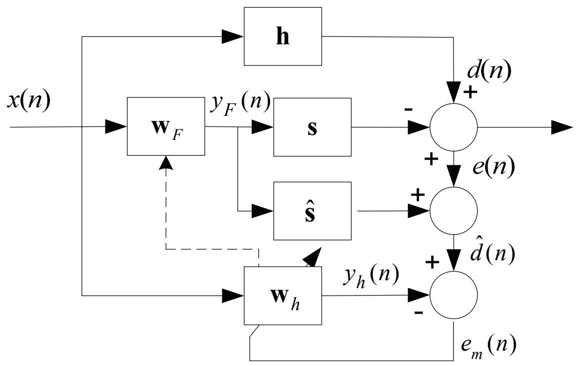

Fig. 2. The block diagram of the CLMS

In order to identify the unknown system we need to estimate the desired signal:

$$\hat{d}(n) = e(n) + \mathbf{y}_F^T(n)\hat{\mathbf{s}} = d(n) - \mathbf{y}_F^T(n)(\mathbf{s} - \hat{\mathbf{s}}), \tag{8}$$

where $\mathbf{y}_F(n)$ is the $M\times 1$ vector of the output signal from the adaptive filter $\mathbf{w}_F(n)$.

The estimated desired signal $\hat{d}(n)$ is used to identify the unknown system $\mathbf{h}$ using the LMS iterative formula:

$$\mathbf{w}_h(n+1) = \mathbf{w}_h(n) + \mu \mathbf{x}(n) e_m(n), \tag{9}$$

where $\mathbf{w}_h(n)$ is the adaptive vector, $\mathbf{x}(n)$ is the vector of the input signal, while $e_m(n)$ is the modified error signal:

$$e_m(n) = \hat{d}(n) - \mathbf{w}_h^T \mathbf{x}(n). \tag{10}$$

In order to minimize the output error, the second adaptive filter $\mathbf{w}_F(n)$ is supposed to converge to (4) in steady state, and it is updated in the following way:

$$\mathbf{w}_F(n) = \hat{\mathbf{S}}_c^+ \mathbf{w}_h(n), \tag{11}$$

where $\hat{\mathbf{S}}_c$ is the convolution matrix defined as:

$$\hat{\mathbf{S}}_c = \begin{bmatrix} \hat{s}_0 & \hat{s}_1 & \dots & \hat{s}_{M-1} & 0 & \dots & 0 \\ 0 & \hat{s}_0 & \hat{s}_1 & \dots & \hat{s}_{M-1} & \dots & 0 \\ 0 & 0 & \hat{s}_0 & \hat{s}_1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \dots & 0 \\ 0 & \dots & 0 & \hat{s}_0 & \dots & \hat{s}_{M-2} & \hat{s}_{M-1} \end{bmatrix}_{N\times(N+M-1)} , \tag{12}$$

and + denotes a pseudo-inverse.

3. FAST CLMS ALGORITHM (FCLMS)

In order to reduce the CLMS computational complexity, the adaptive vectors can be updated every L-th iteration. For $n=kL+i$, $i=0,..,L-1$ the adaptive vectors are constant and will be denoted by $\mathbf{w}_h(k)$ and $\mathbf{w}_F(k)$. On the other hand, in each iteration the gradient is estimated in the following way:

$$\boldsymbol{\varphi}(k) = \sum_{i=0}^{L-1} \mathbf{x}(nL+i)e_m(nL+i), \tag{13}$$

where $e_m(nL+i)$ is the modified error signal defined according to (8) and (10):

$$e_m(nL+i) = \hat{d}(nL+i) - \mathbf{x}(nL+i)^T \mathbf{w}_h(k). \tag{14}$$

In each L-th iteration the LMS filter is updated:

$$\mathbf{w}_h(k+1) = \mathbf{w}_h(k) + \mu\boldsymbol{\varphi}(k), \tag{15}$$

Finally, according to (11) the second adaptive filters is updated as follows:

$$\mathbf{w}_F(k+1) = \hat{\mathbf{S}}_c^{+}\mathbf{w}_h(k+1). \tag{16}$$

A. Mean analysis of the FCLMS

The convergence of the vector $\mathbf{w}_h$ implies:

$$E\{\mathbf{X}(k)(\mathbf{d}(k) - \mathbf{Y}_F^T(k)(\mathbf{s}-\hat{\mathbf{s}}) - \mathbf{X}^T(k)\mathbf{w}_h(k))\} = 0. \tag{17}$$

Taking into account that in the steady state we have:

$$\mathbf{Y}_F^T(k)(\mathbf{s}-\hat{\mathbf{s}}) = (\mathbf{X}_s - \mathbf{X}_{\hat{s}})^T \mathbf{w}_F(k) = (\mathbf{X}_s - \mathbf{X}_{\hat{s}})^T \hat{\mathbf{S}}_c^{+} \mathbf{w}_h(k), \quad (18)$$

where $\mathbf{X}_s^T(k)$ $(\mathbf{X}_{\hat{s}}^T(k))$ is the k-th block of the successive vectors $\mathbf{x}_s(kL+i)$ $(\mathbf{x}_{\hat{s}}(kL+i))$, from (17) we obtain the steady state value of the $\mathbf{w}_h$:

$$\mathbf{w}_h(\infty) = [(\mathbf{R}_k(\mathbf{S}_c - \hat{\mathbf{S}}_c)^T \hat{\mathbf{S}}_c^{+} + \mathbf{I})]^{-1} \mathbf{p}_k, \quad (19)$$

where:

$$\mathbf{R}_k = E[\mathbf{X}(k)\mathbf{X}(k)^T] = LE[\mathbf{x}(n)\mathbf{x}(n)^T] = L\mathbf{R}, \quad (20)$$

$$\mathbf{p}_k = E[\mathbf{X}(k)d(\mathbf{k})] = LE[\mathbf{x}(n)d(n)] = L\mathbf{p} \quad (21)$$

Substituting (19), (20) and (21) into (16) gives the steady state value of the filter $\mathbf{w}_F$:

$$\mathbf{w}_F(\infty) = \hat{\mathbf{S}}_c^{+}[(\mathbf{S}_c - \hat{\mathbf{S}}_c)^T \hat{\mathbf{S}}_c^{+} + \mathbf{I}]^{-1} \mathbf{R}^{-1}\mathbf{p} \quad (22)$$

If we use matrix a identity, [11]

$$\mathbf{B}(\mathbf{I}+\mathbf{BA})^{-1} = (\mathbf{I}+\mathbf{AB})^{-1}\mathbf{B}, \quad (23)$$

after rearranging the expression (22) we obtain

$$\mathbf{w}_F(\infty) = (\hat{\mathbf{S}}_c \mathbf{S}_c{}^T)^{-1} \hat{\mathbf{S}}_c \mathbf{h} = \mathbf{R}_{\hat{s}s}^{-1} \mathbf{p}_{\hat{s}} \quad (24)$$

From (4) and (24) it is obvious that the proposed algorithm converges to the same solution as the FxLMS.
Substituting (14) into (15), some straightforward rearrangements provide:

$$\Delta\mathbf{w}_h(k+1) = (\mathbf{I} + \mu L\mathbf{R}[(\mathbf{S}_c - \hat{\mathbf{S}}_c)^T \hat{\mathbf{S}}_c^{+} + \mathbf{I}])\Delta\mathbf{w}_h(k), \quad (25)$$

where $\Delta\mathbf{w}_h(k)$ is the error vector.

Convergence characteristics of the proposed algorithm depend on the eigenvalues of the matrix $\mathbf{R}(\mathbf{S}_c - \hat{\mathbf{S}}_c)^T \hat{\mathbf{S}}_c^{+} + \mathbf{R}$. The algorithm maximal step size is bounded by, [2]:

$$\mu_{\max} < \frac{1}{Ltr[\mathbf{R}((\mathbf{S}-\hat{\mathbf{S}})^T \hat{\mathbf{S}}^{+} + \mathbf{I})]}. \quad (26)$$

The convergence speed of the filter $\mathbf{w}_h(k)$ depends not only on the characteristics of the input signal, but also on the term $\mathbf{R}(\mathbf{S}-\hat{\mathbf{S}})^T\hat{\mathbf{S}}^+$. The smaller the estimation errors, the algorithm faster converges. In the case when the secondary path is modeled without errors, convergence speed depends only on the autocorrelation matrix of the input signal. For the white inputs, the vector $\mathbf{w}_h(k)$ converges with maximal speed and the algorithm step is limited by:

$$\mu_{\max} < \frac{1}{Ltr[\mathbf{R}]}. \tag{27}$$

Since the second adaptive filter converges with the same speed as the first one, it can be concluded that the proposed algorithm will converge significantly faster than the conventional FxLMS, where the characteristics of the convergence depends on the autocorrelation matrix of the filtered signal.

4. COMPUTATIONAL COMPLEXITY OF THE FCLMS

In each iteration the FCLMS uses N+M-1 multiplication and additions for calculation of the gradient vector. On the other hand, in each L-th iteration the FCLMS uses N+M-1 multiplications and additions for updating the filter $\mathbf{w}_h(k)$. Updating of the vector $\mathbf{w}_F(k)$ requires $N(N+M+1)$ multiplication and $N(N+M)$ addition, respectively.

The total number of real multiplications per iteration is:

$$M(N,M,L) = \frac{N\left(N+M+1\right)+2M+1}{L},$$

whereas the number of real additions is:

$$A(N,M,L) = \frac{N\left(N+M-1\right)+2M-2}{L}.$$

The computational complexity of the FCLMS in every L-th iteration is much higher than in other sample intervals. This increase in complexity can cause delays in adaptation process. In order to overcame this issue, the vector $\mathbf{w}_h(k)$, instead of the vector $\mathbf{w}_h(k+1)$, can be used for the update of the filter. This modification allows to uniformly distribute the calculations over all samples. For example, for L equal to N+M-1 the multiplication of matrix $\hat{\mathbf{S}}_c^+$ with the vector $\mathbf{w}_h(k)$ can be decomposed into N+M-1 iterations. Namely, in each iteration one column of $\hat{\mathbf{S}}_c^+$ will be calculated with the vector $\mathbf{w}_h(k)$.

The FCLMS computational complexity is the most efficient when the block length is L=N+M-1. The computational complexity of the FxLMS, SOFxLMS, CLMS and FCLMS, for L=N+M-1, is given in Table 1. Note that the computational complexity of the FCLMS is of the same order as the computational complexity of the FxLMS and SOFxLMS. On the other hand, the computational complexity of the CLMS in time domain is of the order $O(N(N+M))$.

Table I. The computational complexity of the considered algorithms

Algorithm	+	*
FxLMS	N+M-1	N+M+1
SOFxLMS	2N+M-2	2N+M
CLMS	N(N+M+1)+2M+1	N(N+M+1)+2M-2
FCLMS	2N+M	2N+M+1

5. SIMULATION RESULTS

Performance of the proposed algorithm is verified through numerical simulations for a system identification problem. Here we present two illustrative examples with different estimation of the secondary path. As a performance indicator we use the mean square error (MSE) and simulations are performed by the Monte Carlo method with 1000 averaged simulations. For all the used algorithms, the step producing the fastest convergence (and maximal mean square error) is taken. The secondary path is a FIR filter of order M = 10 and cutoff frequency 0.8, created by the Matlab fir1 function, as in [5]. The plant is an FIR filter of order 25.

Example 1: The perfect estimation of the secondary path

In this example the LMS adaptive filter is of order 30, and the adaptive filters $\mathbf{w}_F$ and $\mathbf{w}$ are of order 20.
Figures 3 and 4 show the MSE characteristics of the considered algorithms in the case when there is no additive noise at the system output, and in the case when the desired signal is noisy ($\sigma_v^2 = 0.01$). It may be observed that the proposed algorithm has faster convergence rate than the FxLMS, and similar convergence speed to the SOFxLMS. Apart from faster convergence, the proposed algorithm has a smaller steady state error, as well.

Example 2: The erroneous estimation of the secondary path

In this example the adaptive filters $\mathbf{w}$ and $\mathbf{w}_F$ are of order N = 64, and the adaptive filter $\mathbf{w}_h$ is of order 128. Secondary path coefficients are corrupted by white noise with variance 0.01. Figure 5 show the MSE characteristics of the considered algorithms ($\sigma_v^2 = 0.01$). Observe that the proposed algorithm converges faster than the FxLMS and SOFxLMS with a smaller steady state error.

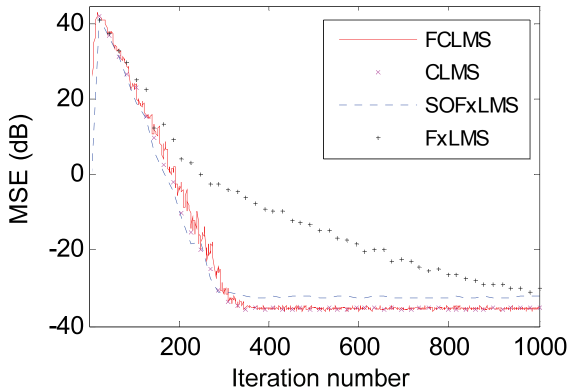

Fig 3. MSE of the considered algorithms

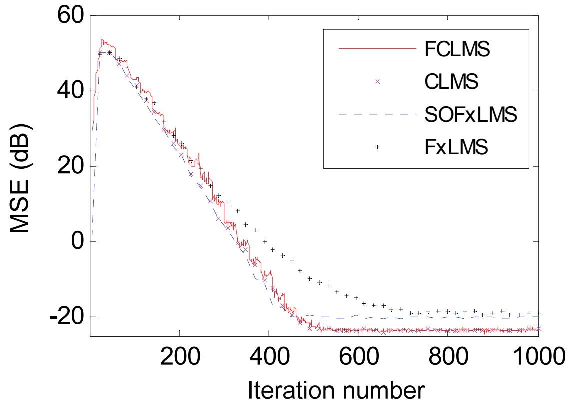

Fig 4. MSE of the considered algorithms

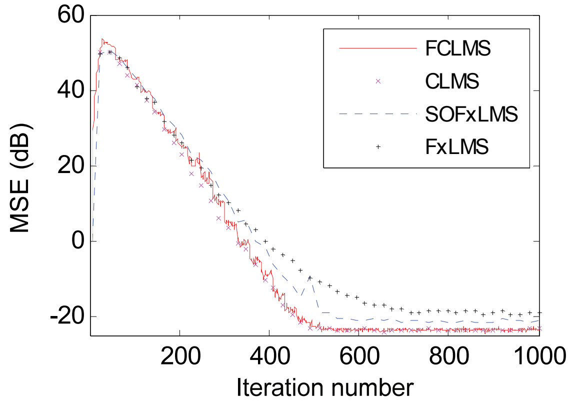

Fig 5. MSE of the considered algorithms

6. CONCLUSION

In this paper a modification of the CLMS algorithm with reduced computational complexity is presented, which is referred to as FCLMS. Mean error analysis shows that the FCLMS exhibits similar performances as the CLMS.

Presented simulation results confirm that the proposed algorithm exhibits the faster convergence speed than the FxLMS and SOFxLMS algorithm. Also, the steady state error of the proposed algorithm is smaller than in the algorithms used for comparison.

A slight increase in calculation complexity of the proposed algortihm is justified by the mentioned advantages.

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