ESTIMATION OF THE FREQUENCY OF A SIGNAL BY MEANS OF INTERPOLATION WITH A QUADRATIC CONVOLUTION KERNEL

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Abstract: In the first part of this paper the parametric quadratic convolution kernels were presented, i.e. Dodgson kernel and the proposed kernel. The analytical expressions for the position of the maximum interpolation function were determined. The second part of this paper represents the results of application of the convolution interpolation with the described parametric kernels in estimation of the frequency of the signal in the spectral domain. In addition to that the efficiency of the estimation of the frequency of the signal in case when AWGN was superimposed to the signal if SNR=0-50dB was analyzed. As a measure of successfulness of estimation the mean square error was used. The analysis of the results shown both graphically and tabular determined the optimal value of the kernel parameter and the corresponding window function.

1. INTRODUCTION

The theory of approximation and interpolation has always been attractive to many mathematicians and scientists. In 1853 a famous Russian mathematician Chebishev considered the issue of the interpolation function while he was working on the connecting device for turning the linear movement into the circular movement of a wheel. In data processing it is often necessary to estimate some values. The measuring results are mainly presented on a discrete set of points. On the base of these data the interdate can be estimated, i.e. a given discrete function is exchanged for a continual function on that set or even wider set. Characteristic cases appear when picture dimensions are changed, when

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picture is rotated etc. In picture processing the convolution interpolation is applied [2]. For the purpose of convolution interpolation many interpolation kernels have been developed. Starting from the quadratic kernel through the cubic kernel and so on [1]-[3]. Parametric kernels are introduced because they can be used at various problems by choosing of an optimal parameter [4], [5]. For picture processing along with the quadratic kernels the parametric kernels are used intensively. The quadratic parametric kernels are characteristic for their very fast performing of the convolution interpolation due to their small numerical complexity. On the other hand because of the small length of the kernel (the number of the interpolation knots) the quadratic kernels are less precise in the estimation of the interpolation value. The cubic kernels in many cases represent a compromise between the numerical complexity and precision characteristic for kernels of higher degrees. In the literature the kernels proposed by Keys in his paper [6] are especially popular. Because of that the kernels derived from the proposed parametric cubic kernel are called Keys kernels. Along with them for the purpose of picture processing a considerable role has the Greville kernels [2], [7]. At discrete electric signals it is often necessary to have an interpolation in the real time (loss of the sample, change of the measuring frequency etc.). Especially present are the problems of estimation of parameters such as frequency and phase, where it is necessary to perform the interpolation.

The estimation of the frequency of the sinusoidal signal in the spectral domain was analyzed in this paper. In order to perform the processing in the spectral domain it is necessary to perform the discrete Fourier transformation, DFT, first. As a result DFT gives the approximation of the time signal spectrum. The basic frequency represents only one component in the frequency domain. However, in the case when the real frequency is different from the frequency of counting DFT, the estimation of the spectrum is wrong. The error is additionally being increased by the appearing of the spectrum escaping. The problem of estimation of the exact position in the spectral domain is solved by the quadratic parametric interpolation kernels. The first kernel was elaborated in [1]. The authors formed the second kernel after the kernel from [1] adding the conditions defined in [8]. Precision of the estimation of the frequency was measured by means of the mean square error, MSE. The increase of precision was accomplished by processing the time discrete signal by means of some classic window functions (Hann, Hamming,...) and by choosing the optimal parameter of the convolution kernel. Presented are the results of the frequency estimation when AWGN for SNR=0-50dB is superimposed to the sinusoidal signal. This paper contains the analytical expressions which are the original results of the authors. Using of the analytical expressions makes convolution unnecessary, which considerably speeds up the process.

Further organization of the paper is as follows. Section 2.A presents Dodgson interpolation kernel. Section 2.B brings the construction of the interpolation kernel. Section 2.C. presents the algorithm for estimation of the frequency. The experimental results and the analysis are given in the section 3.

2. CONVOLUTION INTERPOLATION

Convolution of the continual functions *f*(*x*) and *g*(*x*) is defined by the following expression:

$$f(x) * g(x) = \int f(t) g(x - t) dt \ . \tag{1}$$

For the discrete functions convolution is determined by the expression:

$$f(n) * g(n) = \sum_{k} f(k) g(n - k) . \tag{2}$$

Suppose f is the sampled function and g the corresponding interpolation function. Then in the interpolation knots the value of the sampled function is equal to the value of the interpolation function, i.e. $f(x_k)=g(x_k)$. For uniformly arranged data many interpolation functions can be written down as:

$$g(x) = \sum_{k} c_k u\left(\frac{x - x_k}{h}\right) . \tag{3}$$

In (3) c_k there are parameters that depend on the sampled data and u is the interpolation kernel, h the sampled increment.

A. Dodgson parametric kernel

In paper [1] the quadratic parametric interpolation kernel is defined with:

$$r(f) = \begin{cases} -2\alpha|f|^2 + \frac{1}{2}(\alpha + 1), & |f| \le \frac{1}{2} \\ \alpha|f|^2 + \left(-2\alpha - \frac{1}{2}\right)|f| + \frac{3}{4}(\alpha + 1), & \frac{1}{2} < |f| \le \frac{3}{2} \\ 0 & |f| > \frac{3}{2} \end{cases} . \tag{4}$$

B. The proposed parametric kernel

Let us consider the symmetrical kernel defined in parts by the polynomials of the second degree on the intervals [-2,-1), [-1,1], (1,2]:

$$r(f)=\begin{cases} a|f|^2+b, & |f|\le 1 \\ c|f|^2+d|f|+e, & 1<|f|\le 2 \\ 0 & |f|>2 \end{cases}. \tag{5}$$

Suppose that the interpolation kernel is continual and equal zero in the interpolation knots and suppose it has the value 1 in 0. Since it is:

$$r(0)=1\Rightarrow b=1, \tag{6}$$

$$r(1)=1\Rightarrow a=-1, \tag{7}$$

$$r(2)=0\Rightarrow 4c+2d+e=0, \tag{8}$$

$$\lim_{x\to 1^-} r(f)=\lim_{x\to 1^+} r(f)\Rightarrow a+b=c+d+e. \tag{9}$$

Since the system of the linear equations (6)-(8) has 4 equations and 5 unknowns, one unknown will be arbitrary. Hence it is

$$r(f)=\begin{cases} -|f|^2+1, & |f|\le 1 \\ -2\alpha|f|^2+6\alpha|f|-4\alpha, & 1<|f|\le 2 \\ 0 & |f|>2 \end{cases}. \tag{10}$$

C. Algorithm of the estimation frequency

The algorithm for estimation of the frequency of the discrete sinusoidal signal is realized in the following steps:

Step 1: The time continual signal $x(t)$ is converted into the discrete signal $x(n)$ by the time measurement with the period T:

$$x(nT)=x(t)|_{t=nT}. \tag{11}$$

Step 2: The signal $x(n)$ is divided into frames of the length N:

$$x_l(0:N-1), \tag{12}$$

where l=1:L is the ordinal number of the frame.

Step 3: Processing is performed by the window function $w(n)$ of the length N.

Step 4: Discrete Fourier Transform (DFT) of the length NFT is applied on the frame x_l, which generates the spectrum:

$$X_l(f)=DFT(x_l,NFT). \tag{13}$$

Step 5: The maximal value of the amplitude characteristic of the spectrum, which is found on the frequency f_{max} is determined by the method of peaking in the spectrum X:

$$X_{max} = X(f_{max}). \tag{14}$$

Step 6: Application of interpolation over a part of the spectrum:

$$X(f_{k-1}, f_k, f_{k+1}, f_{k+2}). \tag{15}$$

The reconstructed function is:

$$X_r(f) = \sum_{i=-1}^{2} p_i r(f-i). \tag{16}$$

where is p_i=$X(i)$, $r(f)$ is the interpolation kernel and $k \leq f \leq k+1$. The position of maximum, which represents the estimated basic frequency is:

$$f_e = -B/2A. \tag{17}$$

If the interpolation kernel is applied (10) the coefficients are:

$$A = -2p_{k-1} - p_k - p_{k+1} - 2p_{k+2}, \tag{18}$$

$$B = 2\alpha p_{k-1} + 2p_{k+1} + 2\alpha p_{k+2}, \tag{19}$$

while the coefficient for the interpolation kernel (4) are:

$$A = \alpha(p_{k-1} - 2p_k + p_{k+1}), \tag{20}$$

$$B = \frac{1}{2}(p_{k-1} + p_{k+1}). \tag{21}$$

Step7: Determination of the mean square error between the real f_0 and estimated f_e frequencies

$$MSE = \overline{\left((f_0 - f_e)^2\right)}. \tag{22}$$

In equations (4), (18), (19) exists a parameter α. The optimal values of this parameter will be determined by the minimum value of MSE for the proposed kernel.

$$\alpha_{opt} = \arg\min_{\alpha} MSE. \tag{23}$$

3. THE EXPERIMENTAL RESULTS AND THE ANALYSES

A. Experiment

The algorithm for estimation of the frequency of the time sinusoidal signal described in the section 2.C was applied on the sinusoidal time signal whose frequency changes in the range f_0=125-140 Hz The frequency of sampling is f_s=8kHz, the blocks are 32 ms long, i.e. Dodgson kernel was applied and the quadratic convolution kernel proposed along with the application of Hamming, Hann, Blackman, rectangular, Kaiser and triangular windows. After that the efficiency of estimation of the frequency was analyzed when AWGN was superimposed to the sinusoidal signal. The analysis was performed for SNR=0-50 dB.

B. Results

Values of the mean square error depending on the parameter α for some window functions are represented in: Fig. 1 (Hamming), Fig. 2 (Hann), Fig. 3 (Blackman), Fig. 4 (rectangular), Fig. 5 (Kaiser) and Fig. 6 (triangular). The Tab. 1 contains the optimal values α and the minimal values MSE. In the table 2 the values MSE are represented for the various values SNR and various windows. Fig. 7 represented MSE for various windows depending on SNR.

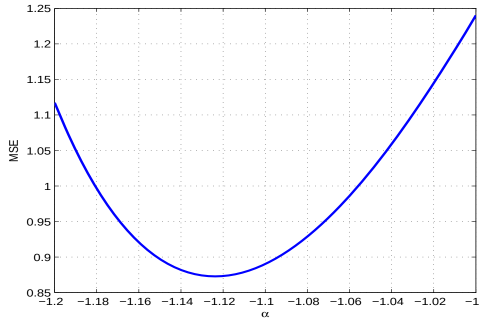

Fig. 1. MSE(α) for the case of Hamming window.

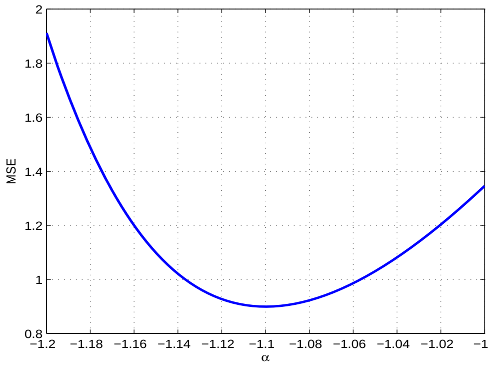

Fig. 2. MSE(α) for the case of Hann window.

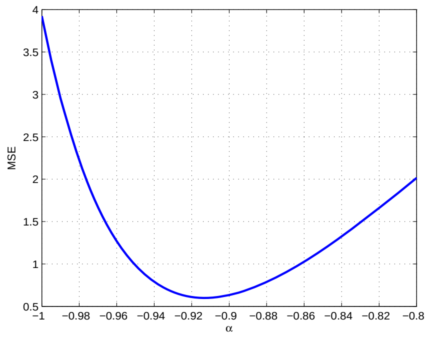

Fig. 3. *MES*(α) for the case of Blackman window.

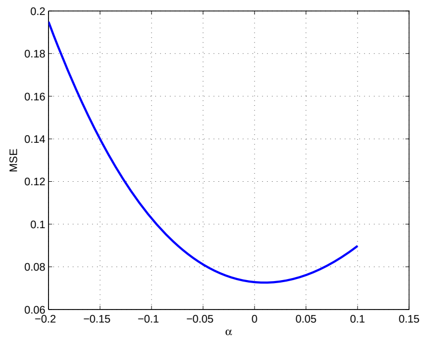

Fig. 4.MSE(α) for the case of rectangular window.

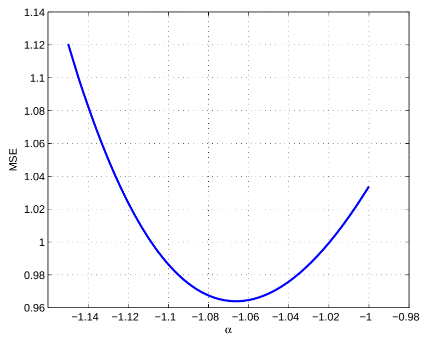

Fig. 5. MSE(α) for the case of Kaiser window.

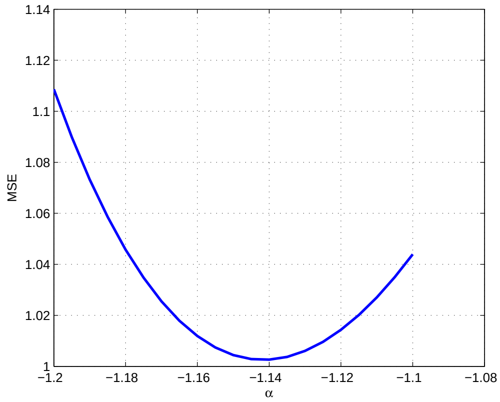

Fig. 6. MSE(α) for the case of triangular window.

Table I
The minimal value MSE if the quadratic proposed kernel is applied.

Window	**MSEmin**	α_{opt}
Hamming	0.8727	-1.125
Hann	0.899	-1.1
Blackman	0.6014	-0.915
Rectangular	0.0726	0.01
Kaiser	0.963	-1.065
Triangular	1.0026	-1.14

TABLE II
MSE depending on SNR.

Window	**SNR [dB]**					
	0dB	**10dB**	**20dB**	**30dB**	**40dB**	**50dB**
Haming	1.8234	1.2292	0.9847	0.9029	0.8800	0.8727
Han	2.5087	1.5753	1.1040	0.9533	0.9127	0.899
Blackman	3.0488	1.2009	0.7050	0.6269	0.6094	0.6014
Rectangular	2.2243	1.0315	0.6554	0.3667	0.1707	0.0728
Kaiser	1.2892	0.9305	0.9245	0.9488	0.9593	0.9642
Triangular	1.7217	1.2501	1.0975	1.0372	1.0133	1.0026

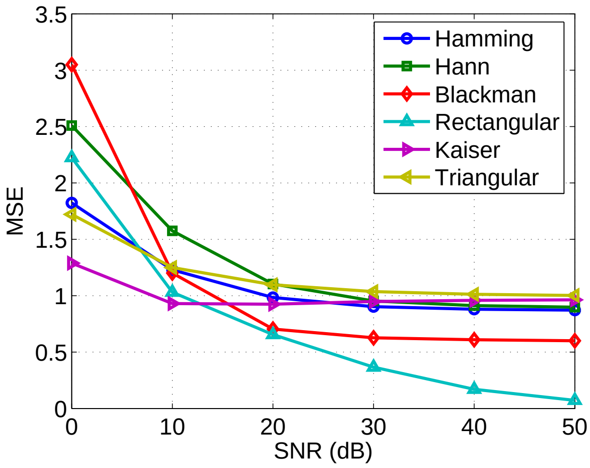

Fig. 7. MSE for various windows depending on SNR.

C. Analysis of the results

On the base of the results presented in Fig. 1-7 and Tab. I-II it can be concluded that:

a) application of Dodgson convolution kernel generates unacceptably high values MSE (MSE>20),

b) application the proposed kernel gave the optimal results for the rectangular window function. In relation to other window functions the rectangular one had better results: 91% (Hamming), 92% (Hann, Kaiser), 88% (Blackman), 93% (triangular),

c) in comparison to the results attained when Keys cubic interpolation kernel [9] was applied, it can be concluded that application of the proposed quadratic kernel causes the mean square error which is: 37.94 (Hamming), 224.75 (Hann), 48.15 (Kaiser), 0.14 (rectangular), 601.4 (Blackman) and 358.07 (triangular) times greater,

d) if SNR decreases MSE increases. For SNR=0dB (Kaiser window) precision is in relation to SNR=50dB (rectangular window) 1.2892/0.0728=17.71 times lesser.

In comparison to Greville two-parametric cubic kernel (Blackman window) [9] the proposed one parametric quadratic kernel (rectangular window) has 0.0728/0.00037=196.76 times lesser precision of the frequency estimation. In relation to Keys one parametric cubic kernel (Blackman window) the proposed kernel has precision 0.0728/0.001=72.8 times lesser. Having great calculating complexity in determination of convolution with two-parametric cubic kernels in mind, the proposed one-parametric quadratic kernel, according to the presented results, can be recommended for implementation in real time systems where great number of calculations is demanded (processing of the speech signals, picture signals etc.).

4. CONCLUSION

The results of the application of parametric quadratic convolution kernels, both Dodgson and the proposed one, are presented in this paper. The detailed analysis showed that

Dodgson kernel produced unacceptably great error (MSE>20), while the proposed kernel gave a better result if the rectangular window function was used. In comparison to Keys cubic interpolation kernel the proposed kernel produced considerably greater errors. Considering the fact that the process of convolution with the quadratic kernel has a less numerical complexity than convolution with the cubic kernels, the use in the systems for work in real time represents a compromise between the demanded speed and precision.

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